1. Let U, V be normal vectors in an inner product space V s. t. $\| U + V \| = 1$. Then $\| U - V \|$ is :

(1) $\sqrt{3}$ (2) 1

(3) 0 (4) $\sqrt{2}$

2. Let $u, v \in V$. Then $\| U + v \| \leq \| U \| + \| v \|$. This result is known as :

(1) Bessel's inequality (2) Cauchy-Schwarz inequality

(3) Triangle inequality (4) None of these

3. Which is an orthogonal set ?

(1) {(1, 0, 1), (1, 0, –1) (0, 1, 0)}

(2) {(1, 0, 1), (1, 0, –1) (–1, 0, 1)}

(3) {(1, 0, 1), (1, 0, –1) (0, 3, 4)}

(4) None of these

4. Let w be a subspace of $R^4(R)$ generated by the vectors u_1 = (1, 2, 3, –2) and u_2 = (2, 4, 5, –1). Then dim $w^{\perp}$ is :

(1) 1 (2) 2

(3) 3 (4) 4

5. Which of the following is ***not*** a subspace of R^3 ?

(1) $\{(x, 0, 0) : x$ is real$\}$

(2) $\{(a, a + b, -a + 2b) : a, b$ real$\}$

(3) $\{(a, a - b, b) : a, b$ real$\}$

(4) $\{(a, b, c) : a, b, c$ integers$\}$

6. Let V be a vector space over the field F of dimension n. Consider the following statements :

(I) Every subset of V containing n elements is a basis of V.

(II) No linearly independent subset of V contains more than n elements.

Which of the above statement is/are ***correct*** ?

(1) (I) only

(2) (II) only

(3) Both (I) and (II)

(4) Neither (I) nor (II)

7. The set W of ordered triplets $(a_1, a_2, 0)$ of R^3 has dimension :

(1) 1 (2) 2

(3) 3 (4) 4

8. Largest Linearly independent subset of R^3 contains .... elements.

(1) 1 (2) 2

(3) 3 (4) 4

9. The finite dimensional vector space $V(F)$ is the direct sum of its subspace W_1 and W_2 such that dim $W_1 = 2$, dim $W_2 = 3$. Then dim V is :

(1) 2 (2) 3

(3) 5 (4) 6

10. Let V be the vector space of all polynomials of degree $\leq n$ over R. Then dim V is :

(1) n (2) $n - 1$

(3) $n + 1$ (4) n^2

11. A one-one linear transformation is called :

(1) homomorphism (2) monomorphism

(3) epimorphism (4) isomorphism

12. Let $T : R^2 \to R^3$ be linear transformation defined by $T(x_1, x_2) = (x_1 - x_2, x_2 - x_1, -x_1)$. Then nullity ($T$) is :

(1) 0 (2) 1

(3) 2 (4) None of these

13. Let X = (1, 2, 1) be relative to standard basis. Then its coordinates relative to a new basis Y_1 = (1, 1, 0), Y_2 = (1, 0, 1), Y_3 = (1, 1, 1) are :

(1) (1, 2, 1) (2) (2, 1, 1)

(3) (0, −1, 2) (4) (1, 1, 3)

14. Let V be the vector space of all 3×3 skew symmetric matrices over R. Then dim V is :

(1) 6 (2) 3

(3) 4 (4) 9

15. Let $T : U \to V$ be a linear transformation where U is finite dimensional. Then $\rho(T) + \mu(T)$ is :

(1) dim U (2) dim $R(T)$

(3) dim $V(T)$ (4) dim $N(T)$

16. If $S = \{(1, 1, 0), (2, 1, 3)\} \subseteq R^3$, then which one of the following vectors of R^3 is ***not*** in the span of S ?

(1) (0, 0, 0) (2) (3, 2, 3)

(3) (1, 2, 3) (4) $\left(\frac{4}{3}, 1, 1\right)$

17. If W_1 and W_2 are linear subspace of a vector space V such that $W_1 \cap W_2 = \{0\}$, then $\dim(W_1 + W_2)$ is equal to :

(1) $\dim W_1$

(2) $\dim W_2$

(3) $\dim W_1 + \dim W_2$

(4) $\dim W_1 - \dim W_2$

18. Which of the following is ***not*** a subspace of R^3 ?

(1) $\{(a, b, c) : a + b = c; a, b, c$ being real$\}$

(2) $\{(0, 0, 0)\}$

(3) $\{(a, a, z + 2b) : a, b$ real$\}$

(4) $\{(a, a - b, 1) : a, b \in$ real number$\}$

19. Let U be n-dimensional vector space over F and v be m-dimensional vector space over F. Then $L(U, V)$ is a vector space of dimension :

(1) m (2) m^2n^2

(3) 1 (4) None of these

20. Dimension of subspace $W = \{(a, b, c) : a = -b = c\}$ of a vector space $R^3(R)$ equals :

(1) 0 (2) 1

(3) 2 (4) 3

21. Let e_1, e_2, e_3 denote the standard basis of R^3. Then $ae_1 + be_2 + ce_3,\ e_2, e_3$ is an orthonormal basis of R^3 iff :

(1) $a \neq 0,\ a^2 + b^2 + c^2 = 1$ (2) $a = 1, b = c = 0$

(3) $a = b = c = 1$ (4) $a = b = c$

22. Let $V(F)$ be the vector space of all polynomials is x in which an inner product is defined by $(f, g) = \int_0^1 f(x)g(x)dx$. Then for $f(x) = x + 2$, $g(x) = x^2 - 2x - 3$, $<f, g>$ is :

(1) $\frac{5}{2}$ (2) $\frac{-37}{4}$

(3) $\frac{5}{8}$ (4) $\frac{37}{4}$

23. The dimension of $C(R)$ is :

(1) 1 (2) 2

(3) 3 (4) 4

24. Let $T : R^2 \to R^3$ be a linear transformation given by $T(x_1 - x_2) = (x_1 + x_2, x_1 - x_2, x_2)$. Then Rank T is :

(1) 0 (2) 1

(3) 2 (4) 3

25. Consider the mapping :

(I) $T : R^3 \to R^2$, $T(x, y, z) = (x + 1, y + z)$

(II) $T : R^3 \to R$, $T(x, y) = xy$

(III) $T : R^3 \to R^2$, $T(x, y, z) = (|x|, 0)$

Which of the above are linear transformation ?

(1) (I), (II) and (III)

(2) (I) and (III) only

(3) (II) and (III) only

(4) None of these

26. Let $U = (1, 1, 1)$, $V = (1, 2, -3)$ and $W = (1, -4, 3)$ in R^3. Then which of the following is ***not*** true ?

(1) U is orthogonal to V

(2) U is orthogonal to W

(3) V is orthogonal to W

(4) V is not orthogonal to W

27. Let M and N be subspaces of a finite dimensional inner product space V. Then show that $(M + N)^{\perp} =$

(1) $M^{\perp} \cup N^{\perp}$

(2) $M^{\perp} \cap N^{\perp}$

(3) $M^{\perp}$

(4) None of these

28. Find the dimension of the vector space $Q(\sqrt{2})$ over Q :

(1) 1 (2) 2

(3) 0 (4) 3

29. The set of vectors (1, 2, 0), (0, 3, 1) and (–1, 0, 1) of $V_3(X)$ is linearly independent if :

(1) X is set of rational number

(2) X is set of irrational number

(3) Neither (1) and nor (2)

(4) None of these

30. For what value of K will the vector u = (1, K, 5) in $V_3(R)$ be a linear combination of vectors v = (1, –3, 2) and w = (2, –1, 1) ?

