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PG-EE-2019

SUBJECT : Mathematics Hons. (Five Year)-(SET-X)

A

Sr. No. 10761

Time : **1¼ Hours (75 minutes)** Total Questions : **100** Max. Marks : **100**

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1. The set A has 3 elements and the Set B has 7 elements. The minimum number of elements in the set $A \cup B$ is :
(1) 21 (2) 10 (3) 7 (4) Can not say

2. If A and B are two sets, then $A \cap (A \cup B)^C$ (where 'C' denotes complement) is equal to :
(1) ϕ (2) A (3) B (4) $A - B$

3. Let $A = \{0, 1, 2, 3, 4, 5\}$ and a relation R is defined by xRy such that $2x + y = 10$. Then R^{-1} is :
(1) $\{(4, 3), (2, 4), (5, 0)\}$
(2) $\{(4, 3), (2, 4), (0, 5)\}$
(3) $\{(3, 4), (4, 2), (5, 0)\}$
(4) $\{(3, 4), (4, 2), (0, 5)\}$

4. If $A + C = B$, then $\tan A \tan B \tan C =$
(1) $\tan A + \tan B + \tan C$
(2) $\tan A + \tan B - \tan C$
(3) $\tan B - \tan C - \tan A$
(4) $\tan B + \tan C - \tan A$

5. If $\sin x + \sin^2 x = 1$, then $\cos^8 x + 2 \cos^6 x + \cos^4 x =$
(1) 0 (2) 1 (3) -1 (4) 2

6. If $4 \sin^2 x = 1$, then the values of x are :
(1) $n\pi \pm \frac{\pi}{3}$
(2) $n\pi \pm \frac{\pi}{4}$
(3) $2n\pi \pm \frac{\pi}{6}$
(4) $n\pi \pm \frac{\pi}{6}$

7. If $n \in N$, then $3^{3n} - 26n - 1$ is divisible by :
(1) 4 (2) 3 (3) 9 (4) 15

8. If $z = (K + 3) + i\sqrt{5 - k^2}$, then the locus of z is :
(1) a straight line
(2) a parabola
(3) an ellipse
(4) a circle

9. If 1, w and w^2 are the three cube roots of unity, then the roots of the equation $(x - 1)^3 - 8 = 0$ are :
(1) $2, 2w, 2w^2$
(2) $3, 2w, 2w^2$
(3) $3, 1 + 2w, 1 + 2w^2$
(4) $2, 1 - 2w, 1 - 2w^2$

10. The smallest positive integer n for which $\left(\frac{1+i}{1-i}\right)^n = 1$, is :

(1) 4 (2) 3 (3) 2 (4) 1

11. The value of K for which one of the roots of $x^2 - 3x + 2K = 0$ is double of one of the roots of $x^2 - x + K = 0$, is :

(1) 2 (2) –2 (3) –1 (4) 1

12. The interior angles of a regular polygon measure 160° each. The number of diagonals of the polygon are :

(1) 105 (2) 135 (3) 145 (4) 147

13. The number of ways in which 9 identical balls can be placed in three identical boxes, is :

(1) 9 (2) 12 (3) 55 (4) 27

14. In the expansion of $\left(x^2 - \frac{1}{3x}\right)^9$, the term independent of x is :

(1) 5th (2) 6th (3) 7th (4) 4th

15. If the coefficients of rth and $(r + 1)$th terms in the expansion of $(3 + 7x)^{29}$ are equal, then $r =$

(1) 14 (2) 15 (3) 18 (4) 21

16. Three numbers forms an increasing G.P. If the middle number is doubled, then the new numbers are in A. P. The common ratio of the G. P. is :

(1) $2 + \sqrt{3}$ (2) $3 + \sqrt{2}$

(3) $\sqrt{3} + 1$ (4) $3 - \sqrt{2}$

17. If $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots\ldots\ldots$ to $\infty = \frac{\pi^2}{6}$, then $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots\ldots =$

(1) $\frac{\pi^2}{3}$ (2) $\frac{\pi^2}{4}$ (3) $\frac{\pi^2}{8}$ (4) $\frac{\pi^2}{12}$

18. If a, b, c are in A.P. as well as G.P., then which of the following is ***true*** ?

(1) $a = b = c$ (2) $a = b \neq c$

(3) $a \neq b = c$ (4) $a \neq b \neq c$

19. If the AM of the roots of a quadratic equation in x is A and their GM is G, then the quadratic equation is :

(1) $x^2 - Ax + G^2 = 0$
(2) $x^2 - Ax + G = 0$
(3) $x^2 - 2Ax + G = 0$
(4) $x^2 - 2Ax + G^2 = 0$

20. A line passes through the point (2, 2) and is perpendicular to the line $3x + y = 3$, then its y-intercept is :

(1) 2/3 (2) 4/3 (3) 4/5 (4) 3/4

21. The line $x + y = 4$ divides the line joining (–1, 1) and (5, 7) in the ratio $K : 1$, then the value of K is :

(1) 1/4 (2) 4/3 (3) 1/2 (4) 2

22. If the foot of the perpendicular from the origin to a straight line is at the point (3, –4). Then the equation of the line is :

(1) $3x - 4y = 25$
(2) $4x - 3y = 25$
(3) $4x + 3y = 25$
(4) $3x + 4y = 25$

23. The distance between the parallel lines $6x - 3y - 5 = 0$ and $2x - y + 4 = 0$ is :

(1) $3/\sqrt{5}$
(2) $\sqrt{5}/3$
(3) $17/3\sqrt{5}$
(4) $17/\sqrt{3}$

24. The points $(K + 1, 1)$, $(2K + 1, 3)$ and $(2K + 2, 2K)$ are collinear, then $K =$

(1) –1 (2) $\frac{1}{3}$ (3) $\frac{1}{2}$ (4) $-\frac{1}{2}$

25. The equation of the circle of radius 5 whose centre lies on x-axis and passing through (2, 3) is :

(1) $x^2 + y^2 - 4x - 21 = 0$
(2) $x^2 + y^2 + 4x - 21 = 0$
(3) $x^2 + y^2 + 4x - 17 = 0$
(4) $x^2 + y^2 - 4x + 21 = 0$

26. If the parabola $y^2 = 4\,ax$ passes through (3, 2), then the length of its latus-rectum is :

(1) 2/3 (2) 3/4 (3) 4 (4) 4/3

27. The eccentricity of the hyperbola $16x^2 - 3y^2 - 32x + 12y - 44 = 0$ is :

(1) $\sqrt{13}$ (2) $\sqrt{7}$ (3) $\sqrt{\frac{17}{3}}$ (4) $\sqrt{\frac{19}{3}}$

28. The eccentricity of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ whose latus-rectum is half of its major axis, is :

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{\sqrt{3}}{4}$ (3) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{2}$

29. The ratio in which the *yz*-plane divides the segment joining the points (–2, 4, 7) and (3, – 5, 8) is :

(1) 7 : 8 (2) –7 : 8 (3) 2 : 3 (4) –3 : 2

30. If α, β, γ are the angles which a directed line makes with the positive directions of the co-ordinate axes, then $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=$

(1) 0 (2) 1 (3) 2 (4) 3

31. $\lim_{x\to 1}(1-x)\tan\left(\frac{\pi x}{2}\right)=$

(1) $\frac{\pi}{2}$ (2) $\frac{2}{\pi}$ (3) $\frac{\pi}{4}$ (4) 1

32. $\lim_{n\to\infty}\frac{(1-2+3-4+5-6.............-2n)}{\sqrt{n^2+1}+\sqrt{4n^2-1}}=$

(1) –2 (2) $\frac{1}{2}$ (3) $\frac{1}{3}$ (4) $-\frac{1}{3}$

33. $\lim_{x\to 0}\frac{\sin(\pi\cos^2 x)}{x^2}=$

(1) π (2) π/2 (3) –π (4) 1

34. If $f(x)=\begin{cases}\frac{x-1}{2x^2-7x+5}, & x\neq 1\\ -\frac{1}{3}, & x=1\end{cases}$

then the derivative of $f(x)$ at $x = 1$, is :

(1) $\frac{9}{2}$ (2) $\frac{-9}{2}$ (3) $\frac{-2}{9}$ (4) $\frac{2}{9}$

35. The mean of *n* terms is $\bar{x}$. If the first term is increased by 1, second by 2 and so on, then the new mean is :

