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A

SET-Y

M.Phil./Ph.D./URS-EE-2020

SUBJECT : Mathematics

Sr. No. 10149

Time : **1¼ Hours** Max. Marks : **100** Total Questions : **100**

Roll No. (in figures)__________ (in words) __________

Name __________ Father's Name__________

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SEAL

1. For which of the following sets, the set of interior points is not empty ?

(1) N (2) Z (3) R (4) I

2. Which of the following is ***incorrect*** ?
Counter set is :

(1) Closed (2) Countable

(3) Measurable (4) Equivalent to [0, 1]

3. If X is a set of even natural numbers less than 8 and Y is a set of odd prime numbers less than equal to 7, then the number of relations from X to Y is :

(1) 2^8 (2) $2^8 - 1$ (3) 2^9 (4) $2^9 - 1$

4. The sequence

$$S_n = \begin{cases} 2, \text{ when } n \text{ is even} \\ \text{lowest prime factor } (\neq 1) \text{ of } n, \text{when } n \text{ is odd} \end{cases}$$

has limit point :

(1) 2 (2) countable in number

(3) 1, 2, 3, 4, ... (4) uncountable in number

5. $\lim_{n \to \infty} \left[\frac{n}{n^2} + \frac{n}{n^2 + 1^2} + \frac{n}{n^2 + 2^2} + \ldots\ldots \right] =$

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{2}$ (4) 0

6. If $\{a_n\}$ is a sequence of elements in the interval (–1, 1), then which of the following is ***true*** ?

(1) Every limit point of $\{a_n\}$ is in (–1, 1)

(2) Every limit point of $\{a_n\}$ is in [–1, 1]

(3) Limit points of $\{a_n\}$ can only be {–1, 0, 1}

(4) None of these

7. The infinite series $\sum \frac{\lfloor n \, x^n}{n^n}$ diverges for :

(1) $x < e$

(2) $x > e$

(3) $x < \frac{1}{e}$

(4) for all values of x

8. The function $f(x) = 2x^3 + 9x^2 + 12x - 7$ is decreasing in the interval :

(1) $(-1, 2)$

(2) $(-2, 1)$

(3) $(1, 2)$

(4) $(-2, -1)$

9. If $f(x + y) = f(x) f(y) \; \forall \; x, y$ and $f(5) = -2$, $f'(0) = 3$, then the value of $f'(5)$ is :

(1) 4

(2) -4

(3) 6

(4) -6

10. The series $\sum \frac{\sin(x^2 + n^2 x)}{n(n+1)}$ is convergent uniformly for :

(1) Countable real numbers only

(2) Finite real numbers only

(3) All real numbers

(4) None of these

11. The abscissa of the point at which tangent to the curve $y = x(x - 1)$ is parallel to the chord joining the extremities of the curve in the interval $[1, 2]$ is :

(1) $\frac{3}{2}$

(2) $\frac{3}{4}$

(3) $\frac{4}{3}$

(4) $\frac{2}{3}$

12. The integral $\int_0^\infty \frac{\sin x}{x} dx$:

(1) converges absolutely

(2) converges but not absolutely

(3) does not converges

(4) does not exist

13. The function $f(x, y) = x^4 + x^2y + y^2$ at the point (0, 0) is :

(1) Maximum
(2) Minimum
(3) Harmonic
(4) Not defined

14. If X is a complete metric space and E is non-empty open subset of X, then E is :

(1) of first category
(2) of second category
(3) a null set
(4) complete

15. If $X = \{ x : 0 < d(0, x) \leq 1 \text{ and } x \in R^2 \}$, where d is the usual metric on X, then which of the following is ***not*** true ?

(1) X is closed
(2) X is bounded
(3) X is compact
(4) X is not compact

16. If $\alpha > 0$ and $\beta > 0$ and $f(x) = \begin{cases} x^\alpha \sin\frac{1}{x^\beta} & , \ x \neq 0 \\ 0 & , \ x = 0 \end{cases}$ then $f(x)$ is of bounded variation in [0, 1] if :

(1) $\alpha + \beta = 1$
(2) $\alpha = \beta$
(3) $\alpha < \beta$
(4) $\alpha > \beta$

17. A function $f : R \to R$ need not be Lebesgue measurable if :

(1) f is monotone
(2) $\{x \in R : f(x) = \alpha\}$ is measurable for each $\alpha \in R$
(3) $\{x \in R : f(x) \geq \alpha\}$ is measurable for each $\alpha \in Q$
(4) for each open set G in R, $f^{-1}(G)$ is measurable

18. If $f(x, y) = \frac{x^2}{y^2}, (x, y) \in \left[\frac{1}{2}, \frac{3}{2}\right] \times \left[\frac{1}{2}, \frac{3}{2}\right]$, then the derivative of f at (1, 1) along the direction (1, 1) is :

(1) 0
(2) $\frac{1}{2}$
(3) 1
(4) 2

19. If V is a vector space of dimension 100 and A and B are two subspaces of dimensions 60 and 63, then minimum dimension of $A \cap B$ is :

(1) 60 (2) 40 (3) 37 (4) 23

20. For the matrix $A = \begin{bmatrix} 2 & -2 & 3 \\ -2 & -1 & 6 \\ 1 & 2 & 0 \end{bmatrix}$, one of the eigen values is 3, the other two eigen values are :

(1) 2, 5
(2) −3, 5
(3) 3, −5
(4) 2, −5

21. Let A be a non-zero upper triangular matrix whose all eigen values are zero, then $I + A$ is :

(1) Singular
(2) Invertible
(3) Idempotent
(4) Nilpotent

22. Let T be a linear transformation on a vector space V such that $T^2 - T + 1 = 0$, then T is :

(1) Idempotent (2) Singular (3) Invertible (4) Not invertible

23. The dimension of the subspace of R^3 spaned by (−3, 0, 1), (1, 2, 1) and (3, 0, −1) is :

(1) 2 (2) 3 (3) 0 (4) 1

24. Let $A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 2 & 3 & a \end{pmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 3 \\ b \end{bmatrix}$, then the system $AX = B$ over the field of real numbers has :

(1) Infinite number of solutions if $a \neq 2$

(2) Infinite number of solutions if $a = 2$ and $b \neq 7$

(3) No solution if $b \neq 7$

(4) Unique solution if $a \neq 2$

25. If the matrix of the quadratic form $(x_1 - x_2 + 2x_3)^2$ is denoted by A, then the trace of A is :

(1) 4 (2) 6 (3) 9 (4) 3

26. $\lim_{z \to 0} \frac{\bar{z}}{z}$ is equal to :

(1) 0
(2) 1
(3) i
(4) does not exist

27. If $w = u + iv$, $z = x + iy$, then the image of the line $x = -3$ under the mapping $w = z^2$ is :

(1) $u = -3v + 1$
(2) $u = 3 - \frac{v^2}{4}$
(3) $u = 9 + \frac{v^2}{36}$
(4) $v = 9 - \frac{u^2}{36}$

28. Solution of $e^{z-1} = -ie^3$ is :

(1) $4 + (2n - 1)\pi i$
(2) $4 + \frac{1}{2}(3n - 1)\pi i$
(3) $4 + \frac{1}{2}(2n - 1)\pi i$
(4) $4 + \frac{1}{2}(4n - 1)\pi i$

29. $|z - 2| = \text{Re}(z)$ represents :

(1) a parabola
(2) a hyperbola
(3) a circle
(4) an ellipse

30. If C is the circle $|z| = 2$, then using Cauchy's integral formula for derivatives, $\int_C \frac{\sin z}{z^4} dz =$

(1) $\frac{\pi i}{3}$
(2) $\frac{-\pi i}{3}$
(3) $\frac{\pi i}{2}$
(4) $\frac{-\pi i}{4}$

31. Residue of $f(z) = z \cos \frac{1}{z}$ at $z = 0$ is :

(1) -1 (2) 1 (3) $\frac{1}{2}$ (4) $-\frac{1}{2}$

32. If $f(z) = u(x, y) + iv(x, y)$ is analytic, then $f'(z) =$

(1) $\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$ (2) $\frac{\partial u}{\partial y} - i\frac{\partial v}{\partial x}$

(3) $\frac{\partial u}{\partial x} - i\frac{\partial v}{\partial y}$ (4) $\frac{\partial u}{\partial x} - i\frac{\partial v}{\partial x}$

33. If the principal part of Laurent's expansion of $f(z)$ at a point, has infinite number of non-zero terms, then the singularity of $f(z)$ at that point is :

(1) removable (2) a pole (3) essential (4) non-essential

34. The mapping $f(z) = ze^{z^2-2}$ is not conformal at $z =$

(1) $\pm\sqrt{2}i$ (2) $\pm\frac{1}{\sqrt{2}}$ (3) $\pm\frac{i}{\sqrt{2}}$ (4) $\pm\frac{3i}{\sqrt{2}}$

35. If C denotes the unit circle centred at the origin in the Argant plane, then the value of the integral $\int_C |1+z+z^2|^2 \, dz$, taken anticlockwise along C, is equal to :

(1) 0 (2) πi (3) $2\pi i$ (4) $4\pi i$

36. The number of roots of the equation $z^5 - 12z^2 + 14 = 0$ those lying in the annulus $2 \leq |z| < \frac{5}{2}$, is :

(1) 3 (2) 1 (3) 2 (4) 0

37. In how many ways can a person can invite his 5 friends for dinner ?

(1) 25 (2) 27 (3) 31 (4) 32

38. The number of integers between 1 and 1000 that are divisible by 2 or 3, are :

(1) 343 (2) 367 (3) 372 (4) 377

39. When 2^{50} is divided by 7, the remainder is :

(1) 4 (2) 5 (3) 2 (4) 3

40. For any integer n > 2, which of the following is true for the Euler's function $\phi(n)$?

(1) $\phi(n)$ is even
(2) $\phi(n)$ is odd
(3) $\phi(n)$ is rational
(4) $\phi(n)$ is prime

41. If N is the set of natural numbers, then under the binary operation $a.b = a + b$, $(N, .)$ is :

(1) group
(2) semi-group
(3) quasi-group
(4) monoid

42. A field having no proper subfield is called :

(1) ordered field
(2) improper field
(3) prime field
(4) modular field

43. Which of the following is *not* true ?

(1) Inverse of an odd permutation is odd
(2) Identity permutation is an even permutation
(3) Product of two odd permutations is an even permutation
(4) Every transformation is always an even permutation

44. Which of the following is *not* true ?

(1) Every finite integral domain is a field
(2) Intersection of two subrings is a subring
(3) The sum of two subrings of a ring R is a subring of R
(4) A finite commutative ring without zero divisors is a field

45. Given field F and the set M of all 2 × 2 matrices of the form $\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}$ for $a, b \in F$, then which of the following is ***not*** true ?

(1) M is subring of F

(2) M is left ideal in F

(3) M is right ideal in F

(4) M is right ideal but not left ideal in F

46. The degree of splitting field of $x^4 - 1$ over Q is :

(1) 0

(2) 3

(3) 4

(4) 2

47. Which of the following is ***incorrect*** ?

(1) Every discrete space is regular

(2) Every discrete space is connected

(3) Every discrete space is a Hausdorff space

(4) Every discrete space is not compact

48. If X is a topological space and A is a proper dense open subset of X, then which of the following is ***correct*** ?

(1) If X is compact, then X/A is compact

(2) If X is connected, then A is connected

(3) If X is compact, then A is compact

(4) If X/A is compact, then X is compact

49. If $X = \{a, b, c\}$, $T = \{\phi, X, \{a, c\}, \{b\}\}$, then the topological space (X, T) is :

(1) a connected space

(2) not a connected space

(3) not a compact space

(4) not a Hausdorff space

50. Which of the following is ***not*** correct ?

(1) Every compact is metric space is complete

(2) Every metric space is compact Hausdorff space

(3) A compact Hausdorff space is normal

(4) A compact subspace of a Hausdorff space is closed

51. The solution of $\frac{dy}{dx} = \frac{y}{x} + x \tan \frac{y}{x}$ is :

(1) $\log \tan \frac{y}{2x} = x + c$

(2) $\log \tan \frac{y}{x} = x + c$

(3) $\log \cot \frac{y}{x} = x + c$

(4) $\log \cot \frac{y}{2x} = x + c$

52. The I. V. P. $x\frac{dy}{dx} = y$, $y(0) = 0$, $x \geq 0$ has :

(1) No solution

(2) Exactly two solutions

(3) A unique solution

(4) Infinitely many solutions

53. Solution of $y = xp - p^2$ is :

(1) $y = 3x - c$

(2) $y = \log x + c$

(3) $y = cx - c^2$

(4) $y = cx - cx^2$

54. Solving $y'' - 2y' + y = e^x \log x$ by variation of parameters, the value of Wronskion is :

(1) xe^{-2x} (2) e^{-2x} (3) e^{2x} (4) xe^{2x}

55. For the system $\frac{dy}{dt} = x - xy,\ \frac{dy}{dt} = xy - y$, the critical point (0, 0) is :

(1) centre (2) saddle point

(3) spiral point (4) node

56. The Green's function $G(x, t)$:

(1) is not defined at $x = t$ (2) is continuous at $x = t$

(3) is discontinuous at $x = t$ (4) is harmonic in the xt-plane

57. For the P. D. E $r - 2s + t = \cos(2x + 3y)$, P. I. is :

(1) $\cos(2x + 3y)$ (2) $-3\cos(2x + 3y)$

(3) $-2\cos(2x + 3y)$ (4) $-\cos(2x + 3y)$

58. The P. D. E $yu_{xx} + 2xyu_{xy} + xu_{yy} = u_x + u_y$ is hyperbolic in the region :

(1) $xy > 1$ (2) $xy < 1$

(3) $xy > 0$ (4) $0 < xy < 1$

59. The eigen values of a SLP are :

(1) real and finite (2) real and positive

(3) real and non-zero (4) real and non-negative

60. The general solution of $u_{xx} + u_{yy} = 0$ is of the form :

(1) $u = f(x + iy)$ (2) $u = f(x + y) + g(x - y)$

(3) $u = f(x + iy) + g(x - iy)$ (4) $u = f(x + iy)\, f(x + iy)$

61. If $y(x)$ is the solution of $y'(x) = ay - by^2$; $a, b > 0$, $y(0) = y_0$, then the limiting value of $y(x)$, as $x \to \infty$, will be :