(1) –8 (2) 8

(3) 0 (4) 4

31. Write the linear transformation corresponding to the matrix $T : R^3 \to R^3 \begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix}$:

(1) $T(x_1, x_2, x_3) = (x_1 + 2x_2 - 3x_3, x_2 + x_3, x_1 - x_3)$

(2) $T(x_1, x_2, x_3) = (x_1 + 2x_2 - 3x_3, x_2 - x_3, x_1 + x_3)$

(3) $T(x_1, x_2, x_3) = (x_1 - 2x_2 + 3x_3, x_2 - x_3, x_1 - x_3)$

(4) $T(x_1, x_2, x_3) = (x_1 - 2x_2 - 3x_3, x_2 - x_3, x_1 + x_3)$

32. Which of the following is ***not*** an orthonormal set ?

(1) $\{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$

(2) $\left\{\left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right), \left(\frac{1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}}\right), (0, 1, 0)\right\}$

(3) $\{(3, 0, 4), (-4, 0, 3), (0, 4, 0)\}$

(4) $\left\{\left(\frac{3}{5}, 0, \frac{4}{5}\right), \left(\frac{-4}{5}, 0, \frac{3}{5}\right), (0, 1, 0)\right\}$

33. Let $T_1 : R^3 \to R^2$ such that $T_1(x, y, z) = (3x, 4y - z)$, $T_2 : R^2 \to R^2$ such that $T_2(x, y) = (-x, y)$. Compute $T_1 T_2$:

(1) $T_1 T_2$ is defined

(2) $T_1 T_2$ is not defined

(3) $T_1 T_2 = (-2x, 3y, z)$

(4) None of these

34. Let $T_1 : U \to V$ and $T_2 : V \to W$ be two linear transformation then :

(1) $\rho(T_2 T_1) \leq \rho(T_2)$

(2) $\rho(T_2 T_1) = \rho(T_2)$

(3) $\rho(T_2 T_1) > \rho(T_2)$

(4) $\rho(T_2 T_1) \geq \rho(T_2)$

35. Let the linear transformation $T_1 : R^3 \to R^2$ such that $T_1(x, y, z) = (4x, 3y, -2z)$ and $T_2 : R^2 \to R^2$ such that $T_2(x, y) = (-2x, y)$. Compute T_2T_1 :

(1) T_2T_1 is not defined

(2) $(-8x, 3y - 2z)$

(3) $(8x, 2y - 3z)$

(4) $(6x, 2y - 2z)$

36. Find the coordinates of vector (1, 1, 1) relative to the basis (1, 1, 2), (2, 2, 1), (1, 2, 2) :

(1) $\left(\frac{1}{3}, 0, 0\right)$

(2) $\left(\frac{1}{3}, 0, \frac{1}{3}\right)$

(3) $\left(\frac{1}{3}, \frac{1}{3}, 0\right)$

(4) $\left(0, 0, \frac{1}{3}\right)$

37. The linear transformation $T : R^2 \to R^2$ defined by $T(1, 0) = (2, 3)$, $T(0, 1) = (5, 6)$ is :

(1) one-one and onto

(2) one-one but not onto

(3) onto but not one-one

(4) neither one-one nor onto

38. The norm of vector $U = (2, -3, 6)$ is :

(1) 8 (2) 6

(3) 7 (4) 5

39. Let $T : U \to V$ be a linear transformation where U is a finite dimension. Then :

(1) $\text{rank}(T) + \text{nullity}(T) = \dim U$

(2) $\text{rank}(T) + \text{nullity}(T) = \dim V$

(3) $\text{rank}(T) = \dim U$

(4) $\text{rank}(T) = \text{nullity}(T)$

40. Normalize the vector $U = (1, -2, 5)$ is :

(1) $\left\{\frac{1}{\sqrt{30}}, \frac{-2}{\sqrt{30}}, \frac{5}{\sqrt{30}}\right\}$

(2) $\left\{\frac{1}{\sqrt{32}}, \frac{-2}{\sqrt{32}}, \frac{5}{\sqrt{32}}\right\}$

(3) $\left\{\frac{1}{\sqrt{25}}, \frac{-2}{\sqrt{25}}, \frac{5}{\sqrt{25}}\right\}$

(4) $\left\{\frac{1}{\sqrt{28}}, \frac{-2}{\sqrt{28}}, \frac{5}{\sqrt{28}}\right\}$

41. Consider the linear transformation in R^3 given by $y = AX$ where $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 5 \\ 1 & 3 & 3 \end{bmatrix}$. Then image of $X = (2, 0, 5)$ is :

(1) (12, 27, 17) (2) (17, 12, 27)

(3) (27, 12, 17) (4) (12, 17, 27)

42. Using Cauchy Schwarz inequality, the absolute value of cosine of an angle is :

(1) atmost 1 (2) at least 1

(3) exactly 1 (4) None of these

43. Let V be a finite dimensional inner product space and W be a subspace of V. Then :

(I) $V = W \oplus W^{\perp}$ (II) $W^{\perp\perp} = W$

(1) Only I is true (2) Only II is true

(3) Both are true (4) None of these

44. Let $T_1 : U \to V$ and $T_2 : V \to W$ be two linear transformation. Then :

(I) If T_2T_1 is one-one, then T_1 is one-one

(II) If T_2T_1 is onto, then T_2 is onto

(1) Only I is true (2) Only II is true

(3) Both are true (4) None of these

45. The finite dimensional vector space $V(F)$ is the direct sum of its subspaces W_1 and W_2 such that dim $W_1 = 2$, dim $W_2 = 3$. Then dim V is :

(1) 2 (2) 3

(3) 5 (4) 6

46. If (m, 3, 1) is a linear combination of vectors (3, 2, 1) and (2, 1, 0) in R^3. Then the value of m is :

(1) 1 (2) 3

(3) 5 (4) None of the above

47. Let U, V be elements of an inner product space such that $\| U \| = 3$, $\| V \| = 5$, $\| U + V \| = 8$, then $\| U - V \|$ is :

(1) 2 (2) $\sqrt{34}$

(3) 0 (4) None of the above

48. Every field F is a vector space over itself of dimension :

(1) 1 (2) 2

(3) 3 (4) 0

49. If W is a subspace of V such that dim $W = m$ *and* dim $V = n$. Then :

(1) $m \neq n$ (2) $m < n$

(3) $m = n$ (4) $m > n$

50. A bijective linear transformation is called :

(1) homomorphism (2) monomorphism

(3) epimorphism (4) isomorphism

1. A one-one linear transformation is called :

(1) homomorphism (2) monomorphism

(3) epimorphism (4) isomorphism

2. Let $T : R^2 \to R^3$ be linear transformation defined by $T(x_1, x_2) = (x_1 - x_2, x_2 - x_1, -x_1)$. Then nullity ($T$) is :

(1) 0 (2) 1

(3) 2 (4) None of these

3. Let X = (1, 2, 1) be relative to standard basis. Then its coordinates relative to a new basis Y_1 = (1, 1, 0), Y_2 = (1, 0, 1), Y_3 = (1, 1, 1) are :

(1) (1, 2, 1) (2) (2, 1, 1)

(3) (0, −1, 2) (4) (1, 1, 3)

4. Let V be the vector space of all 3×3 skew symmetric matrices over R. Then dim V is :

(1) 6 (2) 3

(3) 4 (4) 9

5. Let $T : U \to V$ be a linear transformation where U is finite dimensional. Then $\rho(T) + \mu(T)$ is :

(1) dim U (2) dim $R(T)$

(3) dim $V(T)$ (4) dim $N(T)$

6. If $S = \{(1, 1, 0), (2, 1, 3)\} \subseteq R^3$, then which one of the following vectors of R^3 is ***not*** in the span of S ?