(1) $\bar{x}+\frac{n+1}{2}$ (2) $\bar{x}+\frac{n}{2}$ (3) $\bar{x}+n$ (4) $\bar{x}+\frac{n-1}{2}$

36. The standard deviation of 25 numbers is 40. If each of the numbers which is greater than the standard deviation, is increased by 5, then the new standard deviation will be :
(1) 45 (2) 40 (3) 65 (4) 40.75

37. The sum of 10 items is 12 and the sum of their squares is 18, then the standard deviation is :
(1) 3/5 (2) 4/5 (3) 3/10 (4) 2/5

38. There are n persons sitting in a row. Two of them are selected at random. The probability that two selected persons are not sitting together, is :
(1) $\frac{2}{n-2}$ (2) $\frac{n}{n+2}$ (3) $\frac{2}{n}$ (4) $1-\frac{2}{n}$

39. Seven digits from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 are written in a random order. The probability that this seven digit number is divisible by 9, is :
(1) $\frac{1}{3}$ (2) $\frac{2}{7}$ (3) $\frac{1}{9}$ (4) $\frac{2}{9}$

40. The coefficients of a quadratic equation $ax^2 + bx + c = 0$ $(a \neq b \neq c)$ are chosen from first three prime numbers, the probability that roots of the equation are real, is :
(1) 2/3 (2) 1/3 (3) 1/4 (4) 3/4

41. The graph of the function $y = f(x)$ is symmetrical about the line $x = a$, then which of the following is ***true*** ?
(1) $f(x+a) = f(x-a)$ (2) $f(x) = f(-x)$
(3) $f(a+x) = f(a-x)$ (4) $f(x) = -f(-x)$

42. Let $f: R \rightarrow R$ be a function defined by $f(x) = \frac{x^2+2x+5}{x^2+x+1}$, then f is :
(1) one-one and onto (2) one-one and into
(3) many one and onto (4) many one and into

43. $\sin \lambda x + \cos \lambda x$ and $|\sin x| + |\cos x|$ are periodic of same fundamental period, if $\lambda =$
(1) 4 (2) 0 (3) 2 (4) 1

44. Let R be a relation on the set N of natural numbers defined by $nRm \Leftrightarrow n$ is a factor of m (i.e. n/m). Then R is :
(1) equivalence (2) transitive and symmetric
(3) reflexive and symmetric (4) reflexive, transitive but not symmetric

45. If $\sin^{-1}\left(\frac{x}{5}\right)+\operatorname{cosec}^{-1}\left(\frac{5}{4}\right)=\frac{\pi}{2}$, then $x=$

(1) 4 (2) 3 (3) 5 (4) 2

46. A solution of the equation :

$\tan^{-1}(1+x)+\tan^{-1}(1-x)=\frac{\pi}{2}$, is :

(1) $x=0$ (2) $x=1$ (3) $x=-1$ (4) $x=\pi$

47. The value of $\tan\left(\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right)$ is :

(1) $3+\sqrt{5}$ (2) $3-\sqrt{5}$ (3) $\frac{1}{2}(3-\sqrt{5})$ (4) $\frac{1}{2}(\sqrt{5}+3)$

48. Solution of $\sin^{-1}x-\cos^{-1}x=\cos^{-1}\frac{\sqrt{3}}{2}$ is :

(1) $x=\frac{1}{2}$ (2) $x=\frac{1}{\sqrt{3}}$ (3) $x=\frac{\sqrt{3}}{2}$ (4) $x=1$

49. If $A=\begin{bmatrix}1 & 0\\ -1 & 7\end{bmatrix}$ and $A^2=8A+KI_2$, then the value of K is :

(1) -1 (2) 1 (3) 7 (4) -7

50. If 1, w, w^2 are cube roots of unity, inverse of which of the following matrices exists ?

(1) $\begin{bmatrix}w & w^2\\ w^2 & 1\end{bmatrix}$ (2) $\begin{bmatrix}1 & w\\ w & w^2\end{bmatrix}$ (3) $\begin{bmatrix}w^2 & 1\\ 1 & w\end{bmatrix}$ (4) None of these

51. If A an orthogonal matrix, then which of the following is ***true*** ?

(1) $|A|=0$ (2) $|A|=\pm 1$ (3) $|A|=\pm 2$ (4) $|A|=\pi/2$

52. If $A(\alpha)=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$, then $A(\alpha)\,A(\beta)=$

(1) $A(\alpha)+A(\beta)$ (2) $A(\alpha)-A(\beta)$

(3) $A(\alpha+\beta)$ (4) $A(\alpha-\beta)$

53. If A and B are two matrices such that $AB = B$ and $BA = A$, then $A^2 + B^2 =$

(1) $A + B$ (2) AB (3) $2AB$ (4) I

54. If K is a real cube root of –2, then the value of $\begin{vmatrix} 1 & 2K & 1 \\ K^2 & 1 & 3K^2 \\ 2 & 2K & 1 \end{vmatrix}$ is equal to :

(1) –10 (2) –12 (3) –13 (4) –15

55. The equations $Kx - y = 2$, $2x - 3y = -K$, $3x - 2y = -1$ are consistent if $K =$

(1) 2, –3 (2) –2, 3 (3) 1, –4 (4) –1, 4

56. If $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, then $f(2x) - f(x) =$

(1) $ax(3a + 2x)$ (2) $ax(2a + 3x)$ (3) $a(2a + 3x)$ (4) $x(3a + 2x)$

57. Let $f(x) = \dfrac{1 - \tan x}{4x - \pi}, x \neq \pi/4$ and $x \in [0, \pi/2] = a, x = \pi/4$

If $f(x)$ is continuous in $[0, \pi/2]$, then $a =$

(1) 1/2 (2) –1/2 (3) 1 (4) 0

58. Let $f(x) = 1 + x (\sin x) [\cos x]$, $0 < x \leq \pi/2$, where [.] denotes the greatest integer function. Then which of the following is ***true*** ?

(1) $f(x)$ is continuous in $(0, \pi/2)$ (2) $f(x)$ is strictly increasing in $(0, \pi/2)$

(3) $f(x)$ is strictly decreasing in $(0, \pi/2)$ (4) $f(x)$ has global maximum value 2

59. If $y = \tan^{-1} \sqrt{\dfrac{1 + \sin x}{1 - \sin x}}$, $\pi/2 < x < \pi$, then $\dfrac{dy}{dx} =$

(1) –1 (2) 1 (3) 1/2 (4) –1/2

60. If $x = e^{y + e^{y + e^{y + \ldots\ldots\ldots\infty}}}$, then $\dfrac{dy}{dx} =$

(1) $\dfrac{1-x}{x}$ (2) $\dfrac{x}{1-x}$ (3) $\dfrac{1+x}{x}$ (4) $\dfrac{x}{1+x}$

61. If $x = a\cos^3\theta$, $y = a\sin^3\theta$, then $\sqrt{1+\left(\frac{dy}{dx}\right)^2} =$

(1) $\sec^2\theta$ (2) $\tan^2\theta$ (3) $|\sec\theta|$ (4) $|\cot\theta|$

62. If $e^y + xy = e$, then the value of $\frac{d^2y}{dx^2}$ for $x = 0$, is :

(1) e^2 (2) $\frac{1}{e^2}$ (3) $\frac{1}{e}$ (4) $\frac{1}{e^3}$

63. If $x^y . y^x = 16$, then $\frac{dy}{dx}$ at (2, 2) is :

(1) 0 (2) 1 (3) –1 (4) –4

64. The approximate value of square root of 25.2 is :

(1) 5.01 (2) 5.02 (3) 5.03 (4) 5.04

65. The tangent at (1, 1) on the curve $y^2 = x(2-x)^2$ meets it again at the point :

(1) (–3, 7) (2) (4, 4) (3) $\left(\frac{3}{8}, \frac{9}{4}\right)$ (4) $\left(\frac{9}{4}, \frac{3}{8}\right)$

66. The distance between the origin and the normal to the curve $y = e^{2x} + x^2$ at $x = 0$ is :

(1) $4/\sqrt{5}$ (2) $3/\sqrt{5}$ (3) $2/\sqrt{5}$ (4) $2/\sqrt{7}$

67. The length of longest interval in which Rolle's theorem can be applied for the function $f(x) = |x^2 - a^2|$, $(a > 0)$, is :