(1) $\frac{b}{a}$ (2) $\frac{a}{b}$

(3) 0 (4) y_0

62. If $u(x, t)$ be the solution of the I. V. P. $u_{tt} = u_{xx}; u(x, 0) = x^3$, $u_t(x, 0) = \sin x$, then the value of $u(\pi, \pi)$ is :

(1) $2\pi^2$ (2) $2\pi^3$ (3) $4\pi^2$ (4) $4\pi^3$

63. The order of convergence of Newton-Raphson method is :

(1) linear (2) quadratic

(3) cubic (4) exponential

64. Another name of Hermite's interpolation formula is :

(1) Osculating interpolation formula

(2) Critical interpolation formula

(3) Spline interpolation formula

(4) Lagrange's interpolation formula

65. The second order Runge-Kutta method is applied to the initial value problem $\frac{dy}{dx} = -y, y(0) = y_0$ with step size h, then $y(h) =$

(1) $\frac{y_0}{2}\left(h^2 + 2h - 2\right)$ (2) $\frac{y_0}{2}\left(h^2 - 3h + 6\right)$

(3) $\frac{y_0}{2}\left(y^2 - 2h + 2\right)$ (4) $\frac{y_0}{6}\left(y^2 - 4h + 2\right)$

66. Lagrange's interpolation formula is useful in finding any value of a function when the given values of the independent variable are :

(1) positive
(2) non-zero
(3) equidistant
(4) not equidistant

67. On what curve the function $I = \int_0^1 \left[\left(\frac{dy}{dx}\right)^2 + 12xy\right] dx$ with $y(0) = 0$, $y(1) = 1$ can be extremized ?

(1) $y = x^2$
(2) $y = x^3$
(3) $y = x^4$
(4) $y = e^x$

68. The extremals of $\int_a^b \frac{1}{x}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$ are :

(1) circles
(2) parabole
(3) ellipses
(4) hyperbole

69. The result obtained on simplification of the Euler-Lagrange equation is known as :

(1) Cauchy identity
(2) Beltrani identity
(3) Hamilton identity
(4) Liouville identity

70. For the Kernel $k(x, t) = 1$, $a = 0$, $b = 1$; $D(\lambda) =$

(1) $1 - \frac{\lambda}{2}$
(2) $\lambda - 1$
(3) $1 + \lambda$
(4) $1 - \lambda$

71. Solution of the integral equation $\int_0^x \frac{y(t)}{x-t}\, dt = \sqrt{x}$ is :

(1) $y = 1$

(2) $y = \sqrt{2}$

(3) $y = \frac{1}{2}$

(4) $y = \frac{3}{2}$

72. The eigen values λ of the integral equation $y(x) - \lambda \int_0^{2\pi} \sin(x+t) y(t)\, dt$, are :

(1) $\frac{1}{\pi}, -\frac{1}{\pi}$

(2) $\frac{\pi}{2}, -\frac{\pi}{2}$

(3) $\pi, -\frac{\pi}{2}$

(4) $\pi, -\pi$

73. A load slides without friction on a wire in the slope of the cycloid having equation $x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$, $0 \le \theta \le 2\pi$. Then the Lagrangian is :

(1) $ma^2(1-\cos\theta)\dot{\theta}^2 - mga(1+\cos\theta)$

(2) $ma^2(1-\cos\theta)\dot{\theta}^2 + mga(1+\cos\theta)$

(3) $ma^2(1+\cos\theta)\dot{\theta}^2 - mga(1-\cos\theta)$

(4) $ma^2(1+\cos\theta)\dot{\theta}^2 - mga(1-\sin\theta)$

74. The generalized displacement of a rigid body is a translation with rotation. This result is known as :

(1) Law of displacement

(2) Euler's theorem

(3) Chasle's theorem

(4) Law of rotation

75. Equation of constraints that does not contain time as explicit variable, is referred to as :

(1) holonomic (2) non-holonomic

(3) scleronomous (4) rhenomous

76. Two lines of regression are $x = -\frac{1}{18}y + \mu_1$ and $y = -2x + \mu_2$; (μ_1, μ_2) being unknown and the mean of the distribution is (1, 18). Estimated value of y when $x = 10$ is :

(1) 2 (2) 0 (3) −18 (4) −144

77. If partial correlation coefficient $r_{12.3} = 0$, then :

(1) $r_{12} = r_{13}r_{23}$ (2) $r_{23} = r_{21}r_{13}$

(3) $r_{12} = 1$ (4) $r_{31} = r_{12}r_{23}$

78. If A and B are independent events such that $P(\overline{A}) = 0.7$, $P(\overline{B}) = K$ and $P(A \cup B) = 0.8$, then $K =$

(1) 0 (2) 1

(3) $\frac{2}{7}$ (4) $\frac{5}{7}$

79. If $(X, Y) \sim BvN(0, 0, 1, 1, 0.8)$, then $1 + 2X + 3Y$ is distributed as :

(1) $N(0, 1)$ (2) $N(1, 13)$

(3) $N(1, 19)$ (4) $N(0, 19)$

80. In case of simple random sampling with replacement, the variance of the estimate of population mean is :

(1) $\frac{N-n}{nN}\sigma^2$ (2) $\frac{N-n}{nN}\,\frac{N-1}{N}\sigma^2$

(3) $\frac{\sigma^2}{nN}$ (4) $\frac{\sigma^2}{n}$

81. Which of the following is ***true*** ?

(1) In a moderately asymmetrical distribution, Median = 3 Mode – 2 Mean

(2) Mode is always greater than mean

(3) The Median is most affected by the extreme values

(4) Median lies in between Mean and Mode

82. If f_1 and f_2 are p.d.f.'s and $\theta_1 + \theta_2 = 1$, then $g(x) = \theta_1 f_1(x) + \theta_2 f_2 = (x)$

(1) is a p.d.f.

(2) has the value 2

(3) is a c.d.f.

(4) can never the p.d.f.

83. The joint probability density function of a two dimensional random variable (X, Y) is given by :

$$f(x,y) = \begin{cases} 2 \quad , & 0 < x < 1, 0 < y < x \\ 0 \quad , & \text{elsewhere} \end{cases}$$

Then, the marginal density function of Y is :

(1) $2(1 - Y)$ (2) $2(1 + Y)$

(3) $2Y$ (4) 2

84. The following LPP has the multiple optimal solutions :

Max. : $Z = x + 3y$

Subject to :

$$2x + y \leq 10$$

$$x + 3y \leq 15$$

$$x, y \geq 0$$

One of the points that gives optimal solution for the LPP is :

(1) (5.1, 0)

(2) (2.7, 4.1)

(3) (9.2, 2.1)

(4) (2, 1)

85. Let $\{X_n, n \geq 0\}$ be a Markov Chain with three states 0, 1, 2 and with transition matrix $\begin{pmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{pmatrix}$ and the initial distribution $P\{X_0 = i\} = \frac{1}{3}, \ i = 0, 1, 2.$

Then $P\{X_2 = 1 | X_0 = 0\}$ is :

(1) $\frac{1}{2}$

(2) $\frac{5}{8}$

(3) $\frac{3}{16}$

(4) $\frac{5}{16}$

86. In a binomial distribution consisting of 4 independent trials, probabilities of 1 and 2 successes are 0.4 and 0.2 respectively. Then the parameter p, then probability of success in each trial of the distribution is equal to :

(1) 0.25

(2) 0.4

(3) 0.5

(4) 0.75

87. For a Poisson variate X, $E(X^2) = 6$, then the value of $E(X)$ will be :

(1) 4
(2) 3
(3) 2
(4) $\sqrt{6}$

88. The interval between two successive occurrences of a Poisson process $\{N(t), t \geq 0\}$ having parameter λ has :

(1) Poisson distribution with mean μ

(2) Poisson distribution with mean $\frac{1}{\mu}$

(3) Negative exponential distribution with mean μ

(4) Negative exponential distribution with mean $\frac{1}{\mu}$

89. With 0.8 as the traffic intensity, the expected number of customers in M|M|1 system is :

(1) 4
(2) 5
(3) $\frac{20}{9}$
(4) 3.2

90. A symmetric die is thrown 3600 times. The lower bound for the probability of getting 500 to 700 sixes is :

(1) $\frac{199}{200}$
(2) $\frac{99}{100}$
(3) $\frac{49}{50}$
(4) $\frac{19}{20}$

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91. If $X_1, X_2, \ldots, X_n$ be a random sample from a population with constant density, each X_i is exponential with parameter λ. Then, min $(X_1, X_2, \ldots, X_n)$ is :

(1) exponential with parameter λ

(2) exponential with parameter $n\lambda$

(3) poisson with parameter λ

(4) poisson with parameter $n\lambda$

92. Let $X_1, X_2, \ldots, X_{10}$ be 10 i.i.d. variates, each with p.d.f. $f(x)$ and c.d.f. $F(x)$. Then p.d.f. of the smallest order statistic is :

(1) $10[F(x)]^9 f(x)$ (2) $[F(x)]^9 f(x)$

(3) $10[1-F(x)]^9 f(x)$ (4) $1-[1-F(x)]^{10}$

93. Number of observations saved in a 4×4 L. S. D. over a complete 3-way layout is :

(1) 64 (2) 48

(3) 24 (4) 16

94. Let $X_1, X_2, \ldots, X_n$ be a random sample from the uniform distribution with p.d.f. :

$$f(x,\theta) = \begin{cases} \frac{1}{\theta}, & 0 < x < \infty, \theta > 0 \\ 0, & \text{elesewhere} \end{cases}$$

Then the maximum likelihood estimator for θ is :

(1) the same mean

(2) the same median

(3) the largest sample observation

(4) the smallest sample observation

95. If $x \geq 1$ is the critical region for testing $H_0 : \theta = 2$ against the alternative $H_1 : \theta = 1$, on the basis of single observation from the population $F(x, \theta) = \theta \exp(-\theta x)$, $0 \leq x < \infty$. Then the power of the test is :

(1) $\frac{1}{e}$ (2) $\frac{1}{e^2}$

(3) $\frac{e-1}{e}$ (4) $\frac{e-1}{e^2}$

96. Let X_α be an observation from $N(\beta z_\alpha, \Sigma)$, $\alpha = 1, 2, \ldots\ldots, N$ with $(z_1, z_2, \ldots\ldots z_N)$ *of* rank q. Let $C = \sum_{\alpha=1}^{N} x_\alpha Z'_\alpha$ and $A = \sum_{\alpha=1}^{N} Z_\alpha Z'_\alpha$. Then MLE of β is :

(1) CA (2) AC^{-1}

(3) $C^{-1}A$ (4) CA^{-1}

97. A system is composed of three identical independent elements in parallel, each having the reliability 0.7, then the reliability of the system is :

(1) 0.973 (2) 0.793

(3) 0.73 (4) 0.27

98. In case of large sample single tailed test, the magnitude of the critical value of Z at 5 percent level of significance is :

(1) 2.58 (2) 1.327

(3) 1.645 (4) 1.782

99. If there are 10 symbols of two types, equal in number, the maximum possible runs is :

(1) 5 (2) 10

(3) 9 (4) 25

100. Which one problem out of the following is ***not*** related to stratified sampling ?

(1) Fixing the points of demarcation between strata

(2) Fixing the criterion for stratification

(3) Fixing the number of strata

(4) Fixing the sample size

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B

SET-Y

M.Phil./Ph.D./URS-EE-2020

SUBJECT : Mathematics

10178

Sr. No. ....