(1) (0, 0, 0) (2) (3, 2, 3)

(3) (1, 2, 3) (4) $\left(\frac{4}{3}, 1, 1\right)$

7. If W_1 and W_2 are linear subspace of a vector space V such that $W_1 \cap W_2 = \{0\}$, then $\dim (W_1 + W_2)$ is equal to :

(1) $\dim W_1$

(2) $\dim W_2$

(3) $\dim W_1 + \dim W_2$

(4) $\dim W_1 - \dim W_2$

8. Which of the following is ***not*** a subspace of R^3 ?

(1) $\{(a, b, c) : a + b = c; a, b, c$ being real$\}$

(2) $\{(0, 0, 0)\}$

(3) $\{(a, a, z + 2b) : a, b$ real$\}$

(4) $\{(a, a - b, 1) : a, b \in$ real number$\}$

9. Let U be n-dimensional vector space over F and V be m-dimensional vector space over F. Then $L(U, V)$ is a vector space of dimension :

(1) m (2) m^2n^2

(3) 1 (4) None of these

10. Dimension of subspace $W = \{(a, b, c) : a = -b = c\}$ of a vector space $R^3(R)$ equals :

(1) 0 (2) 1

(3) 2 (4) 3

11. Let e_1, e_2, e_3 denote the standard basis of R^3. Then $ae_1 + be_2 + ce_3,\ e_2, e_3$ is an orthonormal basis of R^3 iff :

(1) $a \neq 0,\ a^2 + b^2 + c^2 = 1$ (2) $a = 1, b = c = 0$

(3) $a = b = c = 1$ (4) $a = b = c$

12. Let $V(F)$ be the vector space of all polynomials is x in which an inner product is defined by $(f, g) = \int_0^1 f(x)g(x)dx$. Then for $f(x) = x + 2$, $g(x) = x^2 - 2x - 3$, $<f, g>$ is :

(1) $\frac{5}{2}$ (2) $\frac{-37}{4}$

(3) $\frac{5}{8}$ (4) $\frac{37}{4}$

13. The dimension of $C(R)$ is :

(1) 1 (2) 2

(3) 3 (4) 4

14. Let $T : R^2 \to R^3$ be a linear transformation given by $T(x_1 - x_2) = (x_1 + x_2, x_1 - x_2, x_2)$. Then Rank T is :

(1) 0 (2) 1

(3) 2 (4) 3

15. Consider the mapping :

(I) $T : R^3 \to R^2$, $T(x, y, z) = (x + 1, y + z)$

(II) $T : R^3 \to R$, $T(x, y) = xy$

(III) $T : R^3 \to R^2$, $T(x, y, z) = (|x|, 0)$

Which of the above are linear transformation ?

(1) (I), (II) and (III)

(2) (I) and (III) only

(3) (II) and (III) only

(4) None of these

16. Let $U = (1, 1, 1)$, $V = (1, 2, -3)$ and $W = (1, -4, 3)$ in R^3. Then which of the following is ***not*** true ?

(1) U is orthogonal to V

(2) U is orthogonal to W

(3) V is orthogonal to W

(4) V is not orthogonal to W

17. Let M and N be subspaces of a finite dimensional inner product space V. Then show that $(M + N)^{\perp} =$

(1) $M^{\perp} \cup N^{\perp}$

(2) $M^{\perp} \cap N^{\perp}$

(3) $M^{\perp}$

(4) None of these

18. Find the dimension of the vector space $Q(\sqrt{2})$ over Q :

(1) 1 (2) 2

(3) 0 (4) 3

19. The set of vectors (1, 2, 0), (0, 3, 1) and (–1, 0, 1) of $V_3(X)$ is linearly independent if :

(1) X is set of rational number

(2) X is set of irrational number

(3) Neither (1) and nor (2)

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20. For what value of K will the vector u = (1, K, 5) in $V_3(R)$ be a linear combination of vectors v = (1, –3, 2) and w = (2, –1, 1) ?

(1) –8 (2) 8

(3) 0 (4) 4

21. Write the linear transformation corresponding to the matrix $T : R^3 \to R^3 \begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix}$:

(1) $T(x_1, x_2, x_3) = (x_1 + 2x_2 - 3x_3, x_2 + x_3, x_1 - x_3)$

(2) $T(x_1, x_2, x_3) = (x_1 + 2x_2 - 3x_3, x_2 - x_3, x_1 + x_3)$

(3) $T(x_1, x_2, x_3) = (x_1 - 2x_2 + 3x_3, x_2 - x_3, x_1 - x_3)$

(4) $T(x_1, x_2, x_3) = (x_1 - 2x_2 - 3x_3, x_2 - x_3, x_1 + x_3)$

22. Which of the following is ***not*** an orthonormal set ?

(1) $\{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$

(2) $\left\{\left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right), \left(\frac{1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}}\right), (0, 1, 0)\right\}$

(3) $\{(3, 0, 4), (-4, 0, 3), (0, 4, 0)\}$

(4) $\left\{\left(\frac{3}{5}, 0, \frac{4}{5}\right), \left(\frac{-4}{5}, 0, \frac{3}{5}\right), (0, 1, 0)\right\}$

23. Let $T_1 : R^3 \to R^2$ such that $T_1(x, y, z) = (3x, 4y - z)$, $T_2 : R^2 \to R^2$ such that $T_2(x, y) = (-x, y)$. Compute T_1T_2 :

(1) T_1T_2 is defined

(2) T_1T_2 is not defined

(3) $T_1T_2 = (-2x, 3y, z)$

(4) None of these

24. Let $T_1 : U \to V$ and $T_2 : V \to W$ be two linear transformation then :

(1) $\rho(T_2T_1) \leq \rho(T_2)$

(2) $\rho(T_2T_1) = \rho(T_2)$

(3) $\rho(T_2T_1) > \rho(T_2)$

(4) $\rho(T_2T_1) \geq \rho(T_2)$

25. Let the linear transformation $T_1 : R^3 \to R^2$ such that $T_1(x, y, z) = (4x, 3y, -2z)$ and $T_2 : R^2 \to R^2$ such that $T_2(x, y) = (-2x, y)$. Compute T_2T_1 :