(1) $2a$ (2) $3a$ (3) $4a$ (4) $a\sqrt{2}$

68. If the function $f(x) = \sqrt{3}\sin x - \cos x - 2ax + b$ decreases for all real values of x, then the value of a is given by :

(1) $a < 1$ (2) $a < \sqrt{2}$ (3) $a \geq \sqrt{2}$ (4) $a \geq 1$

69. The condition that $x^3 + ax^2 + bx + c$ may have no extremum, is :

(1) $a^2 > 3b$ (2) $a^2 < 3b$ (3) $a^2 > 2b$ (4) $a^2 < 2b$

70. $\int \frac{3+2\cos x}{(2+3\cos x)^2} dx =$

(1) $\frac{\sin x}{2+3\cos x} + c$ (2) $\frac{\cos x}{2+3\cos x} + c$ (3) $\frac{2\sin x}{2+3\cos x} + c$ (4) $\frac{2\cos x}{2+3\cos x} + c$

71. $\int x^x (1 + \log x)\, dx =$

(1) $x^x + c$ (2) $x^x \log x + c$ (3) $x \log x + c$ (4) none of these

72. $\int \sin \sqrt{x} dx =$

(1) $(\cos\sqrt{x} - \sin\sqrt{x}) + c$

(2) $(\sqrt{x}\cos\sqrt{x} - \sin\sqrt{x}) + c$

(3) $-2(\sqrt{x}\cos\sqrt{x} - \sin\sqrt{x}) + c$

(4) $2(\sqrt{x}\cos\sqrt{x} - \sin\sqrt{x}) + c$

73. $\int \frac{1}{\sqrt{\sin^3 x \cos x}} dx =$

(1) $2(\tan x)^{-1/2} + c$ (2) $(\tan x)^{1/2} + c$ (3) $(\tan x)^{-1/2} + c$ (4) $2(\tan x)^{1/2} + c$

74. If $I_n = \int_0^1 x^n e^{-x} dx$ for $n \in N$, then $I_n - n\, I_{n-1} =$

(1) $1/e$ (2) $-1/e$ (3) e (4) $-2/e$

75. If $\int_{\pi/2}^{\theta} \sin x\, dx = \sin 2\theta$, then the value of θ satisfying $0 < \theta < \pi$, is :

(1) $\pi/6$ (2) $\pi/4$ (3) $\pi/2$ (4) $5\pi/6$

76. $\int_0^{[x]} (x - [x])\, dx =$

(1) $\frac{1}{2}[x]$ (2) $[x]$ (3) $2[x]$ (4) $-2[x]$

77. $\int_0^{\pi/4} \log(1 + \tan x) dx =$

(1) $\frac{\pi}{4}\log 2$ (2) $\frac{\pi}{8}\log 2$ (3) $\frac{\pi}{2}\log 2$ (4) $\pi \log 2$

78. The area bounded by the curve $y = x \sin x$ and x-axis between $x = 0$ and $x = 2\pi$, is :

(1) π sq. units

(2) $\frac{\pi}{2}$ sq. units

(3) 2π sq. units

(4) 4π sq. units

79. If the area bounded by the curves $y^2 = 4ax$ and $y = mx$ is $a^2/3$ sq. units, then the value of m is :

(1) 2 (2) -2 (3) $1/2$ (4) $3/2$

80. Solution of $\frac{dy}{dx} = \cos(x+y)$ is :

(1) $\sin(x+y) = x + c$

(2) $\tan\left(\frac{x+y}{2}\right) + x = c$

(3) $\cot\left(\frac{x+y}{2}\right) = x + c$

(4) $\tan\left(\frac{x+y}{2}\right) = x + c$

81. Solution of $ydx + (x - y^3)\, dy = 0$ is :

(1) $xy + \frac{y^2}{2} = c$

(2) $xy = \frac{y^2}{2} + c$

(3) $xy = \frac{y^2}{4} + c$

(4) $xy = \frac{x^2}{4} + c$

82. The differential equation $y\frac{dy}{dx} = x + a$ (a being constant) represents a set of :

(1) circles having centre on the x-axis

(2) circles having centre on the y-axis

(3) ellipses

(4) hyperbolas

83. From a bag containing 2 white, 3 red and 4 black balls, two balls are drawn one by one without replacement. The probability that at least ones ball is red, is :

(1) $\frac{5}{12}$

(2) $\frac{7}{12}$

(3) $\frac{5}{8}$

(4) $\frac{3}{7}$

84. A box contains 15 items in which 4 items are defective. The items are selected at random, one by one, and examined. The ones examined are not put back. The probability that 9th one examined is the last defective, is :

(1) $\frac{7}{195}$

(2) $\frac{8}{195}$

(3) $\frac{16}{255}$

(4) $\frac{14}{255}$

85. A person is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. The probability that it is actually a six, is :

(1) $\frac{7}{36}$

(2) $\frac{11}{36}$

(3) $\frac{3}{8}$

(4) $\frac{5}{8}$

86. Four numbers are multiplied together. The probability that the product will be divisible by 5 or 10, is :

(1) $\frac{123}{625}$ (2) $\frac{133}{625}$ (3) $\frac{357}{625}$ (4) $\frac{369}{625}$

87. The least number of times a fair coin must be tossed so that the probability of getting at least one head is at lest 0.8, is :

(1) 3 (2) 5 (3) 6 (4) 8

88. If $P(A \cup B) = \frac{3}{4}$ and $P(\overline{A}) = 2/3$, then $P(\overline{A} \cap B) =$

(1) $\frac{7}{12}$ (2) $\frac{5}{12}$ (3) $\frac{1}{12}$ (4) $\frac{1}{6}$

89. If $\vec{a} + \vec{b} + \vec{c} = \vec{0}, |\vec{a}| = 3, |\vec{b}| = 5, |\vec{c}| = 7$, then the angle between $\vec{a}$ and $\vec{b}$ is :

(1) $\pi/3$ (2) $2\pi/3$ (3) $\pi/6$ (4) $5\pi/3$

90. Let $\vec{a}$ and $\vec{b}$ be two unit vectors and α be the angle between them, then $\vec{a} + \vec{b}$ is a unit vector if $\alpha =$

(1) $\pi/2$ (2) $\pi/3$ (3) $2\pi/3$ (4) $\pi/4$

91. The unit vector perpendicular to the vectors $\hat{i} + \hat{j}$ and $\hat{j} + \hat{k}$ is/are :

(1) $\pm\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} - \hat{k})$ (2) $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} - \hat{k})$

(3) $\pm\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} + \hat{k})$ (4) $\frac{1}{\sqrt{2}}(\hat{i} + \hat{k})$

92. The vector $\hat{i} + x\hat{j} + 3\hat{k}$ is rotated through an angle θ and doubled in magnitude, then it becomes $4\hat{i} + (4x - 2)\hat{j} + 2\hat{k}$. The value of x is :

(1) $\frac{1}{3}$ (2) -3 (3) $\frac{2}{3}$ (4) $-\frac{2}{3}$

93. The vectors $\overrightarrow{AB} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\overrightarrow{BC} = -\hat{i} - 2\hat{k}$ are the adjacent sides of a parallelogram. The angle between its diagonals is :

(1) $\pi/6$ (2) $\pi/3$ (3) $\pi/2$ (4) $\pi/4$

94. Consider a LPP : min $Z = 6x + 10y$
subjected to $x \geq 6, y \geq 2, 2x + y \geq 10; x, y \geq 0$.
Redundand constraints in this LPP are :

(1) $x \geq 0, y \geq 0$ (2) $2x + y \geq 10$

(3) $x \geq 6, 2x + y \geq 10$ (4) None of these

95. The angle between the lines having direction ratios 4, –3, 5 and 3, 4, 5 is :

(1) $\pi/3$ (2) $\pi/4$ (3) $\pi/6$ (4) $2\pi/3$

96. If a plane meets the coordinate axes at A, B and C in such a way that the centroid of triangle ABC it at the point (1, 2, 3), then the equation of the plane is :

(1) $\frac{x}{1}+\frac{y}{2}+\frac{z}{3}=1$ (2) $\frac{x}{3}+\frac{y}{6}+\frac{z}{9}=1$

(3) $\frac{x}{1}+\frac{y}{2}+\frac{z}{3}=\frac{1}{3}$ (4) $\frac{x}{3}+\frac{y}{6}+\frac{z}{9}=3$