Time : **1¼ Hours** Max. Marks : **100** Total Questions : **100**

Roll No. (in figures)____________ (in words) ____________

Name ____________ Father's Name ____________

Mother's Name ____________ Date of Examination ____________

(Signature of the Candidate) (Signature of the Invigilator)

SEAL

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3. Keeping in view the transparency of the examination system, carbonless OMR Sheet is provided to the candidate so that a copy of OMR Sheet may be kept by the candidate.
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7. Use only **Black** or **Blue Ball Point Pen** of good quality in the OMR Answer-Sheet.
8. *Before answering the questions, the candidates should ensure that they have been supplied correct and complete booklet. Complaints, if any, regarding misprinting etc. will not be entertained* **30** *minutes after starting of the examination.*

1. Solution of the integral equation $\int_0^x \frac{y(t)}{x-t}\,dt = \sqrt{x}$ is :

(1) $y = 1$ (2) $y = \sqrt{2}$

(3) $y = \frac{1}{2}$ (4) $y = \frac{3}{2}$

2. The eigen values λ of the integral equation $y(x) - \lambda \int_0^{2\pi} \sin(x+t)y(t)\,dt$, are :

(1) $\frac{1}{\pi}, -\frac{1}{\pi}$ (2) $\frac{\pi}{2}, -\frac{\pi}{2}$

(3) $\pi, -\frac{\pi}{2}$ (4) $\pi, -\pi$

3. A load slides without friction on a wire in the slope of the cycloid having equation $x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$, $0 \le \theta \le 2\pi$. Then the Lagrangian is :

(1) $ma^2(1-\cos\theta)\dot{\theta}^2 - mga(1+\cos\theta)$

(2) $ma^2(1-\cos\theta)\dot{\theta}^2 + mga(1+\cos\theta)$

(3) $ma^2(1+\cos\theta)\dot{\theta}^2 - mga(1-\cos\theta)$

(4) $ma^2(1+\cos\theta)\dot{\theta}^2 - mga(1-\sin\theta)$

4. The generalized displacement of a rigid body is a translation with rotation. This result is known as :

(1) Law of displacement (2) Euler's theorem

(3) Chasle's theorem (4) Law of rotation

5. Equation of constraints that does not contain time as explicit variable, is referred to as :

(1) holonomic
(2) non-holonomic
(3) scleronomous
(4) rhenomous

6. Two lines of regression are $x = -\frac{1}{18}y + \mu_1$ and $y = -2x + \mu_2$; (μ_1, μ_2) being unknown and the mean of the distribution is (1, 18). Estimated value of y when $x = 10$ is :

(1) 2
(2) 0
(3) −18
(4) −144

7. If partial correlation coefficient $r_{12.3} = 0$, then :

(1) $r_{12} = r_{13}r_{23}$
(2) $r_{23} = r_{21}r_{13}$
(3) $r_{12} = 1$
(4) $r_{31} = r_{12}r_{23}$

8. If A and B are independent events such that $P(\overline{A}) = 0.7$, $P(\overline{B}) = K$ and $P(A \cup B) = 0.8$, then $K =$

(1) 0
(2) 1
(3) $\frac{2}{7}$
(4) $\frac{5}{7}$

9. If $(X, Y) \sim B \vee N\,(0, 0, 1, 1, 0.8)$, then $1 + 2X + 3Y$ is distributed as :

(1) $N(0, 1)$
(2) $N(1, 13)$
(3) $N(1, 19)$
(4) $N(0, 19)$

10. In case of simple random sampling with replacement, the variance of the estimate of population mean is :

(1) $\frac{N-n}{nN}\sigma^2$
(2) $\frac{N-n}{nN}\,\frac{N-1}{N}\sigma^2$
(3) $\frac{\sigma^2}{nN}$
(4) $\frac{\sigma^2}{n}$

11. The solution of $\frac{dy}{dx} = \frac{y}{x} + x\tan\frac{y}{x}$ is :

(1) $\log\tan\frac{y}{2x} = x + c$
(2) $\log\tan\frac{y}{x} = x + c$
(3) $\log\cot\frac{y}{x} = x + c$
(4) $\log\cot\frac{y}{2x} = x + c$

12. The I. V. P. $x\frac{dy}{dx} = y$, $y(0) = 0, x \geq 0$ has :

(1) No solution
(2) Exactly two solutions
(3) A unique solution
(4) Infinitely many solutions

13. Solution of $y = xp - p^2$ is :

(1) $y = 3x - c$
(2) $y = \log x + c$
(3) $y = cx - c^2$
(4) $y = cx - cx^2$

14. Solving $y'' - 2y' + y = e^x \log x$ by variation of parameters, the value of Wronskion is :

(1) xe^{-2x}
(2) e^{-2x}
(3) e^{2x}
(4) xe^{2x}

15. For the system $\frac{dy}{dt} = x - xy$, $\frac{dy}{dt} = xy - y$, the critical point (0, 0) is :

(1) centre
(2) saddle point
(3) spiral point
(4) node

16. The Green's function $G(x, t)$:

(1) is not defined at $x = t$
(2) is continuous at $x = t$
(3) is discontinuous at $x = t$
(4) is harmonic in the xt-plane

17. For the P. D. E $r - 2s + t = \cos(2x + 3y)$, P. I. is :

(1) $\cos(2x + 3y)$
(2) $-3\cos(2x + 3y)$
(3) $-2\cos(2x + 3y)$
(4) $-\cos(2x + 3y)$

18. The P. D. E $yu_{xx} + 2xyu_{xy} + xu_{yy} = u_x + u_y$ is hyperbolic in the region :

(1) $xy > 1$ (2) $xy < 1$

(3) $xy > 0$ (4) $0 < xy < 1$

19. The eigen values of a SLP are :

(1) real and finite (2) real and positive

(3) real and non-zero (4) real and non-negative

20. The general solution of $u_{xx} + u_{yy} = 0$ is of the form :

(1) $u = f(x + iy)$ (2) $u = f(x + y) + g(x - y)$

(3) $u = f(x + iy) + g(x - iy)$ (4) $u = f(x + iy) f(x + iy)$

21. Residue of $f(z) = z \cos \frac{1}{z}$ at $z = 0$ is :

(1) -1 (2) 1 (3) $\frac{1}{2}$ (4) $-\frac{1}{2}$

22. If $f(z) = u(x, y) + iv(x, y)$ is analytic, then $f'(z) =$

(1) $\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$ (2) $\frac{\partial u}{\partial y} - i\frac{\partial v}{\partial x}$

(3) $\frac{\partial u}{\partial x} - i\frac{\partial v}{\partial y}$ (4) $\frac{\partial u}{\partial x} - i\frac{\partial v}{\partial x}$

23. If the principal part of Laurent's expansion of $f(z)$ at a point, has infinite number of non-zero terms, then the singularity of $f(z)$ at that point is :

(1) removable (2) a pole (3) essential (4) non-essential

24. The mapping $f(z) = ze^{z^2 - 2}$ is not conformal at $z =$

(1) $\pm\sqrt{2}i$ (2) $\pm\frac{1}{\sqrt{2}}$ (3) $\pm\frac{i}{\sqrt{2}}$ (4) $\pm\frac{3i}{\sqrt{2}}$

25. If C denotes the unit circle centred at the origin in the Argant plane, then the value of the integral $\int_C |1+z+z^2|^2 \, dz$, taken anticlockwise along C, is equal to :

(1) 0 (2) πi (3) $2\pi i$ (4) $4\pi i$

26. The number of roots of the equation $z^5 - 12z^2 + 14 = 0$ those lying in the annulus $2 \le |z| < \frac{5}{2}$, is :

(1) 3 (2) 1 (3) 2 (4) 0

27. In how many ways can a person can invite his 5 friends for dinner ?

(1) 25 (2) 27

(3) 31 (4) 32

28. The number of integers between 1 and 1000 that are divisible by 2 or 3, are :

(1) 343 (2) 367

(3) 372 (4) 377

29. When 2^{50} is divided by 7, the remainder is :

(1) 4 (2) 5 (3) 2 (4) 3

30. For any integer n > 2, which of the following is true for the Euler's function $\phi(n)$?

(1) $\phi(n)$ is even (2) $\phi(n)$ is odd

(3) $\phi(n)$ is rational (4) $\phi(n)$ is prime

31. The abscissa of the point at which tangent to the curve $y = x(x-1)$ is parallel to the chord joining the extremities of the curve in the interval [1, 2] is :

(1) $\frac{3}{2}$ (2) $\frac{3}{4}$

(3) $\frac{4}{3}$ (4) $\frac{2}{3}$

32. The integral $\int_0^\infty \frac{\sin x}{x} dx$:

(1) converges absolutely
(2) converges but not absolutely
(3) does not converges
(4) does not exist

33. The function $f(x, y) = x^4 + x^2y + y^2$ at the point (0, 0) is :

(1) Maximum
(2) Minimum
(3) Harmonic
(4) Not defined

34. If X is a complete metric space and E is non-empty open subset of X, then E is :

(1) of first category
(2) of second category
(3) a null set
(4) complete

35. If $X = \{ x : 0 < d(0, x) \leq 1$ and $x \in R^2 \}$, where d is the usual metric on X, then which of the following is ***not*** true ?

(1) X is closed
(2) X is bounded
(3) X is compact
(4) X is not compact

36. If $\alpha > 0$ and $\beta > 0$ and $f(x) = \begin{cases} x^\alpha \sin\frac{1}{x^\beta} & , \quad x \neq 0 \\ 0 & , \quad x = 0 \end{cases}$ then $f(x)$ is of bounded variation in [0, 1] if :

(1) $\alpha + \beta = 1$
(2) $\alpha = \beta$
(3) $\alpha < \beta$
(4) $\alpha > \beta$

37. A function $f : R \to R$ need not be Lebesgue measurable if :

(1) f is monotone
(2) $\{x \in R : f(x) = \alpha\}$ is measurable for each $\alpha \in R$
(3) $\{x \in R : f(x) \geq \alpha\}$ is measurable for each $\alpha \in Q$
(4) for each open set G in R, $f^{-1}(G)$ is measurable

38. If $f(x,y)=\frac{x^2}{y^2},(x,y)\in\left[\frac{1}{2},\frac{3}{2}\right]\times\left[\frac{1}{2},\frac{3}{2}\right]$, then the derivative of f at (1, 1) along the direction (1, 1) is :

(1) 0 (2) $\frac{1}{2}$ (3) 1 (4) 2

39. If V is a vector space of dimension 100 and A and B are two subspaces of dimensions 60 and 63, then minimum dimension of $A \cap B$ is :

(1) 60 (2) 40 (3) 37 (4) 23

40. For the matrix $A=\begin{bmatrix}2 & -2 & 3\\ -2 & -1 & 6\\ 1 & 2 & 0\end{bmatrix}$, one of the eigen values is 3, the other two eigen values are :

(1) 2, 5 (2) −3, 5

(3) 3, −5 (4) 2, −5

41. If $X_1, X_2, \ldots, X_n$ be a random sample from a population with constant density, each X_i is exponential with parameter λ. Then, min $(X_1, X_2, \ldots, X_n)$ is :

(1) exponential with parameter λ

(2) exponential with parameter $n\lambda$

(3) poisson with parameter λ

(4) poisson with parameter $n\lambda$

42. Let $X_1, X_2, \ldots, X_{10}$ be 10 i.i.d. variates, each with p.d.f. $f(x)$ and c.d.f. $F(x)$. Then p.d.f. of the smallest order statistic is :

(1) $10[F(x)]^9 f(x)$ (2) $[F(x)]^9 f(x)$

(3) $10[1-F(x)]^9 f(x)$ (4) $1-[1-F(x)]^{10}$

43. Number of observations saved in a 4 ×4 L. S. D. over a complete 3-way layout is :

(1) 64 (2) 48

(3) 24 (4) 16

44. Let $X_1, X_2, ..., X_n$ be a random sample from the uniform distribution with p.d.f. :

$$f(x,\theta) = \begin{cases} \frac{1}{\theta} & , \quad 0 < x < \infty, \theta > 0 \\ 0 & , \quad \text{elesewhere} \end{cases}$$

Then the maximum likelihood estimator for θ is :

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(2) the same median

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45. If $x \geq 1$ is the critical region for testing $H_0 : \theta = 2$ against the alternative $H_i : \theta = 1$, on the basis of single observation from the population $F(x, \theta) = \theta \exp(-\theta x)$, $0 \leq x < \infty$. Then the power of the test is :

(1) $\frac{1}{e}$ (2) $\frac{1}{e^2}$

(3) $\frac{e-1}{e}$ (4) $\frac{e-1}{e^2}$

46. Let X_α be an observation from $N(\beta z_\alpha, \Sigma)$, $\alpha = 1, 2, ..., N$ with $(z_1, z_2, ...z_N)$ *of* rank q. Let $C = \sum_{\alpha=1}^{N} x_\alpha Z'_\alpha$ and $A = \sum_{\alpha=1}^{N} Z_\alpha Z'_\alpha$. Then MLE of β is :

(1) CA (2) AC^{-1}

(3) $C^{-1}A$ (4) CA^{-1}

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48. In case of large sample single tailed test, the magnitude of the critical value of Z at 5 percent level of significance is :

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49. If there are 10 symbols of two types, equal in number, the maximum possible runs is :

(1) 5 (2) 10

(3) 9 (4) 25

50. Which one problem out of the following is ***not*** related to stratified sampling ?

(1) Fixing the points of demarcation between strata

(2) Fixing the criterion for stratification

(3) Fixing the number of strata

(4) Fixing the sample size

51. If $y(x)$ is the solution of $y'(x) = ay - by^2$; $a, b > 0$, $y(0) = y_0$, then the limiting value of $y(x)$, as $x \to \infty$, will be :

(1) $\frac{b}{a}$ (2) $\frac{a}{b}$

(3) 0 (4) y_0

52. If $u(x, t)$ be the solution of the I. V. P. $u_{tt} = u_{xx}; u(x, 0) = x^3$, $u_t(x, 0) = \sin x$, then the value of $u(\pi, \pi)$ is :

(1) $2\pi^2$ (2) $2\pi^3$ (3) $4\pi^2$ (4) $4\pi^3$

53. The order of convergence of Newton-Raphson method is :

(1) linear (2) quadratic

(3) cubic (4) exponential

54. Another name of Hermite's interpolation formula is :

(1) Osculating interpolation formula

(2) Critical interpolation formula

(3) Spline interpolation formula

(4) Lagrange's interpolation formula

55. The second order Runge-Kutta method is applied to the initial value problem $\frac{dy}{dx} = -y, y(0) = y_0$ with step size h, then $y(h) =$

(1) $\frac{y_0}{2}\left(h^2 + 2h - 2\right)$ (2) $\frac{y_0}{2}\left(h^2 - 3h + 6\right)$

(3) $\frac{y_0}{2}\left(y^2 - 2h + 2\right)$ (4) $\frac{y_0}{6}\left(y^2 - 4h + 2\right)$

56. Lagrange's interpolation formula is useful in finding any value of a function when the given values of the independent variable are :

(1) positive (2) non-zero

(3) equidistant (4) not equidistant

57. On what curve the function $I = \int_0^1 \left[\left(\frac{dy}{dx}\right)^2 + 12xy\right] dx$ with $y(0) = 0,\ y(1) = 1$ can be extremized ?