(1) T_2T_1 is not defined

(2) $(-8x, 3y - 2z)$

(3) $(8x, 2y - 3z)$

(4) $(6x, 2y - 2z)$

26. Find the coordinates of vector (1, 1, 1) relative to the basis (1, 1, 2), (2, 2, 1), (1, 2, 2) :

(1) $\left(\frac{1}{3}, 0, 0\right)$

(2) $\left(\frac{1}{3}, 0, \frac{1}{3}\right)$

(3) $\left(\frac{1}{3}, \frac{1}{3}, 0\right)$

(4) $\left(0, 0, \frac{1}{3}\right)$

27. The linear transformation $T : R^2 \to R^2$ defined by $T(1, 0) = (2, 3)$, $T(0, 1) = (5, 6)$ is :

(1) one-one and onto

(2) one-one but not onto

(3) onto but not one-one

(4) neither one-one nor onto

28. The norm of vector $U = (2, -3, 6)$ is :

(1) 8 (2) 6

(3) 7 (4) 5

29. Let $T : U \to V$ be a linear transformation where U is a finite dimension. Then :

(1) rank(T) + nullity(T) = dim U

(2) rank(T) + nullity(T) = dim V

(3) rank(T) = dim U

(4) rank(T) = nullity(T)

30. Normalize the vector $U = (1, -2, 5)$ is :

(1) $\left\{\frac{1}{\sqrt{30}}, \frac{-2}{\sqrt{30}}, \frac{5}{\sqrt{30}}\right\}$

(2) $\left\{\frac{1}{\sqrt{32}}, \frac{-2}{\sqrt{32}}, \frac{5}{\sqrt{32}}\right\}$

(3) $\left\{\frac{1}{\sqrt{25}}, \frac{-2}{\sqrt{25}}, \frac{5}{\sqrt{25}}\right\}$

(4) $\left\{\frac{1}{\sqrt{28}}, \frac{-2}{\sqrt{28}}, \frac{5}{\sqrt{28}}\right\}$

31. Consider the linear transformation in R^3 given by $y = AX$ where $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 5 \\ 1 & 3 & 3 \end{bmatrix}$. Then image of $X = (2, 0, 5)$ is :

(1) (12, 27, 17)　　(2) (17, 12, 27)

(3) (27, 12, 17)　　(4) (12, 17, 27)

32. Using Cauchy Schwarz inequality, the absolute value of cosine of an angle is :

(1) atmost 1　　(2) at least 1

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33. Let V be a finite dimensional inner product space and W be a subspace of V. Then :

(I) $V = W \oplus W^{\perp}$　　(II) $W^{\perp\perp} = W$

(1) Only I is true　　(2) Only II is true

(3) Both are true　　(4) None of these

34. Let $T_1 : U \rightarrow V$ and $T_2 : V \rightarrow W$ be two linear transformation. Then :

(I) If T_2T_1 is one-one, then T_1 is one-one

(II) If T_2T_1 is onto, then T_2 is onto

(1) Only I is true　　(2) Only II is true

(3) Both are true　　(4) None of these

35. The finite dimensional vector space $V(F)$ is the direct sum of its subspaces W_1 and W_2 such that dim $W_1 = 2$, dim $W_2 = 3$. Then dim V is :

(1) 2　　(2) 3

(3) 5　　(4) 6

36. If (m, 3, 1) is a linear combination of vectors (3, 2, 1) and (2, 1, 0) in R^3. Then the value of m is :

(1) 1 (2) 3

(3) 5 (4) None of the above

37. Let U, V be elements of an inner product space such that $\|U\| = 3$, $\|V\| = 5$, $\|U + V\| = 8$, then $\|U - V\|$ is :

(1) 2 (2) $\sqrt{34}$

(3) 0 (4) None of the above

38. Every field F is a vector space over itself of dimension :

(1) 1 (2) 2

(3) 3 (4) 0

39. If W is a subspace of V such that dim $W = m$ *and* dim $V = n$. Then :

(1) $m \neq n$ (2) $m < n$

(3) $m = n$ (4) $m > n$

40. A bijective linear transformation is called :

(1) homomorphism

(2) monomorphism

(3) epimorphism

(4) isomorphism

41. Let U, V be normal vectors in an inner product space V s. t. $\| U + V \| = 1$. Then $\| U - V \|$ is :

(1) $\sqrt{3}$ (2) 1

(3) 0 (4) $\sqrt{2}$

42. Let $u, v \in V$. Then $\| U + v \| \leq \| U \| + \| v \|$. This result is known as :

(1) Bessel's inequality (2) Cauchy-Schwarz inequality

(3) Triangle inequality (4) None of these

43. Which is an orthogonal set ?

(1) {(1, 0, 1), (1, 0, –1) (0, 1, 0)}

(2) {(1, 0, 1), (1, 0, –1) (–1, 0, 1)}

(3) {(1, 0, 1), (1, 0, –1) (0, 3, 4)}

(4) None of these

44. Let w be a subspace of $R^4(R)$ generated by the vectors u_1 = (1, 2, 3, –2) and u_2 = (2, 4, 5, –1). Then dim $w^{\perp}$ is :

(1) 1 (2) 2

(3) 3 (4) 4

45. Which of the following is ***not*** a subspace of R^3 ?

(1) $\{(x, 0, 0) : x$ is real$\}$

(2) $\{(a, a + b, -a + 2b) : a, b$ real$\}$

(3) $\{(a, a - b, b) : a, b$ real$\}$

(4) $\{(a, b, c) : a, b, c$ integers$\}$

46. Let V be a vector space over the field F of dimension n. Consider the following statements :

(I) Every subset of V containing n elements is a basis of V.

(II) No linearly independent subset of V contains more than n elements.

Which of the above statement is/are ***correct*** ?

(1) (I) only

(2) (II) only

(3) Both (I) and (II)

(4) Neither (I) nor (II)

47. The set W of ordered triplets $(a_1, a_2, 0)$ of R^3 has dimension :

(1) 1 (2) 2

(3) 3 (4) 4

48. Largest Linearly independent subset of R^3 contains ……………. elements.

(1) 1 (2) 2

(3) 3 (4) 4

49. The finite dimensional vector space $V(F)$ is the direct sum of its subspace W_1 and W_2 such that dim $W_1 = 2$, dim $W_2 = 3$. Then dim V is :

(1) 2 (2) 3

(3) 5 (4) 6

50. Let V be the vector space of all polynomials of degree $\leq n$ over R. Then dim V is :

(1) n (2) $n - 1$

(3) $n + 1$ (4) n^2

1. Let e_1, e_2, e_3 denote the standard basis of R^3. Then $ae_1 + be_2 + ce_3$, e_2, e_3 is an orthonormal basis of R^3 iff :

(1) $a \neq 0,\ a^2 + b^2 + c^2 = 1$

(2) $a = 1, b = c = 0$

(3) $a = b = c = 1$

(4) $a = b = c$

2. Let $V(F)$ be the vector space of all polynomials is x in which an inner product is defined by $(f, g) = \int_0^1 f(x)g(x)dx$. Then for $f(x) = x + 2$, $g(x) = x^2 - 2x - 3$, $<f, g>$ is :

(1) $\frac{5}{2}$

(2) $\frac{-37}{4}$

(3) $\frac{5}{8}$

(4) $\frac{37}{4}$

3. The dimension of $C(R)$ is :

(1) 1 (2) 2

(3) 3 (4) 4

4. Let $T : R^2 \to R^3$ be a linear transformation given by $T(x_1 - x_2) = (x_1 + x_2, x_1 - x_2, x_2)$. Then Rank T is :

(1) 0 (2) 1

(3) 2 (4) 3

5. Consider the mapping :

(I) $T : R^3 \to R^2$, $T(x, y, z) = (x + 1, y + z)$

(II) $T : R^3 \to R$, $T(x, y) = xy$

(III) $T : R^3 \to R^2$, $T(x, y, z) = (|x|, 0)$

Which of the above are linear transformation ?