97. The image of the point (1, 3, 4) in the plane $2x - y + z + 3 = 0$ is :

(1) (3, 5, 2) (2) (3, 5, –2)

(3) (–3, 5, 2) (4) (3, –5, 2)

98. The lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x-1}{-2}=\frac{y-2}{-4}=\frac{z-3}{-6}$ are :

(1) intersecting (2) parallel

(3) coincidental (4) skew

99. The distance between the line $\vec{r} = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda(\hat{i} - \hat{j} + 4\hat{k})$ and the plane $\vec{r}.(\hat{i} + 5\hat{j} + \hat{k}) = 5$ is :

(1) $\frac{10}{3\sqrt{3}}$ (2) $\frac{10}{\sqrt{3}}$ (3) $\frac{10}{3}$ (4) $\frac{5}{3\sqrt{3}}$

100. The lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar if $k =$

(1) 4 (2) 3 (3) –1 (4) –3

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PG-EE-2019

SUBJECT : Mathematics Hons. (Five Year)-(SET-X)

B

Sr. No. 10762

Time : 1¼ **Hours (75 minutes)** Total Questions : **100** Max. Marks : **100**

Roll No. (in figures)__________ (in words) __________

Name __________ Date of Birth __________

Father's Name __________ Mother's Name __________

Date of Exam __________

(Signature of the Candidate) (Signature of the Invigilator)

CANDIDATES MUST READ THE FOLLOWING INFORMATION/INSTRUCTIONS BEFORE STARTING THE QUESTION PAPER.

1. ***All questions are compulsory and carry equal marks. The candidates are required to attempt all questions.***
2. The candidate ***must return*** this question booklet and the OMR Answer-Sheet to the Invigilator concerned before leaving the Examination Hall, failing which a case of use of unfair-means / misbehaviour will be registered against him / her, in addition to lodging of an FIR with the police. Further the answer-sheet of such a candidate will not be evaluated.
3. Keeping in view the transparency of the examination system, carbonless OMR Sheet is provided to the candidate so that a copy of OMR Sheet may be kept by the candidate.
4. Question Booklet along-with answer key of all the A, B, C and D code shall be got uploaded on the University Website immediately after the conduct of Entrance Examination. Candidates may raise valid objection/complaint if any, with regard to discrepancy in the question booklet/answer key within 24 hours of uploading the same on the University website. The complaint be sent by the students to the Controller of Examinations by hand or through email. Thereafter, no complaint in any case will be considered.
5. The candidate ***must not*** do any rough work or writing in the OMR Answer-Sheet. Rough work, if any, may be done in the question booklet itself. Answers ***must not*** be ticked in the question booklet.
6. ***Use only black or blue ball point pen of good quality in the OMR Answer-Sheet.***
7. There will be ***no negative*** marking. Each correct answer will be awarded ***one*** full mark. Cutting, erasing, overwriting and more than one answer in OMR Answer-Sheet will be treated as incorrect answer.
8. *Before answering the questions, the candidates should ensure that they have been supplied correct and complete question booklet. Complaints, if any, regarding misprinting etc. will not be entertained* **30** *minutes after starting of the examination.*

1. $\int x^x (1 + \log x)\, dx =$

(1) $x^x + c$ (2) $x^x \log x + c$ (3) $x \log x + c$ (4) none of these

2. $\int \sin \sqrt{x} dx =$

(1) $(\cos\sqrt{x} - \sin\sqrt{x}) + c$

(2) $(\sqrt{x}\cos\sqrt{x} - \sin\sqrt{x}) + c$

(3) $-2(\sqrt{x}\cos\sqrt{x} - \sin\sqrt{x}) + c$

(4) $2(\sqrt{x}\cos\sqrt{x} - \sin\sqrt{x}) + c$

3. $\int \frac{1}{\sqrt{\sin^3 x \cos x}} dx =$

(1) $2(\tan x)^{-1/2} + c$ (2) $(\tan x)^{1/2} + c$ (3) $(\tan x)^{-1/2} + c$ (4) $2(\tan x)^{1/2} + c$

4. If $I_n = \int_0^1 x^n e^{-x} dx$ for $n \in N$, then $I_n - n\, I_{n-1} =$

(1) $1/e$ (2) $-1/e$ (3) e (4) $-2/e$

5. If $\int_{\pi/2}^{\theta} \sin x\, dx = \sin 2\theta$, then the value of θ satisfying $0 < \theta < \pi$, is :

(1) $\pi/6$ (2) $\pi/4$ (3) $\pi/2$ (4) $5\pi/6$

6. $\int_0^{[x]} (x - [x])\, dx =$

(1) $\frac{1}{2}[x]$ (2) $[x]$ (3) $2[x]$ (4) $-2[x]$

7. $\int_0^{\pi/4} \log(1 + \tan x) dx =$

(1) $\frac{\pi}{4}\log 2$ (2) $\frac{\pi}{8}\log 2$ (3) $\frac{\pi}{2}\log 2$ (4) $\pi \log 2$

8. The area bounded by the curve $y = x \sin x$ and x-axis between $x = 0$ and $x = 2\pi$, is :

(1) π sq. units

(2) $\frac{\pi}{2}$ sq. units

(3) 2π sq. units

(4) 4π sq. units

9. If the area bounded by the curves $y^2 = 4ax$ and $y = mx$ is $a^2/3$ sq. units, then the value of m is :

(1) 2 (2) -2 (3) $1/2$ (4) $3/2$

10. Solution of $\frac{dy}{dx} = \cos(x+y)$ is :

(1) $\sin(x+y) = x + c$

(2) $\tan\left(\frac{x+y}{2}\right) + x = c$

(3) $\cot\left(\frac{x+y}{2}\right) = x + c$

(4) $\tan\left(\frac{x+y}{2}\right) = x + c$

11. If A an orthogonal matrix, then which of the following is ***true*** ?

(1) $|A| = 0$ (2) $|A| = \pm 1$ (3) $|A| = \pm 2$ (4) $|A| = \pi/2$

12. If $A(\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then $A(\alpha)\,A(\beta) =$

(1) $A(\alpha) + A(\beta)$ (2) $A(\alpha) - A(\beta)$ (3) $A(\alpha + \beta)$ (4) $A(\alpha - \beta)$

13. If A and B are two matrices such that $AB = B$ and $BA = A$, then $A^2 + B^2 =$

(1) $A + B$ (2) AB (3) $2AB$ (4) I

14. If K is a real cube root of –2, then the value of $\begin{vmatrix} 1 & 2K & 1 \\ K^2 & 1 & 3K^2 \\ 2 & 2K & 1 \end{vmatrix}$ is equal to :

(1) –10 (2) –12 (3) –13 (4) –15

15. The equations $Kx - y = 2$, $2x - 3y = -K$, $3x - 2y = -1$ are consistent if $K =$

(1) 2, –3 (2) –2, 3 (3) 1, –4 (4) –1, 4

16. If $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, then $f(2x) - f(x) =$

(1) $ax(3a + 2x)$ (2) $ax(2a + 3x)$ (3) $a(2a + 3x)$ (4) $x(3a + 2x)$

17. Let $f(x) = \frac{1 - \tan x}{4x - \pi}, x \neq \pi/4$ and $x \in [0, \pi/2] = a, x = \pi/4$

If $f(x)$ is continuous in $[0, \pi/2]$, then $a =$

(1) 1/2 (2) –1/2 (3) 1 (4) 0

18. Let $f(x) = 1 + x (\sin x) [\cos x]$, $0 < x \leq \pi/2$, where [.] denotes the greatest integer function. Then which of the following is ***true*** ?