(1) $y = x^2$

(2) $y = x^3$

(3) $y = x^4$

(4) $y = e^x$

58. The extremals of $\int_a^b \frac{1}{x}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$ are :

(1) circles

(2) parabole

(3) ellipses

(4) hyperbole

59. The result obtained on simplification of the Euler-Lagrange equation is known as :

(1) Cauchy identity

(2) Beltrani identity

(3) Hamilton identity

(4) Liouville identity

60. For the Kernel $k(x, t) = 1, a = 0, b = 1; D(\lambda) =$

(1) $1 - \frac{\lambda}{2}$

(2) $\lambda - 1$

(3) $1 + \lambda$

(4) $1 - \lambda$

61. Which of the following is ***true*** ?

(1) In a moderately asymmetrical distribution, Median = 3 Mode − 2 Mean

(2) Mode is always greater than mean

(3) The Median is most affected by the extreme values

(4) Median lies in between Mean and Mode

62. If f_1 and f_2 are p.d.f.'s and $\theta_1 + \theta_2 = 1$, then $g(x) = \theta_1 f_1(x) + \theta_2 f_2 = (x)$

(1) is a p.d.f.

(2) has the value 2

(3) is a c.d.f.

(4) can never the p.d.f.

63. The joint probability density function of a two dimensional random variable (X, Y) is given by :

$$f(x, y) = \begin{cases} 2 & , \quad 0 < x < 1, 0 < y < x \\ 0 & , \quad \text{elsewhere} \end{cases}$$

Then, the marginal density function of Y is :

(1) $2(1 - Y)$

(2) $2(1 + Y)$

(3) $2Y$

(4) 2

64. The following LPP has the multiple optimal solutions :

Max. : $Z = x + 3y$

Subject to :

$$2x + y \leq 10$$

$$x + 3y \leq 15$$

$$x, y \geq 0$$

One of the points that gives optimal solution for the LPP is :

(1) (5.1, 0)

(2) (2.7, 4.1)

(3) (9.2, 2.1)

(4) (2, 1)

65. Let $\{X_n, n \geq 0\}$ be a Markov Chain with three states 0, 1, 2 and with transition matrix $\begin{pmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{pmatrix}$ and the initial distribution $P\{X_0 = i\} = \frac{1}{3}$, $i = 0, 1, 2$.

Then $P\{X_2 = 1 | X_0 = 0\}$ is :

(1) $\frac{1}{2}$

(2) $\frac{5}{8}$

(3) $\frac{3}{16}$

(4) $\frac{5}{16}$

66. In a binomial distribution consisting of 4 independent trials, probabilities of 1 and 2 successes are 0.4 and 0.2 respectively. Then the parameter p, then probability of success in each trial of the distribution is equal to :

(1) 0.25

(2) 0.4

(3) 0.5

(4) 0.75

67. For a Poisson variate X, $E(X^2) = 6$, then the value of $E(X)$ will be :

(1) 4

(2) 3

(3) 2

(4) $\sqrt{6}$

68. The interval between two successive occurrences of a Poisson process $\{N(t), t \geq 0\}$ having parameter λ has :

(1) Poisson distribution with mean μ

(2) Poisson distribution with mean $\frac{1}{\mu}$

(3) Negative exponential distribution with mean μ

(4) Negative exponential distribution with mean $\frac{1}{\mu}$

69. With 0.8 as the traffic intensity, the expected number of customers in M|M|1 system is :

(1) 4
(2) 5
(3) $\frac{20}{9}$
(4) 3.2

70. A symmetric die is thrown 3600 times. The lower bound for the probability of getting 500 to 700 sixes is :

(1) $\frac{199}{200}$
(2) $\frac{99}{100}$
(3) $\frac{49}{50}$
(4) $\frac{19}{20}$

71. If N is the set of natural numbers, then under the binary operation $a.b = a + b$, $(N, .)$ is :

(1) group
(2) semi-group
(3) quasi-group
(4) monoid

72. A field having no proper subfield is called :

(1) ordered field
(2) improper field
(3) prime field
(4) modular field

73. Which of the following is ***not*** true ?

(1) Inverse of an odd permutation is odd
(2) Identity permutation is an even permutation
(3) Product of two odd permutations is an even permutation
(4) Every transformation is always an even permutation

B

74. Which of the following is *not* true ?

(1) Every finite integral domain is a field

(2) Intersection of two subrings is a subring

(3) The sum of two subrings of a ring R is a subring of R

(4) A finite commutative ring without zero divisors is a field

75. Given field F and the set M of all 2 × 2 matrices of the form $\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}$ for $a, b \in F$, then which of the following is *not* true ?

(1) M is subring of F

(2) M is left ideal in F

(3) M is right ideal in F

(4) M is right ideal but not left ideal in F

76. The degree of splitting field of $x^4 - 1$ over Q is :

(1) 0 (2) 3

(3) 4 (4) 2

77. Which of the following is *incorrect* ?

(1) Every discrete space is regular

(2) Every discrete space is connected

(3) Every discrete space is a Hausdorff space

(4) Every discrete space is not compact

78. If X is a topological space and A is a proper dense open subset of X, then which of the following is ***correct*** ?

(1) If X is compact, then X/A is compact

(2) If X is connected, then A is connected

(3) If X is compact, then A is compact

(4) If X/A is compact, then X is compact

79. If $X = \{a, b, c\}$, $T = \{\phi, X, \{a, c\}, \{b\}\}$, then the topological space (X, T) is :

(1) a connected space

(2) not a connected space

(3) not a compact space

(4) not a Hausdorff space

80. Which of the following is ***not*** correct ?

(1) Every compact is metric space is complete

(2) Every metric space is compact Hausdorff space

(3) A compact Hausdorff space is normal

(4) A compact subspace of a Hausdorff space is closed

81. Let A be a non-zero upper triangular matrix whose all eigen values are zero, then $I + A$ is :

(1) Singular

(2) Invertible

(3) Idempotent

(4) Nilpotent

B

82. Let T be a linear transformation on a vector space V such that $T^2 - T + 1 = 0$, then T is :

(1) Idempotent

(2) Singular

(3) Invertible

(4) Not invertible

83. The dimension of the subspace of R^3 spaned by (–3, 0, 1), (1, 2, 1) and (3, 0, –1) is :

(1) 2

(2) 3

(3) 0

(4) 1

84. Let $A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 2 & 3 & a \end{pmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 3 \\ b \end{bmatrix}$, then the system $AX = B$ over the field of real numbers has :

(1) Infinite number of solutions if $a \neq 2$

(2) Infinite number of solutions if $a = 2$ and $b \neq 7$

(3) No solution if $b \neq 7$

(4) Unique solution if $a \neq 2$

85. If the matrix of the quadratic form $(x_1 - x_2 + 2x_3)^2$ is denoted by A, then the trace of A is :

(1) 4

(2) 6

(3) 9

(4) 3

86. $\lim_{z \to 0} \frac{\bar{z}}{z}$ is equal to :

(1) 0

(2) 1

(3) i

(4) does not exist

87. If $w = u + iv$, $z = x + iy$, then the image of the line $x = -3$ under the mapping $w = z^2$ is :

(1) $u = -3v + 1$

(2) $u = 3 - \frac{v^2}{4}$

(3) $u = 9 + \frac{v^2}{36}$

(4) $v = 9 - \frac{u^2}{36}$

88. Solution of $e^{z-1} = -ie^3$ is :

(1) $4 + (2n - 1)\pi i$

(2) $4 + \frac{1}{2}(3n - 1)\pi i$

(3) $4 + \frac{1}{2}(2n - 1)\pi i$

(4) $4 + \frac{1}{2}(4n - 1)\pi i$

89. $|z - 2| = \text{Re}(z)$ represents :

(1) a parabola

(2) a hyperbola

(3) a circle

(4) an ellipse

90. If C is the circle $|z| = 2$, then using Cauchy's integral formula for derivatives, $\int_C \frac{\sin z}{z^4} dz =$

(1) $\frac{\pi i}{3}$

(2) $\frac{-\pi i}{3}$

(3) $\frac{\pi i}{2}$

(4) $\frac{-\pi i}{4}$

91. For which of the following sets, the set of interior points is not empty ?

(1) N (2) Z (3) R (4) I

92. Which of the following is ***incorrect*** ?
Counter set is :

(1) Closed

(2) Countable

(3) Measurable

(4) Equivalent to [0, 1]

93. If X is a set of even natural numbers less than 8 and Y is a set of odd prime numbers less than equal to 7, then the number of relations from X to Y is :

(1) 2^8 (2) $2^8 - 1$ (3) 2^9 (4) $2^9 - 1$

94. The sequence

$$S_n = \begin{cases} 2, \text{ when } n \text{ is even} \\ \text{lowest prime factor } (\neq 1) \text{ of } n, \text{when } n \text{ is odd} \end{cases}$$

has limit point :

(1) 2

(2) countable in number

(3) 1, 2, 3, 4, ...

(4) uncountable in number

95. $\lim_{n\to\infty}\left[\frac{n}{n^2}+\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\ldots\ldots\right]=$

(1) $\frac{\pi}{4}$

(2) $\frac{\pi}{3}$

(3) $\frac{\pi}{2}$

(4) 0

96. If $\{a_n\}$ is a sequence of elements in the interval (–1, 1), then which of the following is ***true*** ?

(1) Every limit point of $\{a_n\}$ is in (–1, 1)

(2) Every limit point of $\{a_n\}$ is in [–1, 1]

(3) Limit points of $\{a_n\}$ can only be {–1, 0, 1}

(4) None of these

97. The infinite series $\sum \frac{\lfloor n \, x^n}{n^n}$ diverges for :

(1) $x < e$

(2) $x > e$

(3) $x < \frac{1}{e}$

(4) for all values of x

98. The function $f(x) = 2x^3 + 9x^2 + 12x - 7$ is decreasing in the interval :

(1) $(-1, 2)$
(2) $(-2, 1)$
(3) $(1, 2)$
(4) $(-2, -1)$

99. If $f(x + y) = f(x) f(y)\ \forall\ x, y$ and $f(5) = -2$, $f'(0) = 3$, then the value of $f'(5)$ is :

(1) 4
(2) −4
(3) 6
(4) −6

100. The series $\sum \frac{\sin(x^2 + n^2 x)}{n(n+1)}$ is convergent uniformly for :

(1) Countable real numbers only
(2) Finite real numbers only
(3) All real numbers
(4) None of these

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C

SET-Y

M.Phil./Ph.D./URS-EE-2020

SUBJECT : Mathematics

10147

Sr. No. ....

Time : **1¼ Hours** Max. Marks : **100** Total Questions : **100**

Roll No. (in figures)____________ (in words) ____________

Name ____________ Father's Name ____________

Mother's Name ____________ Date of Examination ____________

(Signature of the Candidate) (Signature of the Invigilator)

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1. ***Candidates are required to attempt any 75 questions out of the given 100 multiple choice questions of 4/3 marks each. No credit will be given for more than 75 correct responses.***
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3. Keeping in view the transparency of the examination system, carbonless OMR Sheet is provided to the candidate so that a copy of OMR Sheet may be kept by the candidate.
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5. The candidate ***must not*** do any rough work or writing in the OMR Answer-Sheet. Rough work, if any, may be done in the question booklet itself. Answers ***must not*** be ticked in the question booklet.
6. ***There will be no negative marking. Each correct answer will be awarded 4/3 mark. Cutting, erasing, overwriting and more than one answer in OMR Answer-Sheet will be treated as incorrect answer.***
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8. *Before answering the questions, the candidates should ensure that they have been supplied correct and complete booklet. Complaints, if any, regarding misprinting etc. will not be entertained* **30** *minutes after starting of the examination.*

1. If N is the set of natural numbers, then under the binary operation $a.b = a + b$, $(N, .)$ is :

(1) group

(2) semi-group

(3) quasi-group

(4) monoid

2. A field having no proper subfield is called :

(1) ordered field

(2) improper field

(3) prime field

(4) modular field

3. Which of the following is ***not*** true ?

(1) Inverse of an odd permutation is odd

(2) Identity permutation is an even permutation

(3) Product of two odd permutations is an even permutation

(4) Every transformation is always an even permutation

4. Which of the following is ***not*** true ?

(1) Every finite integral domain is a field

(2) Intersection of two subrings is a subring

(3) The sum of two subrings of a ring R is a subring of R

(4) A finite commutative ring without zero divisors is a field

5. Given field F and the set M of all 2×2 matrices of the form $\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}$ for $a, b \in F$, then which of the following is ***not*** true ?