(1) (I), (II) and (III)

(2) (I) and (III) only

(3) (II) and (III) only

(4) None of these

6. Let $U = (1, 1, 1)$, $V = (1, 2, -3)$ and $W = (1, -4, 3)$ in R^3. Then which of the following is ***not*** true ?

(1) U is orthogonal to V

(2) U is orthogonal to W

(3) V is orthogonal to W

(4) V is not orthogonal to W

7. Let M and N be subspaces of a finite dimensional inner product space V. Then show that $(M + N)^{\perp} =$

(1) $M^{\perp} \cup N^{\perp}$

(2) $M^{\perp} \cap N^{\perp}$

(3) $M^{\perp}$

(4) None of these

8. Find the dimension of the vector space $Q(\sqrt{2})$ over Q :

(1) 1 (2) 2

(3) 0 (4) 3

9. The set of vectors (1, 2, 0), (0, 3, 1) and (–1, 0, 1) of $V_3(X)$ is linearly independent if :

(1) X is set of rational number

(2) X is set of irrational number

(3) Neither (1) and nor (2)

(4) None of these

10. For what value of K will the vector $u = (1, K, 5)$ in $V_3(R)$ be a linear combination of vectors $v = (1, -3, 2)$ and $w = (2, -1, 1)$?

(1) –8 (2) 8

(3) 0 (4) 4

11. Write the linear transformation corresponding to the matrix $T : R^3 \to R^3 \begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix}$:

(1) $T(x_1, x_2, x_3) = (x_1 + 2x_2 - 3x_3, x_2 + x_3, x_1 - x_3)$

(2) $T(x_1, x_2, x_3) = (x_1 + 2x_2 - 3x_3, x_2 - x_3, x_1 + x_3)$

(3) $T(x_1, x_2, x_3) = (x_1 - 2x_2 + 3x_3, x_2 - x_3, x_1 - x_3)$

(4) $T(x_1, x_2, x_3) = (x_1 - 2x_2 - 3x_3, x_2 - x_3, x_1 + x_3)$

12. Which of the following is ***not*** an orthonormal set ?

(1) $\{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$

(2) $\left\{\left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right), \left(\frac{1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}}\right), (0, 1, 0)\right\}$

(3) $\{(3, 0, 4), (-4, 0, 3), (0, 4, 0)\}$

(4) $\left\{\left(\frac{3}{5}, 0, \frac{4}{5}\right), \left(\frac{-4}{5}, 0, \frac{3}{5}\right), (0, 1, 0)\right\}$

13. Let $T_1 : R^3 \to R^2$ such that $T_1 (x, y, z) = (3x, 4y - z)$, $T_2 : R^2 \to R^2$ such that $T_2 (x, y) = (-x, y)$. Compute $T_1 T_2$:

(1) $T_1 T_2$ is defined

(2) $T_1 T_2$ is not defined

(3) $T_1 T_2 = (-2x, 3y, z)$

(4) None of these

14. Let $T_1 : U \to V$ and $T_2 : V \to W$ be two linear transformation then :

(1) $\rho(T_2 T_1) \leq \rho(T_2)$

(2) $\rho(T_2 T_1) = \rho(T_2)$

(3) $\rho(T_2 T_1) > \rho(T_2)$

(4) $\rho(T_2 T_1) \geq \rho(T_2)$

15. Let the linear transformation $T_1 : R^3 \to R^2$ such that $T_1(x, y, z) = (4x, 3y, -2z)$ and $T_2 : R^2 \to R^2$ such that $T_2(x, y) = (-2x, y)$. Compute T_2T_1 :

(1) T_2T_1 is not defined

(2) $(-8x, 3y - 2z)$

(3) $(8x, 2y - 3z)$

(4) $(6x, 2y - 2z)$

16. Find the coordinates of vector (1, 1, 1) relative to the basis (1, 1, 2), (2, 2, 1), (1, 2, 2) :

(1) $\left(\frac{1}{3}, 0, 0\right)$

(2) $\left(\frac{1}{3}, 0, \frac{1}{3}\right)$

(3) $\left(\frac{1}{3}, \frac{1}{3}, 0\right)$

(4) $\left(0, 0, \frac{1}{3}\right)$

17. The linear transformation $T : R^2 \to R^2$ defined by $T(1, 0) = (2, 3)$, $T(0, 1) = (5, 6)$ is :

(1) one-one and onto

(2) one-one but not onto

(3) onto but not one-one

(4) neither one-one nor onto

18. The norm of vector $U = (2, -3, 6)$ is :

(1) 8 (2) 6

(3) 7 (4) 5

19. Let $T : U \to V$ be a linear transformation where U is a finite dimension. Then :

(1) rank(T) + nullity(T) = dim U

(2) rank(T) + nullity(T) = dim V

(3) rank(T) = dim U

(4) rank(T) = nullity(T)

20. Normalize the vector $U = (1, -2, 5)$ is :

(1) $\left\{\frac{1}{\sqrt{30}}, \frac{-2}{\sqrt{30}}, \frac{5}{\sqrt{30}}\right\}$

(2) $\left\{\frac{1}{\sqrt{32}}, \frac{-2}{\sqrt{32}}, \frac{5}{\sqrt{32}}\right\}$

(3) $\left\{\frac{1}{\sqrt{25}}, \frac{-2}{\sqrt{25}}, \frac{5}{\sqrt{25}}\right\}$

(4) $\left\{\frac{1}{\sqrt{28}}, \frac{-2}{\sqrt{28}}, \frac{5}{\sqrt{28}}\right\}$

21. Consider the linear transformation in R^3 given by $y = AX$ where $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 5 \\ 1 & 3 & 3 \end{bmatrix}$. Then image of $X = (2, 0, 5)$ is :

(1) (12, 27, 17) (2) (17, 12, 27)

(3) (27, 12, 17) (4) (12, 17, 27)

22. Using Cauchy Schwarz inequality, the absolute value of cosine of an angle is :

(1) atmost 1 (2) at least 1

(3) exactly 1 (4) None of these

23. Let V be a finite dimensional inner product space and W be a subspace of V. Then :

(I) $V = W \oplus W^{\perp}$ (II) $W^{\perp\perp} = W$

(1) Only I is true (2) Only II is true

(3) Both are true (4) None of these

24. Let $T_1 : U \to V$ and $T_2 : V \to W$ be two linear transformation. Then :

(I) If T_2T_1 is one-one, then T_1 is one-one

(II) If T_2T_1 is onto, then T_2 is onto

(1) Only I is true (2) Only II is true

(3) Both are true (4) None of these

25. The finite dimensional vector space $V(F)$ is the direct sum of its subspaces W_1 and W_2 such that dim $W_1 = 2$, dim $W_2 = 3$. Then dim V is :