(1) $f(x)$ is continuous in $(0, \pi/2)$ (2) $f(x)$ is strictly increasing in $(0, \pi/2)$

(3) $f(x)$ is strictly decreasing in $(0, \pi/2)$ (4) $f(x)$ has global maximum value 2

19. If $y = \tan^{-1}\sqrt{\dfrac{1+\sin x}{1-\sin x}}$, $\pi/2 < x < \pi$, then $\dfrac{dy}{dx} =$

(1) -1 (2) 1 (3) $1/2$ (4) $-1/2$

20. If $x = e^{y+e^{y+e^{y+\ldots\ldots\ldots\infty}}}$, then $\dfrac{dy}{dx} =$

(1) $\dfrac{1-x}{x}$ (2) $\dfrac{x}{1-x}$ (3) $\dfrac{1+x}{x}$ (4) $\dfrac{x}{1+x}$

21. Solution of $ydx + (x - y^3)\, dy = 0$ is :

(1) $xy + \dfrac{y^2}{2} = c$ (2) $xy = \dfrac{y^2}{2} + c$

(3) $xy = \dfrac{y^2}{4} + c$ (4) $xy = \dfrac{x^2}{4} + c$

22. The differential equation $y\dfrac{dy}{dx} = x + a$ (a being constant) represents a set of :

(1) circles having centre on the x-axis (2) circles having centre on the y-axis

(3) ellipses (4) hyperbolas

23. From a bag containing 2 white, 3 red and 4 black balls, two balls are drawn one by one without replacement. The probability that at least ones ball is red, is :

(1) $\dfrac{5}{12}$ (2) $\dfrac{7}{12}$ (3) $\dfrac{5}{8}$ (4) $\dfrac{3}{7}$

24. A box contains 15 items in which 4 items are defective. The items are selected at random, one by one, and examined. The ones examined are not put back. The probability that 9th one examined is the last defective, is :

(1) $\dfrac{7}{195}$ (2) $\dfrac{8}{195}$ (3) $\dfrac{16}{255}$ (4) $\dfrac{14}{255}$

25. A person is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. The probability that it is actually a six, is :

(1) $\frac{7}{36}$ (2) $\frac{11}{36}$ (3) $\frac{3}{8}$ (4) $\frac{5}{8}$

26. Four numbers are multiplied together. The probability that the product will be divisible by 5 or 10, is :

(1) $\frac{123}{625}$ (2) $\frac{133}{625}$ (3) $\frac{357}{625}$ (4) $\frac{369}{625}$

27. The least number of times a fair coin must be tossed so that the probability of getting at least one head is at lest 0.8, is :

(1) 3 (2) 5 (3) 6 (4) 8

28. If $P(A \cup B) = \frac{3}{4}$ and $P(\overline{A}) = 2/3$, then $P(\overline{A} \cap B) =$

(1) $\frac{7}{12}$ (2) $\frac{5}{12}$ (3) $\frac{1}{12}$ (4) $\frac{1}{6}$

29. If $\vec{a}+\vec{b}+\vec{c}=\vec{0}, |\vec{a}|=3, |\vec{b}|=5, |\vec{c}|=7$, then the angle between $\vec{a}$ and $\vec{b}$ is :

(1) $\pi/3$ (2) $2\pi/3$ (3) $\pi/6$ (4) $5\pi/3$

30. Let $\vec{a}$ and $\vec{b}$ be two unit vectors and α be the angle between them, then $\vec{a}+\vec{b}$ is a unit vector if $\alpha =$

(1) $\pi/2$ (2) $\pi/3$ (3) $2\pi/3$ (4) $\pi/4$

31. The value of K for which one of the roots of $x^2 - 3x + 2K = 0$ is double of one of the roots of $x^2 - x + K = 0$, is :

(1) 2 (2) –2 (3) –1 (4) 1

32. The interior angles of a regular polygon measure 160° each. The number of diagonals of the polygon are :

(1) 105 (2) 135 (3) 145 (4) 147

33. The number of ways in which 9 identical balls can be placed in three identical boxes, is :

(1) 9 (2) 12 (3) 55 (4) 27

34. In the expansion of $\left(x^2 - \frac{1}{3x}\right)^9$, the term independent of x is :

(1) 5th (2) 6th (3) 7th (4) 4th

35. If the coefficients of rth and $(r + 1)$th terms in the expansion of $(3 + 7x)^{29}$ are equal, then $r =$

(1) 14 (2) 15 (3) 18 (4) 21

36. Three numbers forms an increasing G.P. If the middle number is doubled, then the new numbers are in A. P. The common ratio of the G. P. is :

(1) $2 + \sqrt{3}$ (2) $3 + \sqrt{2}$

(3) $\sqrt{3} + 1$ (4) $3 - \sqrt{2}$

37. If $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + ...$ to $\infty = \frac{\pi^2}{6}$, then $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ... =$

(1) $\frac{\pi^2}{3}$ (2) $\frac{\pi^2}{4}$ (3) $\frac{\pi^2}{8}$ (4) $\frac{\pi^2}{12}$

38. If a, b, c are in A.P. as well as G.P., then which of the following is ***true*** ?

(1) $a = b = c$ (2) $a = b \neq c$

(3) $a \neq b = e$ (4) $a \neq b \neq c$

39. If the AM of the roots of a quadratic equation in x is A and their GM is G, then the quadratic equation is :

(1) $x^2 - Ax + G^2 = 0$ (2) $x^2 - Ax + G = 0$

(3) $x^2 - 2Ax + G = 0$ (4) $x^2 - 2Ax + G^2 = 0$

40. A line passes through the point (2, 2) and is perpendicular to the line $3x + y = 3$, then its y-intercept is :

(1) 2/3 (2) 4/3 (3) 4/5 (4) 3/4

41. The unit vector perpendicular to the vectors $\hat{i} + \hat{j}$ and $\hat{j} + \hat{k}$ is/are :

(1) $\pm\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} - \hat{k})$ (2) $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} - \hat{k})$

(3) $\pm\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} + \hat{k})$ (4) $\frac{1}{\sqrt{2}}(\hat{i} + \hat{k})$

42. The vector $\hat{i} + x\hat{j} + 3\hat{k}$ is rotated through an angle θ and doubled in magnitude, then it becomes $4\hat{i} + (4x - 2)\hat{j} + 2\hat{k}$. The value of x is :

(1) $\frac{1}{3}$ (2) –3 (3) $\frac{2}{3}$ (4) $-\frac{2}{3}$

43. The vectors $\overrightarrow{AB} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\overrightarrow{BC} = -\hat{i} - 2\hat{k}$ are the adjacent sides of a parallelogram. The angle between its diagonals is :

(1) π/6 (2) π/3 (3) π/2 (4) π/4

44. Consider a LPP : min $Z = 6x + 10y$
subjected to $x \geq 6, y \geq 2, 2x + y \geq 10; x, y \geq 0$.
Redundand constraints in this LPP are :

(1) $x \geq 0, y \geq 0$ (2) $2x + y \geq 10$

(3) $x \geq 6, 2x + y \geq 10$ (4) None of these

45. The angle between the lines having direction ratios 4, –3, 5 and 3, 4, 5 is :

(1) π/3 (2) π/4 (3) π/6 (4) 2π/3

46. If a plane meets the coordinate axes at A, B and C in such a way that the centroid of triangle ABC it at the point (1, 2, 3), then the equation of the plane is :

(1) $\frac{x}{1} + \frac{y}{2} + \frac{z}{3} = 1$ (2) $\frac{x}{3} + \frac{y}{6} + \frac{z}{9} = 1$

(3) $\frac{x}{1} + \frac{y}{2} + \frac{z}{3} = \frac{1}{3}$ (4) $\frac{x}{3} + \frac{y}{6} + \frac{z}{9} = 3$

47. The image of the point (1, 3, 4) in the plane $2x - y + z + 3 = 0$ is :

(1) (3, 5, 2) (2) (3, 5, –2)

(3) (–3, 5, 2) (4) (3, –5, 2)

48. The lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x-1}{-2} = \frac{y-2}{-4} = \frac{z-3}{-6}$ are :

(1) intersecting (2) parallel

(3) coincidental (4) skew

49. The distance between the line $\vec{r} = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda(\hat{i} - \hat{j} + 4\hat{k})$ and the plane $\vec{r}.(\hat{i} + 5\hat{j} + \hat{k}) = 5$ is :

(1) $\frac{10}{3\sqrt{3}}$ (2) $\frac{10}{\sqrt{3}}$ (3) $\frac{10}{3}$ (4) $\frac{5}{3\sqrt{3}}$