(1) M is subring of F

(2) M is left ideal in F

(3) M is right ideal in F

(4) M is right ideal but not left ideal in F

6. The degree of splitting field of $x^4 - 1$ over Q is :

(1) 0
(2) 3
(3) 4
(4) 2

7. Which of the following is ***incorrect*** ?

(1) Every discrete space is regular

(2) Every discrete space is connected

(3) Every discrete space is a Hausdorff space

(4) Every discrete space is not compact

8. If X is a topological space and A is a proper dense open subset of X, then which of the following is ***correct*** ?

(1) If X is compact, then X/A is compact

(2) If X is connected, then A is connected

(3) If X is compact, then A is compact

(4) If X/A is compact, then X is compact

9. If $X = \{a, b, c\}$, $T = \{\phi, X, \{a, c\}, \{b\}\}$, then the topological space (X, T) is :

(1) a connected space

(2) not a connected space

(3) not a compact space

(4) not a Hausdorff space

10. Which of the following is *not* correct ?

(1) Every compact is metric space is complete

(2) Every metric space is compact Hausdorff space

(3) A compact Hausdorff space is normal

(4) A compact subspace of a Hausdorff space is closed

11. Let A be a non-zero upper triangular matrix whose all eigen values are zero, then $I + A$ is :

(1) Singular

(2) Invertible

(3) Idempotent

(4) Nilpotent

12. Let T be a linear transformation on a vector space V such that $T^2 - T + 1 = 0$, then T is :

(1) Idempotent

(2) Singular

(3) Invertible

(4) Not invertible

13. The dimension of the subspace of R^3 spaned by (–3, 0, 1), (1, 2, 1) and (3, 0, –1) is :

(1) 2

(2) 3

(3) 0

(4) 1

14. Let $A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 2 & 3 & a \end{pmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 3 \\ b \end{bmatrix}$, then the system $AX = B$ over the field of real numbers has :

(1) Infinite number of solutions if $a \neq 2$

(2) Infinite number of solutions if $a = 2$ and $b \neq 7$

(3) No solution if $b \neq 7$

(4) Unique solution if $a \neq 2$

15. If the matrix of the quadratic form $(x_1 - x_2 + 2x_3)^2$ is denoted by A, then the trace of A is :

(1) 4 (2) 6 (3) 9 (4) 3

16. $\lim_{z \to 0} \frac{\bar{z}}{z}$ is equal to :

(1) 0 (2) 1

(3) i (4) does not exist

17. If $w = u + iv$, $z = x + iy$, then the image of the line $x = -3$ under the mapping $w = z^2$ is :

(1) $u = -3v + 1$ (2) $u = 3 - \frac{v^2}{4}$

(3) $u = 9 + \frac{v^2}{36}$ (4) $v = 9 - \frac{u^2}{36}$

18. Solution of $e^{z-1} = -ie^3$ is :

(1) $4 + (2n - 1)\pi i$ (2) $4 + \frac{1}{2}(3n - 1)\pi i$

(3) $4 + \frac{1}{2}(2n - 1)\pi i$ (4) $4 + \frac{1}{2}(4n - 1)\pi i$

19. $|z - 2| = \text{Re}(z)$ represents :

(1) a parabola (2) a hyperbola

(3) a circle (4) an ellipse

20. If C is the circle $|z| = 2$, then using Cauchy's integral formula for derivatives, $\int_C \frac{\sin z}{z^4} dz =$

(1) $\frac{\pi i}{3}$ (2) $\frac{-\pi i}{3}$

(3) $\frac{\pi i}{2}$ (4) $\frac{-\pi i}{4}$

21. For which of the following sets, the set of interior points is not empty ?

(1) N (2) Z (3) R (4) I

22. Which of the following is ***incorrect*** ?
Counter set is :

(1) Closed (2) Countable

(3) Measurable (4) Equivalent to [0, 1]

23. If X is a set of even natural numbers less than 8 and Y is a set of odd prime numbers less than equal to 7, then the number of relations from X to Y is :

(1) 2^8 (2) $2^8 - 1$ (3) 2^9 (4) $2^9 - 1$

24. The sequence

$$S_n = \begin{cases} 2, \text{ when } n \text{ is even} \\ \text{lowest prime factor} (\neq 1) \text{ of } n, \text{when } n \text{ is odd} \end{cases}$$

has limit point :

(1) 2 (2) countable in number

(3) 1, 2, 3, 4, ... (4) uncountable in number

25. $$\lim_{n\to\infty}\left[\frac{n}{n^2}+\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\ldots\ldots\right]=$$

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{2}$ (4) 0

26. If $\{a_n\}$ is a sequence of elements in the interval $(-1, 1)$, then which of the following is ***true*** ?

(1) Every limit point of $\{a_n\}$ is in $(-1, 1)$

(2) Every limit point of $\{a_n\}$ is in $[-1, 1]$

(3) Limit points of $\{a_n\}$ can only be $\{-1, 0, 1\}$

(4) None of these

27. The infinite series $\sum \frac{\lfloor n \, x^n}{n^n}$ diverges for :

(1) $x < e$

(2) $x > e$

(3) $x < \frac{1}{e}$

(4) for all values of x

28. The function $f(x) = 2x^3 + 9x^2 + 12x - 7$ is decreasing in the interval :

(1) $(-1, 2)$

(2) $(-2, 1)$

(3) $(1, 2)$

(4) $(-2, -1)$

29. If $f(x + y) = f(x) f(y) \; \forall \; x, y$ and $f(5) = -2$, $f'(0) = 3$, then the value of $f'(5)$ is :

(1) 4

(2) −4

(3) 6

(4) −6

30. The series $\sum \frac{\sin(x^2 + n^2 x)}{n(n+1)}$ is convergent uniformly for :

(1) Countable real numbers only

(2) Finite real numbers only

(3) All real numbers

(4) None of these

31. Which of the following is ***true*** ?

(1) In a moderately asymmetrical distribution, Median = 3 Mode – 2 Mean

(2) Mode is always greater than mean

(3) The Median is most affected by the extreme values

(4) Median lies in between Mean and Mode

C

32. If f_1 and f_2 are p.d.f.'s and $\theta_1 + \theta_2 = 1$, then $g(x) = \theta_1 f_1(x) + \theta_2 f_2 = (x)$

(1) is a p.d.f.

(2) has the value 2

(3) is a c.d.f.

(4) can never the p.d.f.

33. The joint probability density function of a two dimensional random variable (X, Y) is given by :

$$f(x,y) = \begin{cases} 2 , & 0 < x < 1, 0 < y < x \\ 0 , & \text{elsewhere} \end{cases}$$

Then, the marginal density function of Y is :

(1) $2(1 - Y)$

(2) $2(1 + Y)$

(3) $2Y$

(4) 2

34. The following LPP has the multiple optimal solutions :

Max. : $Z = x + 3y$

Subject to :

$$2x + y \leq 10$$

$$x + 3y \leq 15$$

$$x, y \geq 0$$

One of the points that gives optimal solution for the LPP is :

(1) (5.1, 0)

(2) (2.7, 4.1)

(3) (9.2, 2.1)

(4) (2, 1)

35. Let $\{X_n, n \geq 0\}$ be a Markov Chain with three states 0, 1, 2 and with transition matrix $\begin{pmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{pmatrix}$ and the initial distribution $P\{X_0 = i\} = \frac{1}{3}$, $i = 0, 1, 2.$

Then $P\{X_2 = 1 | X_0 = 0\}$ is :

(1) $\frac{1}{2}$ (2) $\frac{5}{8}$

(3) $\frac{3}{16}$ (4) $\frac{5}{16}$

36. In a binomial distribution consisting of 4 independent trials, probabilities of 1 and 2 successes are 0.4 and 0.2 respectively. Then the parameter p, then probability of success in each trial of the distribution is equal to :

(1) 0.25 (2) 0.4

(3) 0.5 (4) 0.75

37. For a Poisson variate X, $E(X^2) = 6$, then the value of $E(X)$ will be :

(1) 4 (2) 3

(3) 2 (4) $\sqrt{6}$

38. The interval between two successive occurrences of a Poisson process $\{N(t), t \geq 0\}$ having parameter λ has :

(1) Poisson distribution with mean μ

(2) Poisson distribution with mean $\frac{1}{\mu}$

(3) Negative exponential distribution with mean μ

(4) Negative exponential distribution with mean $\frac{1}{\mu}$

39. With 0.8 as the traffic intensity, the expected number of customers in M|M|1 system is :

(1) 4 (2) 5

(3) $\frac{20}{9}$ (4) 3.2

40. A symmetric die is thrown 3600 times. The lower bound for the probability of getting 500 to 700 sixes is :

(1) $\frac{199}{200}$ (2) $\frac{99}{100}$

(3) $\frac{49}{50}$ (4) $\frac{19}{20}$

41. If $y(x)$ is the solution of $y'(x) = ay - by^2$; $a, b > 0, y(0) = y_0$, then the limiting value of $y(x)$, as $x \to \infty$, will be :

(1) $\frac{b}{a}$ (2) $\frac{a}{b}$

(3) 0 (4) y_0

42. If $u(x, t)$ be the solution of the I. V. P. $u_{tt} = u_{xx}; u(x, 0) = x^3$, $u_t(x, 0) = \sin x$, then the value of $u(\pi, \pi)$ is :

(1) $2\pi^2$ (2) $2\pi^3$

(3) $4\pi^2$ (4) $4\pi^3$

43. The order of convergence of Newton-Raphson method is :

(1) linear (2) quadratic

(3) cubic (4) exponential

44. Another name of Hermite's interpolation formula is :

(1) Osculating interpolation formula

(2) Critical interpolation formula

(3) Spline interpolation formula

(4) Lagrange's interpolation formula

45. The second order Runge-Kutta method is applied to the initial value problem $\frac{dy}{dx} = -y, y(0) = y_0$ with step size h, then $y(h) =$

(1) $\frac{y_0}{2}\left(h^2 + 2h - 2\right)$

(2) $\frac{y_0}{2}\left(h^2 - 3h + 6\right)$

(3) $\frac{y_0}{2}\left(y^2 - 2h + 2\right)$

(4) $\frac{y_0}{6}\left(y^2 - 4h + 2\right)$

46. Lagrange's interpolation formula is useful in finding any value of a function when the given values of the independent variable are :

(1) positive

(2) non-zero

(3) equidistant

(4) not equidistant

47. On what curve the function $I = \int_0^1 \left[\left(\frac{dy}{dx}\right)^2 + 12xy\right] dx$ with $y(0) = 0$, $y(1) = 1$ can be extremized ?

(1) $y = x^2$

(2) $y = x^3$

(3) $y = x^4$

(4) $y = e^x$

48. The extremals of $\int_a^b \frac{1}{x}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$ are :

(1) circles
(2) parabole
(3) ellipses
(4) hyperbole

49. The result obtained on simplification of the Euler-Lagrange equation is known as :

(1) Cauchy identity
(2) Beltrani identity
(3) Hamilton identity
(4) Liouville identity

50. For the Kernel $k(x, t) = 1$, $a = 0$, $b = 1$; $D(\lambda) =$

(1) $1-\frac{\lambda}{2}$
(2) $\lambda - 1$
(3) $1+\lambda$
(4) $1-\lambda$

51. Residue of $f(z) = z\cos\frac{1}{z}$ at $z = 0$ is :

(1) -1
(2) 1
(3) $\frac{1}{2}$
(4) $-\frac{1}{2}$

52. If $f(z) = u(x, y) + iv(x, y)$ is analytic, then $f'(z) =$

(1) $\frac{\partial u}{\partial x}+i\frac{\partial v}{\partial x}$
(2) $\frac{\partial u}{\partial y}-i\frac{\partial v}{\partial x}$
(3) $\frac{\partial u}{\partial x}-i\frac{\partial v}{\partial y}$
(4) $\frac{\partial u}{\partial x}-i\frac{\partial v}{\partial x}$

53. If the principal part of Laurent's expansion of $f(z)$ at a point, has infinite number of non-zero terms, then the singularity of $f(z)$ at that point is :

(1) removable
(2) a pole
(3) essential
(4) non-essential

54. The mapping $f(z) = ze^{z^2-2}$ is not conformal at $z =$

(1) $\pm\sqrt{2}i$ (2) $\pm\frac{1}{\sqrt{2}}$ (3) $\pm\frac{i}{\sqrt{2}}$ (4) $\pm\frac{3i}{\sqrt{2}}$

55. If C denotes the unit circle centred at the origin in the Argant plane, then the value of the integral $\int_C |1+z+z^2|^2\, dz$, taken anticlockwise along C, is equal to :

(1) 0 (2) πi (3) $2\pi i$ (4) $4\pi i$

56. The number of roots of the equation $z^5 - 12z^2 + 14 = 0$ those lying in the annulus $2 \le |z| < \frac{5}{2}$, is :

(1) 3 (2) 1 (3) 2 (4) 0

57. In how many ways can a person can invite his 5 friends for dinner ?

(1) 25 (2) 27 (3) 31 (4) 32

58. The number of integers between 1 and 1000 that are divisible by 2 or 3, are :

(1) 343 (2) 367 (3) 372 (4) 377

59. When 2^{50} is divided by 7, the remainder is :

(1) 4 (2) 5 (3) 2 (4) 3

60. For any integer n > 2, which of the following is true for the Euler's function $\phi(n)$?

(1) $\phi(n)$ is even (2) $\phi(n)$ is odd

(3) $\phi(n)$ is rational (4) $\phi(n)$ is prime

61. Solution of the integral equation $\int_0^x \frac{y(t)}{x-t}\, dt = \sqrt{x}$ is :

(1) $y = 1$ (2) $y = \sqrt{2}$

(3) $y = \frac{1}{2}$ (4) $y = \frac{3}{2}$

62. The eigen values λ of the integral equation $y(x) - \lambda \int_0^{2\pi} \sin(x+t) y(t)\, dt$, are :

(1) $\frac{1}{\pi}, -\frac{1}{\pi}$

(2) $\frac{\pi}{2}, -\frac{\pi}{2}$

(3) $\pi, -\frac{\pi}{2}$

(4) $\pi, -\pi$

63. A load slides without friction on a wire in the slope of the cycloid having equation $x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$, $0 \le \theta \le 2\pi$. Then the Lagrangian is :