(1) 2 (2) 3

(3) 5 (4) 6

26. If (m, 3, 1) is a linear combination of vectors (3, 2, 1) and (2, 1, 0) in R^3. Then the value of m is :

(1) 1 (2) 3

(3) 5 (4) None of the above

27. Let U, V be elements of an inner product space such that $\| U \| = 3$, $\| V \| = 5$, $\| U + V \| = 8$, then $\| U - V \|$ is :

(1) 2 (2) $\sqrt{34}$

(3) 0 (4) None of the above

28. Every field F is a vector space over itself of dimension :

(1) 1 (2) 2

(3) 3 (4) 0

29. If W is a subspace of V such that dim $W = m$ *and* dim $V = n$. Then :

(1) $m \neq n$ (2) $m < n$

(3) $m = n$ (4) $m > n$

30. A bijective linear transformation is called :

(1) homomorphism

(2) monomorphism

(3) epimorphism

(4) isomorphism

31. Let U, V be normal vectors in an inner product space V s. t. $\| U + V \| = 1$. Then $\| U - V \|$ is :

(1) $\sqrt{3}$ (2) 1

(3) 0 (4) $\sqrt{2}$

32. Let $u, v \in V$. Then $\| U + v \| \leq \| U \| + \| v \|$. This result is known as :

(1) Bessel's inequality (2) Cauchy-Schwarz inequality

(3) Triangle inequality (4) None of these

33. Which is an orthogonal set ?

(1) {(1, 0, 1), (1, 0, –1) (0, 1, 0)}

(2) {(1, 0, 1), (1, 0, –1) (–1, 0, 1)}

(3) {(1, 0, 1), (1, 0, –1) (0, 3, 4)}

(4) None of these

34. Let w be a subspace of $R^4(R)$ generated by the vectors u_1 = (1, 2, 3, –2) and u_2 = (2, 4, 5, –1). Then dim $w^{\perp}$ is :

(1) 1 (2) 2

(3) 3 (4) 4

35. Which of the following is ***not*** a subspace of R^3 ?

(1) $\{(x, 0, 0) : x$ is real$\}$

(2) $\{(a, a + b, -a + 2b) : a, b$ real$\}$

(3) $\{(a, a - b, b) : a, b$ real$\}$

(4) $\{(a, b, c) : a, b, c$ integers$\}$

36. Let V be a vector space over the field F of dimension n. Consider the following statements :

(I) Every subset of V containing n elements is a basis of V.

(II) No linearly independent subset of V contains more than n elements.

Which of the above statement is/are correct ?

(1) (I) only

(2) (II) only

(3) Both (I) and (II)

(4) Neither (I) nor (II)

37. The set W of ordered triplets $(a_1, a_2, 0)$ of R^3 has dimension :

(1) 1 (2) 2

(3) 3 (4) 4

38. Largest Linearly independent subset of R^3 contains ……………. elements.

(1) 1 (2) 2

(3) 3 (4) 4

39. The finite dimensional vector space $V(F)$ is the direct sum of its subspace W_1 and W_2 such that dim $W_1 = 2$, dim $W_2 = 3$. Then dim V is :

(1) 2 (2) 3

(3) 5 (4) 6

40. Let V be the vector space of all polynomials of degree $\leq n$ over R. Then dim V is :

(1) n (2) $n - 1$

(3) $n + 1$ (4) n^2

41. A one-one linear transformation is called :

(1) homomorphism (2) monomorphism

(3) epimorphism (4) isomorphism

42. Let $T : R^2 \rightarrow R^3$ be linear transformation defined by $T(x_1, x_2) = (x_1 - x_2, x_2 - x_1, -x_1)$. Then nullity ($T$) is :

(1) 0 (2) 1

(3) 2 (4) None of these

43. Let X = (1, 2, 1) be relative to standard basis. Then its coordinates relative to a new basis Y_1 = (1, 1, 0), Y_2 = (1, 0, 1), Y_3 = (1, 1, 1) are :

(1) (1, 2, 1) (2) (2, 1, 1)

(3) (0, −1, 2) (4) (1, 1, 3)

44. Let V be the vector space of all 3×3 skew symmetric matrices over R. Then dim V is :

(1) 6 (2) 3

(3) 4 (4) 9

45. Let $T : U \rightarrow V$ be a linear transformation where U is finite dimensional. Then $\rho(T) + \mu\,(T)$ is :

(1) dim U (2) dim $R(T)$

(3) dim $V(T)$ (4) dim $N(T)$

46. If $S = \{(1, 1, 0), (2, 1, 3)\} \subseteq R^3$, then which one of the following vectors of R^3 is ***not*** in the span of S ?

(1) (0, 0, 0) (2) (3, 2, 3)

(3) (1, 2, 3) (4) $\left(\frac{4}{3}, 1, 1\right)$

47. If W_1 and W_2 are linear subspace of a vector space V such that $W_1 \cap W_2 = \{0\}$, then dim $(W_1 + W_2)$ is equal to :

(1) dim W_1 (2) dim W_2

(3) dim W_1 + dim W_2 (4) dim W_1 – dim W_2

48. Which of the following is ***not*** a subspace of R^3 ?

(1) $\{(a, b, c) : a + b = c; a, b, c$ being real$\}$

(2) $\{(0, 0, 0)\}$

(3) $\{(a, a, z + 2b) : a, b$ real$\}$

(4) $\{(a, a - b, 1) : a, b \in$ real number$\}$

49. Let U be n-dimensional vector space over F and v be m-dimensional vector space over F. Then $L(U, V)$ is a vector space of dimension :

(1) m (2) m^2n^2

(3) 1 (4) None of these

50. Dimension of subspace $W = \{(a, b, c) : a = -b = c\}$ of a vector space $R^3(R)$ equals :

(1) 0 (2) 1

(3) 2 (4) 3

1. Write the linear transformation corresponding to the matrix $T : R^3 \to R^3 \begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix}$:

(1) $T(x_1, x_2, x_3) = (x_1 + 2x_2 - 3x_3, x_2 + x_3, x_1 - x_3)$

(2) $T(x_1, x_2, x_3) = (x_1 + 2x_2 - 3x_3, x_2 - x_3, x_1 + x_3)$

(3) $T(x_1, x_2, x_3) = (x_1 - 2x_2 + 3x_3, x_2 - x_3, x_1 - x_3)$

(4) $T(x_1, x_2, x_3) = (x_1 - 2x_2 - 3x_3, x_2 - x_3, x_1 + x_3)$

2. Which of the following is ***not*** an orthonormal set ?

(1) $\{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$

(2) $\left\{\left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right), \left(\frac{1}{\sqrt{2}}, 0, \frac{-1}{\sqrt{2}}\right), (0, 1, 0)\right\}$

(3) $\{(3, 0, 4), (-4, 0, 3), (0, 4, 0)\}$

(4) $\left\{\left(\frac{3}{5}, 0, \frac{4}{5}\right), \left(\frac{-4}{5}, 0, \frac{3}{5}\right), (0, 1, 0)\right\}$