50. The lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar if $k =$

(1) 4 (2) 3 (3) –1 (4) –3

51. If $x = a \cos^3 \theta, y = a \sin^3 \theta$, then $\sqrt{1+\left(\frac{dy}{dx}\right)^2} =$

(1) $\sec^2\theta$ (2) $\tan^2\theta$ (3) $|\sec \theta|$ (4) $|\cot \theta|$

52. If $e^y + xy = e$, then the value of $\frac{d^2y}{dx^2}$ for $x = 0$, is :

(1) e^2 (2) $\frac{1}{e^2}$ (3) $\frac{1}{e}$ (4) $\frac{1}{e^3}$

53. If $x^y . y^x = 16$, then $\frac{dy}{dx}$ at (2, 2) is :

(1) 0 (2) 1 (3) –1 (4) –4

54. The approximate value of square root of 25.2 is :

(1) 5.01 (2) 5.02 (3) 5.03 (4) 5.04

55. The tangent at (1, 1) on the curve $y^2 = x(2-x)^2$ meets it again at the point :

(1) (–3, 7) (2) (4, 4) (3) $\left(\frac{3}{8}, \frac{9}{4}\right)$ (4) $\left(\frac{9}{4}, \frac{3}{8}\right)$

56. The distance between the origin and the normal to the curve $y = e^{2x} + x^2$ at $x = 0$ is :

(1) $4/\sqrt{5}$ (2) $3/\sqrt{5}$ (3) $2/\sqrt{5}$ (4) $2/\sqrt{7}$

57. The length of longest interval in which Rolle's theorem can be applied for the function $f(x) = |x^2 - a^2|, (a > 0)$, is :

(1) $2a$ (2) $3a$ (3) $4a$ (4) $a\sqrt{2}$

58. If the function $f(x) = \sqrt{3}\sin x - \cos x - 2ax + b$ decreases for all real values of x, then the value of a is given by :

(1) $a < 1$ (2) $a < \sqrt{2}$ (3) $a \geq \sqrt{2}$ (4) $a \geq 1$

59. The condition that $x^3 + ax^2 + bx + c$ may have no extremum, is :

(1) $a^2 > 3b$ (2) $a^2 < 3b$ (3) $a^2 > 2b$ (4) $a^2 < 2b$

60. $\int \frac{3+2\cos x}{(2+3\cos x)^2} dx =$

(1) $\frac{\sin x}{2+3\cos x}+c$ (2) $\frac{\cos x}{2+3\cos x}+c$ (3) $\frac{2\sin x}{2+3\cos x}+c$ (4) $\frac{2\cos x}{2+3\cos x}+c$

61. $\lim_{x\to 1} (1-x)\tan\left(\frac{\pi x}{2}\right) =$

(1) $\frac{\pi}{2}$ (2) $\frac{2}{\pi}$ (3) $\frac{\pi}{4}$ (4) 1

62. $\lim_{n\to\infty} \frac{(1-2+3-4+5-6 \ldots\ldots\ldots\ldots -2n)}{\sqrt{n^2+1}+\sqrt{4n^2-1}} =$

(1) –2 (2) $\frac{1}{2}$ (3) $\frac{1}{3}$ (4) $-\frac{1}{3}$

63. $\lim_{x\to 0} \frac{\sin(\pi\cos^2 x)}{x^2} =$

(1) π (2) $\pi/2$ (3) $-\pi$ (4) 1

64. If $f(x) = \begin{cases} \frac{x-1}{2x^2-7x+5}, & x \neq 1 \\ -\frac{1}{3}, & x = 1 \end{cases}$

then the derivative of $f(x)$ at $x = 1$, is :

(1) $\frac{9}{2}$ (2) $\frac{-9}{2}$ (3) $\frac{-2}{9}$ (4) $\frac{2}{9}$

65. The mean of n terms is $\bar{x}$. If the first term is increased by 1, second by 2 and so on, then the new mean is :

(1) $\bar{x}+\frac{n+1}{2}$ (2) $\bar{x}+\frac{n}{2}$ (3) $\bar{x}+n$ (4) $\bar{x}+\frac{n-1}{2}$

66. The standard deviation of 25 numbers is 40. If each of the numbers which is greater than the standard deviation, is increased by 5, then the new standard deviation will be :

(1) 45 (2) 40 (3) 65 (4) 40.75

67. The sum of 10 items is 12 and the sum of their squares is 18, then the standard deviation is :

(1) 3/5 (2) 4/5 (3) 3/10 (4) 2/5

68. There are n persons sitting in a row. Two of them are selected at random. The probability that two selected persons are not sitting together, is :

(1) $\frac{2}{n-2}$ (2) $\frac{n}{n+2}$ (3) $\frac{2}{n}$ (4) $1-\frac{2}{n}$

69. Seven digits from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 are written in a random order. The probability that this seven digit number is divisible by 9, is :

(1) $\frac{1}{3}$ (2) $\frac{2}{7}$ (3) $\frac{1}{9}$ (4) $\frac{2}{9}$

70. The coefficients of a quadratic equation $ax^2 + bx + c = 0$ $(a \neq b \neq c)$ are chosen from first three prime numbers, the probability that roots of the equation are real, is :

(1) 2/3 (2) 1/3 (3) 1/4 (4) 3/4

71. The graph of the function $y = f(x)$ is symmetrical about the line $x = a$, then which of the following is ***true*** ?

(1) $f(x+a) = f(x-a)$ (2) $f(x) = f(-x)$

(3) $f(a+x) = f(a-x)$ (4) $f(x) = -f(-x)$

72. Let $f : R \to R$ be a function defined by $f(x) = \frac{x^2+2x+5}{x^2+x+1}$, then f is :

(1) one-one and onto (2) one-one and into

(3) many one and onto (4) many one and into

73. $\sin \lambda x + \cos \lambda x$ and $|\sin x| + |\cos x|$ are periodic of same fundamental period, if $\lambda =$

(1) 4 (2) 0 (3) 2 (4) 1

74. Let R be a relation on the set N of natural numbers defined by $nRm \Leftrightarrow n$ is a factor of m (i.e. n/m). Then R is :

(1) equivalence (2) transitive and symmetric

(3) reflexive and symmetric (4) reflexive, transitive but not symmetric

75. If $\sin^{-1}\left(\frac{x}{5}\right)+\operatorname{cosec}^{-1}\left(\frac{5}{4}\right)=\frac{\pi}{2}$, then $x=$

(1) 4 (2) 3 (3) 5 (4) 2

76. A solution of the equation :

$\tan^{-1}(1+x)+\tan^{-1}(1-x)=\frac{\pi}{2}$, is :

(1) $x=0$ (2) $x=1$ (3) $x=-1$ (4) $x=\pi$

77. The value of $\tan\left(\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right)$ is :

(1) $3+\sqrt{5}$ (2) $3-\sqrt{5}$ (3) $\frac{1}{2}(3-\sqrt{5})$ (4) $\frac{1}{2}(\sqrt{5}+3)$

78. Solution of $\sin^{-1}x-\cos^{-1}x=\cos^{-1}\frac{\sqrt{3}}{2}$ is :

(1) $x=\frac{1}{2}$ (2) $x=\frac{1}{\sqrt{3}}$ (3) $x=\frac{\sqrt{3}}{2}$ (4) $x=1$

79. If $A=\begin{bmatrix}1 & 0\\ -1 & 7\end{bmatrix}$ and $A^2=8A+KI_2$, then the value of K is :

(1) –1 (2) 1 (3) 7 (4) –7

80. If 1, w, w^2 are cube roots of unity, inverse of which of the following matrices exists ?

(1) $\begin{bmatrix}w & w^2\\ w^2 & 1\end{bmatrix}$ (2) $\begin{bmatrix}1 & w\\ w & w^2\end{bmatrix}$ (3) $\begin{bmatrix}w^2 & 1\\ 1 & w\end{bmatrix}$ (4) None of these

81. The line $x+y=4$ divides the line joining (–1, 1) and (5, 7) in the ratio $K:1$, then the value of K is :

(1) 1/4 (2) 4/3 (3) 1/2 (4) 2

82. If the foot of the perpendicular from the origin to a straight line is at the point (3, –4). Then the equation of the line is :