(1) $ma^2(1-\cos\theta)\dot{\theta}^2 - mga(1+\cos\theta)$

(2) $ma^2(1-\cos\theta)\dot{\theta}^2 + mga(1+\cos\theta)$

(3) $ma^2(1+\cos\theta)\dot{\theta}^2 - mga(1-\cos\theta)$

(4) $ma^2(1+\cos\theta)\dot{\theta}^2 - mga(1-\sin\theta)$

64. The generalized displacement of a rigid body is a translation with rotation. This result is known as :

(1) Law of displacement

(2) Euler's theorem

(3) Chasle's theorem

(4) Law of rotation

65. Equation of constraints that does not contain time as explicit variable, is referred to as :

(1) holonomic

(2) non-holonomic

(3) scleronomous

(4) rhenomous

66. Two lines of regression are $x = -\frac{1}{18}y + \mu_1$ and $y = -2x + \mu_2$; (μ_1, μ_2) being unknown and the mean of the distribution is (1, 18). Estimated value of y when $x = 10$ is :

(1) 2
(2) 0
(3) −18
(4) −144

67. If partial correlation coefficient $r_{12.3} = 0$, then :

(1) $r_{12} = r_{13} r_{23}$
(2) $r_{23} = r_{21} r_{13}$
(3) $r_{12} = 1$
(4) $r_{31} = r_{12} r_{23}$

68. If A and B are independent events such that $P(\overline{A}) = 0.7$, $P(\overline{B}) = K$ and $P(A \cup B) = 0.8$, then $K =$

(1) 0
(2) 1
(3) $\frac{2}{7}$
(4) $\frac{5}{7}$

69. If $(X, Y) \sim B \vee N(0, 0, 1, 1, 0.8)$, then $1 + 2X + 3Y$ is distributed as :

(1) $N(0, 1)$
(2) $N(1, 13)$
(3) $N(1, 19)$
(4) $N(0, 19)$

70. In case of simple random sampling with replacement, the variance of the estimate of population mean is :

(1) $\frac{N-n}{nN}\sigma^2$
(2) $\frac{N-n}{nN}\frac{N-1}{N}\sigma^2$
(3) $\frac{\sigma^2}{nN}$
(4) $\frac{\sigma^2}{n}$

71. If $X_1, X_2, ..., X_n$ be a random sample from a population with constant density, each X_i is exponential with parameter λ. Then, min $(X_1, X_2, ..., X_n)$ is :

(1) exponential with parameter λ

(2) exponential with parameter $n\lambda$

(3) poisson with parameter λ

(4) poisson with parameter $n\lambda$

72. Let $X_1, X_2, ..., X_{10}$ be 10 i.i.d. variates, each with p.d.f. $f(x)$ and c.d.f. $F(x)$. Then p.d.f. of the smallest order statistic is :

(1) $10[F(x)]^9 f(x)$

(2) $[F(x)]^9 f(x)$

(3) $10[1 - F(x)]^9 f(x)$

(4) $1 - [1 - F(x)]^{10}$

73. Number of observations saved in a 4 ×4 L. S. D. over a complete 3-way layout is :

(1) 64

(2) 48

(3) 24

(4) 16

74. Let $X_1, X_2, ..., X_n$ be a random sample from the uniform distribution with p.d.f. :

$$f(x,\theta) = \begin{cases} \frac{1}{\theta} & , \quad 0 < x < \infty, \theta > 0 \\ 0 & , \quad \text{elesewhere} \end{cases}$$

Then the maximum likelihood estimator for θ is :

(1) the same mean

(2) the same median

(3) the largest sample observation

(4) the smallest sample observation

75. If $x \geq 1$ is the critical region for testing $H_0 : \theta = 2$ against the alternative $H_1 : \theta = 1$, on the basis of single observation from the population $F(x, \theta) = \theta \exp(-\theta x)$, $0 \leq x < \infty$. Then the power of the test is :

(1) $\frac{1}{e}$ (2) $\frac{1}{e^2}$

(3) $\frac{e-1}{e}$ (4) $\frac{e-1}{e^2}$

76. Let X_α be an observation from $N(\beta z_\alpha, \Sigma)$, $\alpha = 1, 2, \ldots\ldots, N$ with $(z_1, z_2, \ldots\ldots z_N)$ *of* rank q. Let $C = \sum_{\alpha=1}^{N} x_\alpha Z'_\alpha$ and $A = \sum_{\alpha=1}^{N} Z_\alpha Z'_\alpha$. Then MLE of β is :

(1) CA (2) AC^{-1}

(3) $C^{-1}A$ (4) CA^{-1}

77. A system is composed of three identical independent elements in parallel, each having the reliability 0.7, then the reliability of the system is :

(1) 0.973 (2) 0.793

(3) 0.73 (4) 0.27

78. In case of large sample single tailed test, the magnitude of the critical value of Z at 5 percent level of significance is :

(1) 2.58 (2) 1.327

(3) 1.645 (4) 1.782

79. If there are 10 symbols of two types, equal in number, the maximum possible runs is :

(1) 5 (2) 10

(3) 9 (4) 25

80. Which one problem out of the following is *not* related to stratified sampling ?

(1) Fixing the points of demarcation between strata

(2) Fixing the criterion for stratification

(3) Fixing the number of strata

(4) Fixing the sample size

81. The abscissa of the point at which tangent to the curve $y = x\,(x-1)$ is parallel to the chord joining the extremities of the curve in the interval [1, 2] is :

(1) $\frac{3}{2}$ (2) $\frac{3}{4}$

(3) $\frac{4}{3}$ (4) $\frac{2}{3}$

82. The integral $\int_0^\infty \frac{\sin x}{x} dx$:

(1) converges absolutely (2) converges but not absolutely

(3) does not converges (4) does not exist

83. The function $f(x,y) = x^4 + x^2y + y^2$ at the point (0, 0) is :

(1) Maximum (2) Minimum

(3) Harmonic (4) Not defined

84. If X is a complete metric space and E is non-empty open subset of X, then E is :

(1) of first category (2) of second category

(3) a null set (4) complete

85. If $X = \{ x : 0 < d\,(0, x) \le 1 \text{ and } x \in R^2 \}$, where d is the usual metric on X, then which of the following is *not* true ?

(1) X is closed (2) X is bounded

(3) X is compact (4) X is not compact

86. If $\alpha > 0$ and $\beta > 0$ and $f(x) = \begin{cases} x^{\alpha} \sin \frac{1}{x^{\beta}} & , \quad x \neq 0 \\ 0 & , \quad x = 0 \end{cases}$ then $f(x)$ is of bounded variation in [0, 1] if :

(1) $\alpha + \beta = 1$

(2) $\alpha = \beta$

(3) $\alpha < \beta$

(4) $\alpha > \beta$

87. A function $f: R \to R$ need not be Lebesgue measurable if :

(1) f is monotone

(2) $\{x \in R : f(x) = \alpha\}$ is measurable for each $\alpha \in R$

(3) $\{x \in R : f(x) \geq \alpha\}$ is measurable for each $\alpha \in Q$

(4) for each open set G in R, $f^{-1}(G)$ is measurable

88. If $f(x, y) = \frac{x^2}{y^2}, (x, y) \in \left[\frac{1}{2}, \frac{3}{2}\right] \times \left[\frac{1}{2}, \frac{3}{2}\right]$, then the derivative of f at (1, 1) along the direction (1, 1) is :

(1) 0

(2) $\frac{1}{2}$

(3) 1

(4) 2

89. If V is a vector space of dimension 100 and A and B are two subspaces of dimensions 60 and 63, then minimum dimension of $A \cap B$ is :

(1) 60

(2) 40

(3) 37

(4) 23

90. For the matrix $A = \begin{bmatrix} 2 & -2 & 3 \\ -2 & -1 & 6 \\ 1 & 2 & 0 \end{bmatrix}$, one of the eigen values is 3, the other two eigen values are :

(1) 2, 5

(2) –3, 5

(3) 3, –5

(4) 2, –5

91. The solution of $\frac{dy}{dx} = \frac{y}{x} + x\tan\frac{y}{x}$ is :

(1) $\log\tan\frac{y}{2x} = x + c$
(2) $\log\tan\frac{y}{x} = x + c$
(3) $\log\cot\frac{y}{x} = x + c$
(4) $\log\cot\frac{y}{2x} = x + c$

92. The I. V. P. $x\frac{dy}{dx} = y$, $y(0) = 0$, $x \geq 0$ has :

(1) No solution
(2) Exactly two solutions
(3) A unique solution
(4) Infinitely many solutions

93. Solution of $y = xp - p^2$ is :

(1) $y = 3x - c$
(2) $y = \log x + c$
(3) $y = cx - c^2$
(4) $y = cx - cx^2$

94. Solving $y'' - 2y' + y = e^x \log x$ by variation of parameters, the value of Wronskion is :

(1) xe^{-2x}
(2) e^{-2x}
(3) e^{2x}
(4) xe^{2x}

95. For the system $\frac{dy}{dt} = x - xy$, $\frac{dy}{dt} = xy - y$, the critical point (0, 0) is :

(1) centre
(2) saddle point
(3) spiral point
(4) node

96. The Green's function $G(x, t)$:

(1) is not defined at $x = t$
(2) is continuous at $x = t$
(3) is discontinuous at $x = t$
(4) is harmonic in the xt-plane

97. For the P. D. E $r - 2s + t = \cos(2x + 3y)$, P. I. is :

(1) $\cos(2x + 3y)$
(2) $-3\cos(2x + 3y)$
(3) $-2\cos(2x + 3y)$
(4) $-\cos(2x + 3y)$

98. The P. D. E $yu_{xx} + 2xyu_{xy} + xu_{yy} = u_x + u_y$ is hyperbolic in the region :

(1) $xy > 1$

(2) $xy < 1$

(3) $xy > 0$

(4) $0 < xy < 1$

99. The eigen values of a SLP are :

(1) real and finite

(2) real and positive

(3) real and non-zero

(4) real and non-negative

100. The general solution of $u_{xx} + u_{yy} = 0$ is of the form :

(1) $u = f(x + iy)$

(2) $u = f(x + y) + g(x - y)$

(3) $u = f(x + iy) + g(x - iy)$

(4) $u = f(x + iy) f(x + iy)$

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SET-Y

M.Phil./Ph.D./URS-EE-2020

SUBJECT : Mathematics

10184

Sr. No. ....

Time : **1¼ Hours** Max. Marks : **100** Total Questions : **100**

Roll No. (in figures)____________ (in words) ____________

Name ____________ Father's Name ____________

Mother's Name ____________ Date of Examination ____________

(Signature of the Candidate) (Signature of the Invigilator)

CANDIDATES MUST READ THE FOLLOWING INFORMATION/INSTRUCTIONS BEFORE STARTING THE QUESTION PAPER.

1. ***Candidates are required to attempt any 75 questions out of the given 100 multiple choice questions of 4/3 marks each. No credit will be given for more than 75 correct responses.***
2. The candidates ***must return*** the question booklet as well as OMR Answer-Sheet to the Invigilator concerned before leaving the Examination Hall, failing which a case of use of unfair-means / mis-behaviour will be registered against him / her, in addition to lodging of an FIR with the police. Further the answer-sheet of such a candidate will not be evaluated.
3. Keeping in view the transparency of the examination system, carbonless OMR Sheet is provided to the candidate so that a copy of OMR Sheet may be kept by the candidate.
4. Question Booklet along with answer key of all the A, B, C & D code will be got uploaded on the university website after the conduct of Entrance Examination. In case there is any discrepancy in the Question Booklet/Answer Key, the same may be brought to the notice of the Controller of Examinations in writing/through E. Mail within 24 hours of uploading the same on the University Website. Thereafter, no complaint in any case, will be considered.
5. The candidate ***must not*** do any rough work or writing in the OMR Answer-Sheet. Rough work, if any, may be done in the question booklet itself. Answers ***must not*** be ticked in the question booklet.
6. ***There will be no negative marking. Each correct answer will be awarded 4/3 mark. Cutting, erasing, overwriting and more than one answer in OMR Answer-Sheet will be treated as incorrect answer.***
7. Use only **Black** or **Blue Ball Point Pen** of good quality in the OMR Answer-Sheet.
8. *Before answering the questions, the candidates should ensure that they have been supplied correct and complete booklet. Complaints, if any, regarding misprinting etc. will not be entertained* **30** *minutes after starting of the examination.*

1. The abscissa of the point at which tangent to the curve $y = x\ (x - 1)$ is parallel to the chord joining the extremities of the curve in the interval [1, 2] is :

(1) $\frac{3}{2}$ (2) $\frac{3}{4}$

(3) $\frac{4}{3}$ (4) $\frac{2}{3}$

2. The integral $\int_0^\infty \frac{\sin x}{x} dx$:

(1) converges absolutely (2) converges but not absolutely

(3) does not converges (4) does not exist

3. The function $f(x,y) = x^4 + x^2 y + y^2$ at the point (0, 0) is :

(1) Maximum (2) Minimum

(3) Harmonic (4) Not defined

4. If X is a complete metric space and E is non-empty open subset of X, then E is :

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5. If $X = \{\ x : 0 < d\ (0, x) \leq 1$ and $x \in R^2\}$, where d is the usual metric on X, then which of the following is ***not*** true ?