3. Let $T_1 : R^3 \to R^2$ such that $T_1(x, y, z) = (3x, 4y - z)$, $T_2 : R^2 \to R^2$ such that $T_2(x, y) = (-x, y)$. Compute $T_1 T_2$:

(1) $T_1 T_2$ is defined

(2) $T_1 T_2$ is not defined

(3) $T_1 T_2 = (-2x, 3y, z)$

(4) None of these

4. Let $T_1 : U \to V$ and $T_2 : V \to W$ be two linear transformation then :

(1) $\rho(T_2 T_1) \le \rho(T_2)$

(2) $\rho(T_2 T_1) = \rho(T_2)$

(3) $\rho(T_2 T_1) > \rho(T_2)$

(4) $\rho(T_2 T_1) \ge \rho(T_2)$

5. Let the linear transformation $T_1 : R^3 \to R^2$ such that $T_1(x, y, z) = (4x, 3y, -2z)$ and $T_2 : R^2 \to R^2$ such that $T_2(x, y) = (-2x, y)$. Compute $T_2 T_1$:

(1) $T_2 T_1$ is not defined

(2) $(-8x, 3y - 2z)$

(3) $(8x, 2y - 3z)$

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6. Find the coordinates of vector (1, 1, 1) relative to the basis (1, 1, 2), (2, 2, 1), (1, 2, 2) :

(1) $\left(\frac{1}{3}, 0, 0\right)$

(2) $\left(\frac{1}{3}, 0, \frac{1}{3}\right)$

(3) $\left(\frac{1}{3}, \frac{1}{3}, 0\right)$

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7. The linear transformation $T : R^2 \rightarrow R^2$ defined by $T(1, 0) = (2, 3)$, $T(0, 1) = (5, 6)$ is :

(1) one-one and onto

(2) one-one but not onto

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8. The norm of vector $U = (2, -3, 6)$ is :

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(3) 7 (4) 5

9. Let $T : U \rightarrow V$ be a linear transformation where U is a finite dimension. Then :

(1) rank(T) + nullity(T) = dim U

(2) rank(T) + nullity(T) = dim V

(3) rank(T) = dim U

(4) rank(T) = nullity(T)

10. Normalize the vector $U = (1, -2, 5)$ is :

(1) $\left\{\frac{1}{\sqrt{30}}, \frac{-2}{\sqrt{30}}, \frac{5}{\sqrt{30}}\right\}$

(2) $\left\{\frac{1}{\sqrt{32}}, \frac{-2}{\sqrt{32}}, \frac{5}{\sqrt{32}}\right\}$

(3) $\left\{\frac{1}{\sqrt{25}}, \frac{-2}{\sqrt{25}}, \frac{5}{\sqrt{25}}\right\}$

(4) $\left\{\frac{1}{\sqrt{28}}, \frac{-2}{\sqrt{28}}, \frac{5}{\sqrt{28}}\right\}$

11. Consider the linear transformation in R^3 given by $y = AX$ where $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 5 \\ 1 & 3 & 3 \end{bmatrix}$. Then image of $X = (2, 0, 5)$ is :

(1) (12, 27, 17)　　(2) (17, 12, 27)

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(I) $V = W \oplus W^{\perp}$　　(II) $W^{\perp\perp} = W$

(1) Only I is true　　(2) Only II is true

(3) Both are true　　(4) None of these

14. Let $T_1 : U \to V$ and $T_2 : V \to W$ be two linear transformation. Then :

(I) If T_2T_1 is one-one, then T_1 is one-one

(II) If T_2T_1 is onto, then T_2 is onto

(1) Only I is true　　(2) Only II is true

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15. The finite dimensional vector space $V(F)$ is the direct sum of its subspaces W_1 and W_2 such that dim $W_1 = 2$, dim $W_2 = 3$. Then dim V is :

(1) 2　　(2) 3

(3) 5　　(4) 6

16. If (m, 3, 1) is a linear combination of vectors (3, 2, 1) and (2, 1, 0) in R^3. Then the value of m is :

(1) 1 (2) 3

(3) 5 (4) None of the above

17. Let U, V be elements of an inner product space such that $\| U \| = 3$, $\| V \| = 5$, $\| U + V \| = 8$, then $\| U - V \|$ is :

(1) 2 (2) $\sqrt{34}$

(3) 0 (4) None of the above

18. Every field F is a vector space over itself of dimension :

(1) 1 (2) 2

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19. If W is a subspace of V such that dim $W = m$ *and* dim $V = n$. Then :

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(1) homomorphism

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21. Let U, V be normal vectors in an inner product space V s. t. $\| U + V \| = 1$. Then $\| U - V \|$ is :

(1) $\sqrt{3}$ (2) 1

(3) 0 (4) $\sqrt{2}$

22. Let $u, v \in V$. Then $\| U + v \| \leq \| U \| + \| v \|$. This result is known as :

(1) Bessel's inequality (2) Cauchy-Schwarz inequality

(3) Triangle inequality (4) None of these

23. Which is an orthogonal set ?

(1) $\{(1, 0, 1), (1, 0, -1) (0, 1, 0)\}$

(2) $\{(1, 0, 1), (1, 0, -1) (-1, 0, 1)\}$

(3) $\{(1, 0, 1), (1, 0, -1) (0, 3, 4)\}$

(4) None of these

24. Let w be a subspace of $R^4(R)$ generated by the vectors $u_1 = (1, 2, 3, -2)$ and $u_2 = (2, 4, 5, -1)$. Then dim $w^{\perp}$ is :

(1) 1 (2) 2

(3) 3 (4) 4

25. Which of the following is ***not*** a subspace of R^3 ?

(1) $\{(x, 0, 0) : x$ is real$\}$

(2) $\{(a, a + b, -a + 2b) : a, b$ real$\}$

(3) $\{(a, a - b, b) : a, b$ real$\}$

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26. Let V be a vector space over the field F of dimension n. Consider the following statements :

(I) Every subset of V containing n elements is a basis of V.

(II) No linearly independent subset of V contains more than n elements.

Which of the above statement is/are correct ?

(1) (I) only

(2) (II) only

(3) Both (I) and (II)

(4) Neither (I) nor (II)

27. The set W of ordered triplets $(a_1, a_2, 0)$ of R^3 has dimension :

(1) 1 (2) 2

(3) 3 (4) 4

28. Largest Linearly independent subset of R^3 contains .... elements.

(1) 1 (2) 2

(3) 3 (4) 4

29. The finite dimensional vector space $V(F)$ is the direct sum of its subspace W_1 and W_2 such that dim $W_1 = 2$, dim $W_2 = 3$. Then dim V is :

(1) 2 (2) 3

(3) 5 (4) 6

30. Let V be the vector space of all polynomials of degree $\leq n$ over R. Then dim V is :

(1) n (2) $n - 1$

(3) $n + 1$ (4) n^2

31. A one-one linear transformation is called :

(1) homomorphism (2) monomorphism

(3) epimorphism (4) isomorphism

32. Let $T : R^2 \rightarrow R^3$ be linear transformation defined by $T(x_1, x_2) = (x_1 - x_2, x_2 - x_1, -x_1)$. Then nullity (*T*) is :

(1) 0 (2) 1

(3) 2 (4) None of these

33. Let $X = (1, 2, 1)$ be relative to standard basis. Then its coordinates relative to a new basis $Y_1 = (1, 1, 0)$, $Y_2 = (1, 0, 1)$, $Y_3 = (1, 1, 1)$ are :

(1) (1, 2, 1) (2) (2, 1, 1)

(3) (0, −1, 2) (4) (1, 1, 3)

34. Let V be the vector space of all 3×3 skew symmetric matrices over *R*. Then dim *V* is :

(1) 6 (2) 3

(3) 4 (4) 9

35. Let $T : U \rightarrow V$ be a linear transformation where U is finite dimensional. Then $\rho(T) + \mu(T)$ is :

(1) dim *U* (2) dim *R*(*T*)

(3) dim *V*(*T*) (4) dim *N*(*T*)

36. If $S = \{(1, 1, 0), (2, 1, 3)\} \subseteq R^3$, then which one of the following vectors of R^3 is ***not*** in the span of S ?