(1) $3x-4y=25$ (2) $4x-3y=25$

(3) $4x+3y=25$ (4) $3x+4y=25$

83. The distance between the parallel lines $6x - 3y - 5 = 0$ and $2x - y + 4 = 0$ is :

(1) $3/\sqrt{5}$ (2) $\sqrt{5}/3$

(3) $17/3\sqrt{5}$ (4) $17/\sqrt{3}$

84. The points $(K + 1, 1)$, $(2K + 1, 3)$ and $(2K + 2, 2K)$ are collinear, then $K =$

(1) -1 (2) $\frac{1}{3}$ (3) $\frac{1}{2}$ (4) $-\frac{1}{2}$

85. The equation of the circle of radius 5 whose centre lies on x-axis and passing through (2, 3) is :

(1) $x^2 + y^2 - 4x - 21 = 0$ (2) $x^2 + y^2 + 4x - 21 = 0$

(3) $x^2 + y^2 + 4x - 17 = 0$ (4) $x^2 + y^2 - 4x + 21 = 0$

86. If the parabola $y^2 = 4\,ax$ passes through (3, 2), then the length of its latus-rectum is :

(1) 2/3 (2) 3/4 (3) 4 (4) 4/3

87. The eccentricity of the hyperbola $16x^2 - 3y^2 - 32x + 12y - 44 = 0$ is :

(1) $\sqrt{13}$ (2) $\sqrt{7}$ (3) $\sqrt{\frac{17}{3}}$ (4) $\sqrt{\frac{19}{3}}$

88. The eccentricity of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose latus-rectum is half of its major axis, is :

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{\sqrt{3}}{4}$ (3) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{2}$

89. The ratio in which the yz-plane divides the segment joining the points (–2, 4, 7) and (3, – 5, 8) is :

(1) 7 : 8 (2) –7 : 8 (3) 2 : 3 (4) –3 : 2

90. If α, β, γ are the angles which a directed line makes with the positive directions of the co-ordinate axes, then $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma =$

(1) 0 (2) 1 (3) 2 (4) 3

91. The set A has 3 elements and the Set B has 7 elements. The minimum number of elements in the set $A \cup B$ is :

(1) 21 (2) 10 (3) 7 (4) Can not say

92. If A and B are two sets, then $A \cap (A \cup B)^C$ (where 'C' denotes complement) is equal to :

(1) ϕ (2) A (3) B (4) A – B

93. Let $A = \{0, 1, 2, 3, 4, 5\}$ and a relation R is defined by xRy such that $2x + y = 10$. Then R^{-1} is :

(1) $\{(4, 3), (2, 4), (5, 0)\}$ (2) $\{(4, 3), (2, 4), (0, 5)\}$

(3) $\{(3, 4), (4, 2), (5, 0)\}$ (4) $\{(3, 4), (4, 2), (0, 5)\}$

94. If A + C = B, then tan A tan B tan C =

(1) tan A + tan B + tan C (2) tan A + tan B – tan C

(3) tan B – tan C – tan A (4) tan B + tan C – tan A

95. If $\sin x + \sin^2 x = 1$, then $\cos^8 x + 2\cos^6 x + \cos^4 x =$

(1) 0 (2) 1 (3) –1 (4) 2

96. If $4\sin^2 x = 1$, then the values of x are :

(1) $n\pi \pm \frac{\pi}{3}$ (2) $n\pi \pm \frac{\pi}{4}$

(3) $2n\pi \pm \frac{\pi}{6}$ (4) $n\pi \pm \frac{\pi}{6}$

97. If $n \in N$, then $3^{3n} - 26n - 1$ is divisible by :

(1) 4 (2) 3 (3) 9 (4) 15

98. If $z = (K + 3) + i\sqrt{5 - k^2}$, then the locus of z is :

(1) a straight line (2) a parabola

(3) an ellipse (4) a circle

99. If 1, w and w^2 are the three cube roots of unity, then the roots of the equation $(x - 1)^3 - 8 = 0$ are :

(1) $2, 2w, 2w^2$ (2) $3, 2w, 2w^2$

(3) $3, 1 + 2w, 1 + 2w^2$ (4) $2, 1 - 2w, 1 - 2w^2$

100. The smallest positive integer n for which $\left(\frac{1+i}{1-i}\right)^n = 1$, is :

(1) 4 (2) 3 (3) 2 (4) 1

Opened at 01:45 pm for evaluation

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PG-EE-2019

SUBJECT : Mathematics Hons. (Five Year)-(SET-X)

C

10763

Sr. No. ....

Time : **1¼ Hours (75 minutes)** Total Questions : **100** Max. Marks : **100**

Roll No. (in figures)____________ (in words) ____________

Name ____________ Date of Birth ____________

Father's Name ____________ Mother's Name ____________

Date of Exam ____________

(Signature of the Candidate) (Signature of the Invigilator)

SAMPLE

CANDIDATES MUST READ THE FOLLOWING INFORMATION/INSTRUCTIONS BEFORE STARTING THE QUESTION PAPER.

1. ***All questions are compulsory and carry equal marks. The candidates are required to attempt all questions.***
2. The candidate ***must return*** this question booklet and the OMR Answer-Sheet to the Invigilator concerned before leaving the Examination Hall, failing which a case of use of unfair-means / misbehaviour will be registered against him / her, in addition to lodging of an FIR with the police. Further the answer-sheet of such a candidate will not be evaluated.
3. Keeping in view the transparency of the examination system, carbonless OMR Sheet is provided to the candidate so that a copy of OMR Sheet may be kept by the candidate.
4. Question Booklet along-with answer key of all the A, B, C and D code shall be got uploaded on the University Website immediately after the conduct of Entrance Examination. Candidates may raise valid objection/complaint if any, with regard to discrepancy in the question booklet/answer key within 24 hours of uploading the same on the University website. The complaint be sent by the students to the Controller of Examinations by hand or through email. Thereafter, no complaint in any case will be considered.
5. The candidate ***must not*** do any rough work or writing in the OMR Answer-Sheet. Rough work, if any, may be done in the question booklet itself. Answers ***must not*** be ticked in the question booklet.
6. ***Use only black or blue ball point pen of good quality in the OMR Answer-Sheet.***
7. There will be ***no negative*** marking. Each correct answer will be awarded ***one*** full mark. Cutting, erasing, overwriting and more than one answer in OMR Answer-Sheet will be treated as incorrect answer.
8. *Before answering the questions, the candidates should ensure that they have been supplied correct and complete question booklet. Complaints, if any, regarding misprinting etc. will not be entertained* **30** *minutes after starting of the examination.*

1. The graph of the function $y = f(x)$ is symmetrical about the line $x = a$, then which of the following is ***true*** ?
(1) $f(x+a) = f(x-a)$ (2) $f(x) = f(-x)$
(3) $f(a+x) = f(a-x)$ (4) $f(x) = -f(-x)$

2. Let $f : R \to R$ be a function defined by $f(x) = \dfrac{x^2+2x+5}{x^2+x+1}$, then f is :
(1) one-one and onto (2) one-one and into
(3) many one and onto (4) many one and into

3. $\sin \lambda x + \cos \lambda x$ and $|\sin x| + |\cos x|$ are periodic of same fundamental period, if $\lambda =$
(1) 4 (2) 0 (3) 2 (4) 1

4. Let R be a relation on the set N of natural numbers defined by $nRm \Leftrightarrow n$ is a factor of m (i.e. n/m). Then R is :
(1) equivalence (2) transitive and symmetric
(3) reflexive and symmetric (4) reflexive, transitive but not symmetric

5. If $\sin^{-1}\left(\dfrac{x}{5}\right) + \operatorname{cosec}^{-1}\left(\dfrac{5}{4}\right) = \dfrac{\pi}{2}$, then $x =$
(1) 4 (2) 3 (3) 5 (4) 2

6. A solution of the equation :
$\tan^{-1}(1+x) + \tan^{-1}(1-x) = \dfrac{\pi}{2}$, is :
(1) $x = 0$ (2) $x = 1$ (3) $x = -1$ (4) $x = \pi$

7. The value of $\tan\left(\dfrac{1}{2}\cos^{-1}\left(\dfrac{\sqrt{5}}{3}\right)\right)$ is :
(1) $3+\sqrt{5}$ (2) $3-\sqrt{5}$ (3) $\dfrac{1}{2}(3-\sqrt{5})$ (4) $\dfrac{1}{2}(\sqrt{5}+3)$

8. Solution of $\sin^{-1} x - \cos^{-1} x = \cos^{-1}\dfrac{\sqrt{3}}{2}$ is :
(1) $x = \dfrac{1}{2}$ (2) $x = \dfrac{1}{\sqrt{3}}$ (3) $x = \dfrac{\sqrt{3}}{2}$ (4) $x = 1$