(1) X is closed (2) X is bounded

(3) X is compact (4) X is not compact

6. If $\alpha > 0$ and $\beta > 0$ and $f(x) = \begin{cases} x^\alpha \sin \frac{1}{x^\beta} & , \quad x \neq 0 \\ 0 & , \quad x = 0 \end{cases}$ then $f(x)$ is of bounded variation in [0, 1] if :

(1) $\alpha + \beta = 1$ (2) $\alpha = \beta$

(3) $\alpha < \beta$ (4) $\alpha > \beta$

D

7. A function $f : R \to R$ need not be Lebesgue measurable if :

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(2) $\{x \in \mathrm{R} : f(x) = \alpha\}$ is measurable for each $\alpha \in \mathrm{R}$

(3) $\{x \in \mathrm{R} : f(x) \geq \alpha\}$ is measurable for each $\alpha \in \mathrm{Q}$

(4) for each open set G in R, $f^{-1}(G)$ is measurable

8. If $f(x,y) = \frac{x^2}{y^2}, (x,y) \in \left[\frac{1}{2}, \frac{3}{2}\right] \times \left[\frac{1}{2}, \frac{3}{2}\right]$, then the derivative of f at (1, 1) along the direction (1, 1) is :

(1) 0 (2) $\frac{1}{2}$ (3) 1 (4) 2

9. If V is a vector space of dimension 100 and A and B are two subspaces of dimensions 60 and 63, then minimum dimension of $A \cap B$ is :

(1) 60 (2) 40 (3) 37 (4) 23

10. For the matrix $A = \begin{bmatrix} 2 & -2 & 3 \\ -2 & -1 & 6 \\ 1 & 2 & 0 \end{bmatrix}$, one of the eigen values is 3, the other two eigen values are :

(1) 2, 5 (2) –3, 5

(3) 3, –5 (4) 2, –5

11. If $X_1, X_2, \ldots, X_n$ be a random sample from a population with constant density, each X_i is exponential with parameter λ. Then, min $(X_1, X_2, \ldots, X_n)$ is :

(1) exponential with parameter λ

(2) exponential with parameter $n\lambda$

(3) poisson with parameter λ

(4) poisson with parameter $n\lambda$

12. Let $X_1, X_2, ..., X_{10}$ be 10 i.i.d. variates, each with p.d.f. $f(x)$ and c.d.f. $F(x)$. Then p.d.f. of the smallest order statistic is :

(1) $10[F(x)]^9 f(x)$ (2) $[F(x)]^9 f(x)$

(3) $10[1-F(x)]^9 f(x)$ (4) $1-[1-F(x)]^{10}$

13. Number of observations saved in a 4 ×4 L. S. D. over a complete 3-way layout is :

(1) 64 (2) 48

(3) 24 (4) 16

14. Let $X_1, X_2, ..., X_n$ be a random sample from the uniform distribution with p.d.f. :

$$f(x,\theta) = \begin{cases} \frac{1}{\theta} & , \quad 0 < x < \infty, \theta > 0 \\ 0 & , \quad \text{elesewhere} \end{cases}$$

Then the maximum likelihood estimator for θ is :

(1) the same mean

(2) the same median

(3) the largest sample observation

(4) the smallest sample observation

15. If $x \geq 1$ is the critical region for testing $H_0 : \theta = 2$ against the alternative $H_i : \theta = 1$, on the basis of single observation from the population $F(x, \theta) = \theta \exp(-\theta x)$, $0 \leq x < \infty$. Then the power of the test is :

(1) $\frac{1}{e}$ (2) $\frac{1}{e^2}$

(3) $\frac{e-1}{e}$ (4) $\frac{e-1}{e^2}$

16. Let X_α be an observation from $N(\beta z_\alpha, \Sigma)$, $\alpha = 1, 2, \ldots\ldots, N$ with $(z_1, z_2, \ldots\ldots z_N)$ *of* rank q. Let $C = \sum_{\alpha=1}^{N} x_\alpha Z'_\alpha$ and $A = \sum_{\alpha=1}^{N} Z_\alpha Z'_\alpha$. Then MLE of β is :

(1) CA

(2) AC^{-1}

(3) $C^{-1}A$

(4) CA^{-1}

17. A system is composed of three identical independent elements in parallel, each having the reliability 0.7, then the reliability of the system is :

(1) 0.973

(2) 0.793

(3) 0.73

(4) 0.27

18. In case of large sample single tailed test, the magnitude of the critical value of Z at 5 percent level of significance is :

(1) 2.58

(2) 1.327

(3) 1.645

(4) 1.782

19. If there are 10 symbols of two types, equal in number, the maximum possible runs is :

(1) 5

(2) 10

(3) 9

(4) 25

20. Which one problem out of the following is ***not*** related to stratified sampling ?

(1) Fixing the points of demarcation between strata

(2) Fixing the criterion for stratification

(3) Fixing the number of strata

(4) Fixing the sample size

21. Solution of the integral equation $\int_0^x \frac{y(t)}{x-t}\,dt = \sqrt{x}$ is :

(1) $y = 1$

(2) $y = \sqrt{2}$

(3) $y = \frac{1}{2}$

(4) $y = \frac{3}{2}$

22. The eigen values λ of the integral equation $y(x) - \lambda \int_0^{2\pi} \sin(x+t)y(t)\,dt$, are :

(1) $\frac{1}{\pi}, -\frac{1}{\pi}$

(2) $\frac{\pi}{2}, -\frac{\pi}{2}$

(3) $\pi, -\frac{\pi}{2}$

(4) $\pi, -\pi$

23. A load slides without friction on a wire in the slope of the cycloid having equation $x = a(\theta - \sin\theta), y = a(1 + \cos\theta), 0 \leq \theta \leq 2\pi$. Then the Lagrangian is :

(1) $ma^2(1-\cos\theta)\dot{\theta}^2 - mga(1+\cos\theta)$

(2) $ma^2(1-\cos\theta)\dot{\theta}^2 + mga(1+\cos\theta)$

(3) $ma^2(1+\cos\theta)\dot{\theta}^2 - mga(1-\cos\theta)$

(4) $ma^2(1+\cos\theta)\dot{\theta}^2 - mga(1-\sin\theta)$

24. The generalized displacement of a rigid body is a translation with rotation. This result is known as :

(1) Law of displacement

(2) Euler's theorem

(3) Chasle's theorem

(4) Law of rotation

25. Equation of constraints that does not contain time as explicit variable, is referred to as :

(1) holonomic (2) non-holonomic

(3) scleronomous (4) rhenomous

26. Two lines of regression are $x = -\frac{1}{18}y + \mu_1$ and $y = -2x + \mu_2$; (μ_1, μ_2) being unknown and the mean of the distribution is (1, 18). Estimated value of y when $x = 10$ is :

(1) 2 (2) 0 (3) −18 (4) −144

27. If partial correlation coefficient $r_{12.3} = 0$, then :

(1) $r_{12} = r_{13}r_{23}$ (2) $r_{23} = r_{21}r_{13}$

(3) $r_{12} = 1$ (4) $r_{31} = r_{12}r_{23}$

28. If A and B are independent events such that $P(\overline{A}) = 0.7$, $P(\overline{B}) = K$ and $P(A \cup B) = 0.8$, then $K =$

(1) 0 (2) 1

(3) $\frac{2}{7}$ (4) $\frac{5}{7}$

29. If $(X, Y) \sim B \vee N(0, 0, 1, 1, 0.8)$, then $1 + 2X + 3Y$ is distributed as :

(1) $N(0, 1)$ (2) $N(1, 13)$

(3) $N(1, 19)$ (4) $N(0, 19)$

30. In case of simple random sampling with replacement, the variance of the estimate of population mean is :

(1) $\frac{N-n}{nN}\sigma^2$ (2) $\frac{N-n}{nN}\frac{N-1}{N}\sigma^2$

(3) $\frac{\sigma^2}{nN}$ (4) $\frac{\sigma^2}{n}$

31. The solution of $\frac{dy}{dx} = \frac{y}{x} + x\tan\frac{y}{x}$ is :

(1) $\log\tan\frac{y}{2x} = x + c$

(2) $\log\tan\frac{y}{x} = x + c$

(3) $\log\cot\frac{y}{x} = x + c$

(4) $\log\cot\frac{y}{2x} = x + c$

32. The I. V. P. $x\frac{dy}{dx} = y$, $y(0) = 0, x \geq 0$ has :

(1) No solution

(2) Exactly two solutions

(3) A unique solution

(4) Infinitely many solutions

33. Solution of $y = xp - p^2$ is :

(1) $y = 3x - c$

(2) $y = \log x + c$

(3) $y = cx - c^2$

(4) $y = cx - cx^2$

34. Solving $y'' - 2y' + y = e^x \log x$ by variation of parameters, the value of Wronskion is :

(1) xe^{-2x}

(2) e^{-2x}

(3) e^{2x}

(4) xe^{2x}

35. For the system $\frac{dy}{dt} = x - xy$, $\frac{dy}{dt} = xy - y$, the critical point (0, 0) is :

(1) centre

(2) saddle point

(3) spiral point

(4) node

36. The Green's function $G(x, t)$:

(1) is not defined at $x = t$

(2) is continuous at $x = t$

(3) is discontinuous at $x = t$

(4) is harmonic in the xt-plane

37. For the P. D. E $r - 2s + t = \cos(2x + 3y)$, P. I. is :

(1) $\cos(2x + 3y)$ (2) $-3\cos(2x + 3y)$

(3) $-2\cos(2x + 3y)$ (4) $-\cos(2x + 3y)$

38. The P. D. E $yu_{xx} + 2xyu_{xy} + xu_{yy} = u_x + u_y$ is hyperbolic in the region :

(1) $xy > 1$ (2) $xy < 1$

(3) $xy > 0$ (4) $0 < xy < 1$

39. The eigen values of a SLP are :

(1) real and finite (2) real and positive

(3) real and non-zero (4) real and non-negative

40. The general solution of $u_{xx} + u_{yy} = 0$ is of the form :

(1) $u = f(x + iy)$ (2) $u = f(x + y) + g(x - y)$

(3) $u = f(x + iy) + g(x - iy)$ (4) $u = f(x + iy) f(x + iy)$

41. Residue of $f(z) = z\cos\frac{1}{z}$ at $z = 0$ is :

(1) -1 (2) 1 (3) $\frac{1}{2}$ (4) $-\frac{1}{2}$

42. If $f(z) = u(x, y) + iv(x, y)$ is analytic, then $f'(z) =$

(1) $\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$ (2) $\frac{\partial u}{\partial y} - i\frac{\partial v}{\partial x}$

(3) $\frac{\partial u}{\partial x} - i\frac{\partial v}{\partial y}$ (4) $\frac{\partial u}{\partial x} - i\frac{\partial v}{\partial x}$

43. If the principal part of Laurent's expansion of $f(z)$ at a point, has infinite number of non-zero terms, then the singularity of $f(z)$ at that point is :

(1) removable (2) a pole (3) essential (4) non-essential

44. The mapping $f(z) = ze^{z^2 - 2}$ is not conformal at $z =$

(1) $\pm\sqrt{2}i$ (2) $\pm\frac{1}{\sqrt{2}}$ (3) $\pm\frac{i}{\sqrt{2}}$ (4) $\pm\frac{3i}{\sqrt{2}}$

45. If C denotes the unit circle centred at the origin in the Argant plane, then the value of the integral $\int_C |1+z+z^2|^2 \, dz$, taken anticlockwise along C, is equal to :

(1) 0 (2) πi (3) $2\pi i$ (4) $4\pi i$

46. The number of roots of the equation $z^5 - 12z^2 + 14 = 0$ those lying in the annulus $2 \le |z| < \frac{5}{2}$, is :

(1) 3 (2) 1 (3) 2 (4) 0

47. In how many ways can a person can invite his 5 friends for dinner ?

(1) 25 (2) 27 (3) 31 (4) 32

48. The number of integers between 1 and 1000 that are divisible by 2 or 3, are :

(1) 343 (2) 367 (3) 372 (4) 377

49. When 2^{50} is divided by 7, the remainder is :

(1) 4 (2) 5 (3) 2 (4) 3

50. For any integer $n > 2$, which of the following is true for the Euler's function $\phi(n)$?

(1) $\phi(n)$ is even (2) $\phi(n)$ is odd

(3) $\phi(n)$ is rational (4) $\phi(n)$ is prime

51. Let A be a non-zero upper triangular matrix whose all eigen values are zero, then $I + A$ is :

(1) Singular (2) Invertible

(3) Idempotent (4) Nilpotent

52. Let T be a linear transformation on a vector space V such that $T^2 - T + 1 = 0$, then T is :

(1) Idempotent
(2) Singular
(3) Invertible
(4) Not invertible

53. The dimension of the subspace of R^3 spaned by (–3, 0, 1), (1, 2, 1) and (3, 0, –1) is :

(1) 2
(2) 3
(3) 0
(4) 1

54. Let $A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 2 & 3 & a \end{pmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 3 \\ b \end{bmatrix}$, then the system $AX = B$ over the field of real numbers has :