(1) (0, 0, 0)

(2) (3, 2, 3)

(3) (1, 2, 3)

(4) $\left(\frac{4}{3}, 1, 1\right)$

37. If W_1 and W_2 are linear subspace of a vector space V such that $W_1 \cap W_2 = \{0\}$ then dim $(W_1 + W_2)$ is equal to :

(1) dim W_1

(2) dim W_2

(3) dim W_1 + dim W_2

(4) dim W_1 − dim W_2

38. Which of the following is ***not*** a subspace of R^3 ?

(1) $\{(a, b, c) : a + b = c; a, b, c$ being real$\}$

(2) $\{(0, 0, 0)\}$

(3) $\{(a, a, z + 2b) : a, b$ real$\}$

(4) $\{(a, a - b, 1) : a, b \in$ real number$\}$

39. Let U be n-dimensional vector space over F and v be m-dimensional vector space over F. Then $L(U, V)$ is a vector space of dimension :

(1) m

(2) m^2n^2

(3) 1

(4) None of these

40. Dimension of subspace $W = \{(a, b, c) : a = -b = c\}$ of a vector space $R^3(R)$ equals :

(1) 0 (2) 1

(3) 2 (4) 3

41. Let e_1, e_2, e_3 denote the standard basis of R^3. Then $a e_1 + b e_2 + c e_3,\ e_2, e_3$ is an orthonormal basis of R^3 iff :

(1) $a \neq 0,\ a^2 + b^2 + c^2 = 1$ (2) $a = 1, b = c = 0$

(3) $a = b = c = 1$ (4) $a = b = c$

42. Let $V(F)$ be the vector space of all polynomials is x in which an inner product is defined by $(f, g) = \int_0^1 f(x) g(x) dx$. Then for $f(x) = x + 2$, $g(x) = x^2 - 2x - 3$, $<f, g>$ is :

(1) $\frac{5}{2}$ (2) $\frac{-37}{4}$

(3) $\frac{5}{8}$ (4) $\frac{37}{4}$

43. The dimension of $C(R)$ is :

(1) 1 (2) 2

(3) 3 (4) 4

44. Let $T : R^2 \to R^3$ be a linear transformation given by $T(x_1 - x_2) = (x_1 + x_2, x_1 - x_2, x_2)$. Then Rank T is :

(1) 0 (2) 1

(3) 2 (4) 3

45. Consider the mapping :

(I) $T : R^3 \to R^2$, $T(x, y, z) = (x + 1, y + z)$

(II) $T : R^3 \to R$, $T(x, y) = xy$

(III) $T : R^3 \to R^2$, $T(x, y, z) = (|x|, 0)$

Which of the above are linear transformation ?

(1) (I), (II) and (III)

(2) (I) and (III) only

(3) (II) and (III) only

(4) None of these

46. Let $U = (1, 1, 1)$, $V = (1, 2, -3)$ and $W = (1, -4, 3)$ in R^3. Then which of the following is ***not*** true ?

(1) U is orthogonal to V

(2) U is orthogonal to W

(3) V is orthogonal to W

(4) V is not orthogonal to W

47. Let M and N be subspaces of a finite dimensional inner product space V. Then show that $(M + N)^{\perp} =$

(1) $M^{\perp} \cup N^{\perp}$

(2) $M^{\perp} \cap N^{\perp}$

(3) $M^{\perp}$

(4) None of these

48. Find the dimension of the vector space $Q(\sqrt{2})$ over Q :

(1) 1 (2) 2

(3) 0 (4) 3

49. The set of vectors (1, 2, 0), (0, 3, 1) and (−1, 0, 1) of $V_3(X)$ is linearly independent if :

(1) X is set of rational number

(2) X is set of irrational number

(3) Neither (1) and nor (2)

(4) None of these

50. For what value of K will the vector $u = (1, K, 5)$ in $V_3(R)$ be a linear combination of vectors $v = (1, -3, 2)$ and $w = (2, -1, 1)$?

(1) −8 (2) 8

(3) 0 (4) 4

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Subject Mathematics (Hons) Linear Algebra

Set A

ANSWER – KEY

1	2	3	4	5	6	7	8	9	10
2	3	1	2	4	2	2	3	3	3

11	12	13	14	15	16	17	18	19	20
2	1	3	2	1	3	3	4	1	2

21	22	23	24	25	26	27	28	29	30
2	2	2	3	4	3	2	2	1	1

31	32	33	34	35	36	37	38	39	40
1	3	4	1	2	3	1	3	1	1

41	42	43	44	45	46	47	48	49	50
1	1	3	3	3	3	1	1	2	4

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Subject Mathematics (Hons) Set B

Linear Algebra

ANSWER – KEY

1	2	3	4	5	6	7	8	9	10
2	1	3	2	1	3	3	4	1	2
11	12	13	14	15	16	17	18	19	20
2	2	2	3	4	3	2	2	1	1
21	22	23	24	25	26	27	28	29	30
1	3	4	1	2	3	1	3	1	1
31	32	33	34	35	36	37	38	39	40
1	1	3	3	3	3	1	1	2	4
41	42	43	44	45	46	47	48	49	50
2	3	1	2	4	2	2	3	3	3

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Subject Mathematics (Hons) Set C
Linear Algebra

ANSWER – KEY

1	2	3	4	5	6	7	8	9	10
2	2	2	3	4	3	2	2	1	1

11	12	13	14	15	16	17	18	19	20
1	3	4	1	2	3	1	3	1	1

21	22	23	24	25	26	27	28	29	30
1	1	3	3	3	3	1	1	2	4

31	32	33	34	35	36	37	38	39	40
2	3	1	2	4	2	2	3	3	3

41	42	43	44	45	46	47	48	49	50
2	1	3	2	1	3	3	4	1	2

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Set: D

Subject: Mathematics Hons / Linear Algebra

ANSWER – KEY

1	2	3	4	5	6	7	8	9	10
1	3	4	1	2	3	1	3	1	1
11	12	13	14	15	16	17	18	19	20
1	1	3	3	3	3	1	1	2	4
21	22	23	24	25	26	27	28	29	30
2	3	1	2	4	2	2	3	3	3
31	32	33	34	35	36	37	38	39	40
2	1	3	2	1	3	3	4	1	2
41	42	43	44	45	46	47	48	49	50
2	2	2	3	4	3	2	2	1	1

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