(1) Infinite number of solutions if $a \neq 2$

(2) Infinite number of solutions if $a = 2$ and $b \neq 7$

(3) No solution if $b \neq 7$

(4) Unique solution if $a \neq 2$

55. If the matrix of the quadratic form $(x_1 - x_2 + 2x_3)^2$ is denoted by A, then the trace of A is :

(1) 4
(2) 6
(3) 9
(4) 3

56. $\lim\limits_{z \to 0} \frac{\bar{z}}{z}$ is equal to :

(1) 0
(2) 1
(3) i
(4) does not exist

57. If $w = u + iv$, $z = x + iy$, then the image of the line $x = -3$ under the mapping $w = z^2$ is :

(1) $u = -3v + 1$
(2) $u = 3 - \frac{v^2}{4}$
(3) $u = 9 + \frac{v^2}{36}$
(4) $v = 9 - \frac{u^2}{36}$

58. Solution of $e^{z-1} = -ie^3$ is :

(1) $4 + (2n - 1)\pi i$

(2) $4 + \frac{1}{2}(3n - 1)\pi i$

(3) $4 + \frac{1}{2}(2n - 1)\pi i$

(4) $4 + \frac{1}{2}(4n - 1)\pi i$

59. $|z - 2| = \text{Re}(z)$ represents :

(1) a parabola

(2) a hyperbola

(3) a circle

(4) an ellipse

60. If C is the circle $|z| = 2$, then using Cauchy's integral formula for derivatives, $\int_C \frac{\sin z}{z^4} dz =$

(1) $\frac{\pi i}{3}$

(2) $\frac{-\pi i}{3}$

(3) $\frac{\pi i}{2}$

(4) $\frac{-\pi i}{4}$

61. If N is the set of natural numbers, then under the binary operation $a.b = a + b$, $(N, .)$ is :

(1) group

(2) semi-group

(3) quasi-group

(4) monoid

62. A field having no proper subfield is called :

(1) ordered field

(2) improper field

(3) prime field

(4) modular field

63. Which of the following is ***not*** true ?

(1) Inverse of an odd permutation is odd

(2) Identity permutation is an even permutation

(3) Product of two odd permutations is an even permutation

(4) Every transformation is always an even permutation

64. Which of the following is ***not*** true ?

(1) Every finite integral domain is a field

(2) Intersection of two subrings is a subring

(3) The sum of two subrings of a ring R is a subring of R

(4) A finite commutative ring without zero divisors is a field

65. Given field F and the set M of all 2×2 matrices of the form $\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}$ for $a, b \in F$, then which of the following is ***not*** true ?

(1) M is subring of F

(2) M is left ideal in F

(3) M is right ideal in F

(4) M is right ideal but not left ideal in F

66. The degree of splitting field of $x^4 - 1$ over Q is :

(1) 0 (2) 3 (3) 4 (4) 2

67. Which of the following is ***incorrect*** ?

(1) Every discrete space is regular

(2) Every discrete space is connected

(3) Every discrete space is a Hausdorff space

(4) Every discrete space is not compact

68. If X is a topological space and A is a proper dense open subset of X, then which of the following is ***correct*** ?

(1) If X is compact, then X/A is compact

(2) If X is connected, then A is connected

(3) If X is compact, then A is compact

(4) If X/A is compact, then X is compact

69. If $X = \{a, b, c\}$, $T = \{\phi, X, \{a, c\}, \{b\}\}$, then the topological space (X, T) is :

(1) a connected space

(2) not a connected space

(3) not a compact space

(4) not a Hausdorff space

70. Which of the following is ***not*** correct ?

(1) Every compact is metric space is complete

(2) Every metric space is compact Hausdorff space

(3) A compact Hausdorff space is normal

(4) A compact subspace of a Hausdorff space is closed

71. If $y(x)$ is the solution of $y'(x) = ay - by^2$; $a, b > 0$, $y(0) = y_0$, then the limiting value of $y(x)$, as $x \to \infty$, will be :

(1) $\frac{b}{a}$

(2) $\frac{a}{b}$

(3) 0

(4) y_0

72. If $u(x, t)$ be the solution of the I. V. P. $u_{tt} = u_{xx}; u(x, 0) = x^3$, $u_t(x, 0) = \sin x$, then the value of $u(\pi, \pi)$ is :

(1) $2\pi^2$ (2) $2\pi^3$ (3) $4\pi^2$ (4) $4\pi^3$

73. The order of convergence of Newton-Raphson method is :

(1) linear (2) quadratic

(3) cubic (4) exponential

74. Another name of Hermite's interpolation formula is :

(1) Osculating interpolation formula

(2) Critical interpolation formula

(3) Spline interpolation formula

(4) Lagrange's interpolation formula

75. The second order Runge-Kutta method is applied to the initial value problem $\frac{dy}{dx} = -y, y(0) = y_0$ with step size h, then $y(h) =$

(1) $\frac{y_0}{2}\left(h^2 + 2h - 2\right)$ (2) $\frac{y_0}{2}\left(h^2 - 3h + 6\right)$

(3) $\frac{y_0}{2}\left(y^2 - 2h + 2\right)$ (4) $\frac{y_0}{6}\left(y^2 - 4h + 2\right)$

76. Lagrange's interpolation formula is useful in finding any value of a function when the given values of the independent variable are :

(1) positive (2) non-zero

(3) equidistant (4) not equidistant

77. On what curve the function $I = \int_0^1 \left[\left(\frac{dy}{dx}\right)^2 + 12xy\right] dx$ with $y(0) = 0,\ y(1) = 1$ can be extremized ?

(1) $y = x^2$ (2) $y = x^3$

(3) $y = x^4$ (4) $y = e^x$

78. The extremals of $\int_a^b \frac{1}{x}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$ are :

(1) circles (2) parabole

(3) ellipses (4) hyperbole

79. The result obtained on simplification of the Euler-Lagrange equation is known as :

(1) Cauchy identity (2) Beltrani identity

(3) Hamilton identity (4) Liouville identity

80. For the Kernel $k(x, t) = 1,\ a = 0,\ b = 1;\ D(\lambda) =$

(1) $1 - \frac{\lambda}{2}$ (2) $\lambda - 1$

(3) $1 + \lambda$ (4) $1 - \lambda$

81. For which of the following sets, the set of interior points is not empty ?

(1) N (2) Z (3) R (4) I

82. Which of the following is ***incorrect*** ?
Counter set is :

(1) Closed (2) Countable

(3) Measurable (4) Equivalent to [0, 1]

83. If X is a set of even natural numbers less than 8 and Y is a set of odd prime numbers less than equal to 7, then the number of relations from X to Y is :

(1) 2^8 (2) 2^8-1 (3) 2^9 (4) 2^9-1

84. The sequence

$$S_n = \begin{cases} 2, \text{ when } n \text{ is even} \\ \text{lowest prime factor } (\neq 1) \text{ of } n, \text{when } n \text{ is odd} \end{cases}$$

has limit point :

(1) 2 (2) countable in number

(3) 1, 2, 3, 4, ... (4) uncountable in number

85. $\lim_{n\to\infty}\left[\frac{n}{n^2}+\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+......\right]=$

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{3}$

(3) $\frac{\pi}{2}$ (4) 0

86. If $\{a_n\}$ is a sequence of elements in the interval $(-1, 1)$, then which of the following is ***true*** ?

(1) Every limit point of $\{a_n\}$ is in $(-1, 1)$

(2) Every limit point of $\{a_n\}$ is in $[-1, 1]$

(3) Limit points of $\{a_n\}$ can only be $\{-1, 0, 1\}$

(4) None of these

87. The infinite series $\sum\frac{\underline{|n}\, x^n}{n^n}$ diverges for :

(1) $x < e$ (2) $x > e$

(3) $x < \frac{1}{e}$ (4) for all values of x

88. The function $f(x) = 2x^3 + 9x^2 + 12x - 7$ is decreasing in the interval :

(1) $(-1, 2)$

(2) $(-2, 1)$

(3) $(1, 2)$

(4) $(-2, -1)$

89. If $f(x + y) = f(x) f(y)\ \forall\ x, y$ and $f(5) = -2$, $f'(0) = 3$, then the value of $f'(5)$ is :

(1) 4

(2) −4

(3) 6

(4) −6

90. The series $\sum \frac{\sin(x^2 + n^2 x)}{n(n+1)}$ is convergent uniformly for :

(1) Countable real numbers only

(2) Finite real numbers only

(3) All real numbers

(4) None of these

91. Which of the following is ***true*** ?

(1) In a moderately asymmetrical distribution, Median = 3 Mode – 2 Mean

(2) Mode is always greater than mean

(3) The Median is most affected by the extreme values

(4) Median lies in between Mean and Mode

92. If f_1 and f_2 are p.d.f.'s and $\theta_1 + \theta_2 = 1$, then $g(x) = \theta_1 f_1(x) + \theta_2 f_2 = (x)$

(1) is a p.d.f.

(2) has the value 2

(3) is a c.d.f.

(4) can never the p.d.f.

93. The joint probability density function of a two dimensional random variable (X, Y) is given by :

$$f(x,y) = \begin{cases} 2 & , \quad 0 < x < 1, 0 < y < x \\ 0 & , \quad \text{elsewhere} \end{cases}$$

Then, the marginal density function of Y is :

(1) $2(1 - Y)$

(2) $2(1 + Y)$

(3) $2Y$

(4) 2

94. The following LPP has the multiple optimal solutions :

Max. : $Z = x + 3y$

Subject to :

$$2x + y \leq 10$$

$$x + 3y \leq 15$$

$$x, y \geq 0$$

One of the points that gives optimal solution for the LPP is :

(1) (5.1, 0)

(2) (2.7, 4.1)

(3) (9.2, 2.1)

(4) (2, 1)

95. Let $\{X_n, n \geq 0\}$ be a Markov Chain with three states 0, 1, 2 and with transition matrix $\begin{pmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{pmatrix}$ and the initial distribution $P\{X_0 = i\} = \frac{1}{3}$, $i = 0, 1, 2.$ Then $P\{X_2 = 1 | X_0 = 0\}$ is :

(1) $\frac{1}{2}$ (2) $\frac{5}{8}$

(3) $\frac{3}{16}$ (4) $\frac{5}{16}$

96. In a binomial distribution consisting of 4 independent trials, probabilities of 1 and 2 successes are 0.4 and 0.2 respectively. Then the parameter p, then probability of success in each trial of the distribution is equal to :

(1) 0.25 (2) 0.4

(3) 0.5 (4) 0.75

97. For a Poisson variate X, $E(X^2) = 6$, then the value of $E(X)$ will be :

(1) 4 (2) 3

(3) 2 (4) $\sqrt{6}$

98. The interval between two successive occurrences of a Poisson process $\{N(t), t \geq 0\}$ having parameter λ has :

(1) Poisson distribution with mean μ

(2) Poisson distribution with mean $\frac{1}{\mu}$

(3) Negative exponential distribution with mean μ

(4) Negative exponential distribution with mean $\frac{1}{\mu}$

99. With 0.8 as the traffic intensity, the expected number of customers in M|M|1 system is :

(1) 4

(2) 5

(3) $\frac{20}{9}$

(4) 3.2

100. A symmetric die is thrown 3600 times. The lower bound for the probability of getting 500 to 700 sixes is :

(1) $\frac{199}{200}$

(2) $\frac{99}{100}$

(3) $\frac{49}{50}$

(4) $\frac{19}{20}$

Answer Key for Entrance Examination-2020-2021
M.Phil/Ph.D./URS

Question No.	Code A	Code B	Code C	Code D
1	3	3	2	1
2	2	1	3	2
3	3	1	4	2
4	2	3	3	1
5	1	3	2	3
6	2	2	4	4
7	1	1	4	1
8	4	3	1	1
9	4	3	2	4
10	3	4	2	3
11	1	1	2	2
12	2	4	3	3
13	2	3	1	2
14	1	3	4	3
15	3	2	2	1
16	4	2	4	4
17	1	4	3	1
18	1	1	4	3
19	4	4	1	2
20	3	3	2	4
21	2	4	3	3
22	3	2	2	1
23	1	3	3	1
24	4	3	2	3
25	2	4	1	3
26	4	1	2	2
27	3	3	1	1
28	4	2	4	3
29	1	1	4	3
30	2	1	3	4
31	4	1	4	1
32	2	2	1	4
33	3	2	2	3
34	3	1	2	3
35	4	3	4	2
36	1	4	1	2
37	3	1	3	4
38	2	1	4	1
39	1	4	1	4
40	1	3	4	3
41	2	2	2	4

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42	3	3	4	2
43	4	2	2	3
44	3	3	1	3
45	2	1	3	4
46	4	4	4	1
47	4	1	2	3
48	1	3	1	2
49	2	2	2	1
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63	2	2	1	4
64	1	2	3	3
65	3	4	3	2
66	4	1	2	4
67	2	3	1	4
68	1	4	3	1
69	2	1	3	2
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71	3	2	2	2
72	1	3	3	4
73	1	4	2	2
74	3	3	3	1
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76	2	4	4	4
77	1	4	1	2
78	3	1	3	1
79	3	2	2	2
80	4	2	4	4
81	4	2	1	3
82	1	3	2	2
83	2	1	2	3
84	2	4	1	2
85	4	2	3	1

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86	1	4	4	2
87	3	3	1	1
88	4	4	1	4
89	1	1	4	4
90	4	2	3	3
91	2	3	1	4
92	3	2	4	1
93	2	3	3	2
94	3	2	3	2
95	1	1	2	4
96	4	2	2	1
97	1	1	4	3
98	3	4	1	4
99	2	4	4	1
100	4	3	3	4

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