9. If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$ and $A^2 = 8A + KI_2$, then the value of K is :

(1) –1 (2) 1 (3) 7 (4) –7

10. If 1, w, w^2 are cube roots of unity, inverse of which of the following matrices exists ?

(1) $\begin{bmatrix} w & w^2 \\ w^2 & 1 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & w \\ w & w^2 \end{bmatrix}$ (3) $\begin{bmatrix} w^2 & 1 \\ 1 & w \end{bmatrix}$ (4) None of these

11. The line $x + y = 4$ divides the line joining (–1, 1) and (5, 7) in the ratio $K : 1$, then the value of K is :

(1) 1/4 (2) 4/3 (3) 1/2 (4) 2

12. If the foot of the perpendicular from the origin to a straight line is at the point (3, –4). Then the equation of the line is :

(1) $3x - 4y = 25$ (2) $4x - 3y = 25$

(3) $4x + 3y = 25$ (4) $3x + 4y = 25$

13. The distance between the parallel lines $6x - 3y - 5 = 0$ and $2x - y + 4 = 0$ is :

(1) $3/\sqrt{5}$ (2) $\sqrt{5}/3$

(3) $17/3\sqrt{5}$ (4) $17/\sqrt{3}$

14. The points $(K + 1, 1)$, $(2K + 1, 3)$ and $(2K + 2, 2K)$ are collinear, then $K =$

(1) –1 (2) $\frac{1}{3}$ (3) $\frac{1}{2}$ (4) $-\frac{1}{2}$

15. The equation of the circle of radius 5 whose centre lies on x-axis and passing through (2, 3) is :

(1) $x^2 + y^2 - 4x - 21 = 0$ (2) $x^2 + y^2 + 4x - 21 = 0$

(3) $x^2 + y^2 + 4x - 17 = 0$ (4) $x^2 + y^2 - 4x + 21 = 0$

16. If the parabola $y^2 = 4\,ax$ passes through (3, 2), then the length of its latus-rectum is :

(1) 2/3 (2) 3/4 (3) 4 (4) 4/3

17. The eccentricity of the hyperbola $16x^2 - 3y^2 - 32x + 12y - 44 = 0$ is :

(1) $\sqrt{13}$ (2) $\sqrt{7}$ (3) $\sqrt{\frac{17}{3}}$ (4) $\sqrt{\frac{19}{3}}$

18. The eccentricity of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ whose latus-rectum is half of its major axis, is :

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{\sqrt{3}}{4}$ (3) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{2}$

19. The ratio in which the *yz*-plane divides the segment joining the points (–2, 4, 7) and (3, – 5, 8) is :

(1) 7 : 8 (2) –7 : 8 (3) 2 : 3 (4) –3 : 2

20. If α, β, γ are the angles which a directed line makes with the positive directions of the co-ordinate axes, then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma =$

(1) 0 (2) 1 (3) 2 (4) 3

21. The set A has 3 elements and the Set B has 7 elements. The minimum number of elements in the set A ∪ B is :

(1) 21 (2) 10 (3) 7 (4) Can not say

22. If A and B are two sets, then $A \cap (A \cup B)^C$ (where 'C' denotes complement) is equal to :

(1) ϕ (2) A (3) B (4) A – B

23. Let $A = \{0, 1, 2, 3, 4, 5\}$ and a relation R is defined by xRy such that $2x + y = 10$. Then R^{-1} is :

(1) {(4, 3), (2, 4), (5, 0)} (2) {(4, 3), (2, 4), (0, 5)}

(3) {(3, 4), (4, 2), (5, 0)} (4) {(3, 4), (4, 2), (0, 5)}

24. If A + C = B, then tan A tan B tan C =

(1) tan A + tan B + tan C (2) tan A + tan B – tan C

(3) tan B – tan C – tan A (4) tan B + tan C – tan A

25. If $\sin x + \sin^2 x = 1$, then $\cos^8 x + 2\cos^6 x + \cos^4 x =$

(1) 0 (2) 1 (3) –1 (4) 2

26. If $4\sin^2 x = 1$, then the values of x are :

(1) $n\pi \pm \frac{\pi}{3}$ (2) $n\pi \pm \frac{\pi}{4}$

(3) $2n\pi \pm \frac{\pi}{6}$ (4) $n\pi \pm \frac{\pi}{6}$

27. If $n \in N$, then $3^{3n} - 26n - 1$ is divisible by :
(1) 4 (2) 3 (3) 9 (4) 15

28. If $z = (K+3) + i\sqrt{5-k^2}$, then the locus of z is :
(1) a straight line (2) a parabola (3) an ellipse (4) a circle

29. If 1, w and w^2 are the three cube roots of unity, then the roots of the equation $(x-1)^3 - 8 = 0$ are :
(1) $2, 2w, 2w^2$
(2) $3, 2w, 2w^2$
(3) $3, 1+2w, 1+2w^2$
(4) $2, 1-2w, 1-2w^2$

30. The smallest positive integer n for which $\left(\frac{1+i}{1-i}\right)^n = 1$, is :
(1) 4 (2) 3 (3) 2 (4) 1

31. The unit vector perpendicular to the vectors $\hat{i}+\hat{j}$ and $\hat{j}+\hat{k}$ is/are :
(1) $\pm\frac{1}{\sqrt{3}}(\hat{i}-\hat{j}-\hat{k})$
(2) $\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}-\hat{k})$
(3) $\pm\frac{1}{\sqrt{3}}(\hat{i}-\hat{j}+\hat{k})$
(4) $\frac{1}{\sqrt{2}}(\hat{i}+\hat{k})$

32. The vector $\hat{i}+x\hat{j}+3\hat{k}$ is rotated through an angle θ and doubled in magnitude, then it becomes $4\hat{i}+(4x-2)\hat{j}+2\hat{k}$. The value of x is :
(1) $\frac{1}{3}$ (2) -3 (3) $\frac{2}{3}$ (4) $-\frac{2}{3}$

33. The vectors $\overrightarrow{AB} = 3\hat{i}+2\hat{j}+2\hat{k}$ and $\overrightarrow{BC} = -\hat{i}-2\hat{k}$ are the adjacent sides of a parallelogram. The angle between its diagonals is :
(1) $\pi/6$ (2) $\pi/3$ (3) $\pi/2$ (4) $\pi/4$

34. Consider a LPP : min $Z = 6x + 10y$
subjected to $x \ge 6, y \ge 2, 2x + y \ge 10; x, y \ge 0$.
Redundand constraints in this LPP are :
(1) $x \ge 0, y \ge 0$
(2) $2x + y \ge 10$
(3) $x \ge 6, 2x + y \ge 10$
(4) None of these

35. The angle between the lines having direction ratios 4, –3, 5 and 3, 4, 5 is :

(1) $\pi/3$ (2) $\pi/4$ (3) $\pi/6$ (4) $2\pi/3$

36. If a plane meets the coordinate axes at A, B and C in such a way that the centroid of triangle ABC it at the point (1, 2, 3), then the equation of the plane is :

(1) $\frac{x}{1}+\frac{y}{2}+\frac{z}{3}=1$ (2) $\frac{x}{3}+\frac{y}{6}+\frac{z}{9}=1$

(3) $\frac{x}{1}+\frac{y}{2}+\frac{z}{3}=\frac{1}{3}$ (4) $\frac{x}{3}+\frac{y}{6}+\frac{z}{9}=3$

37. The image of the point (1, 3, 4) in the plane $2x-y+z+3=0$ is :

(1) (3, 5, 2) (2) (3, 5, –2)

(3) (–3, 5, 2) (4) (3, –5, 2)

38. The lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x-1}{-2}=\frac{y-2}{-4}=\frac{z-3}{-6}$ are :

(1) intersecting (2) parallel

(3) coincidental (4) skew

39. The distance between the line $\vec{r}=2\hat{i}-2\hat{j}+3\hat{k}+\lambda(\hat{i}-\hat{j}+4\hat{k})$ and the plane $\vec{r}.(\hat{i}+5\hat{j}+\hat{k})=5$ is :

(1) $\frac{10}{3\sqrt{3}}$ (2) $\frac{10}{\sqrt{3}}$ (3) $\frac{10}{3}$ (4) $\frac{5}{3\sqrt{3}}$

40. The lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar if $k=$

(1) 4 (2) 3 (3) –1 (4) –3

41. If $x = a\cos^3\theta$, $y = a\sin^3\theta$, then $\sqrt{1+\left(\frac{dy}{dx}\right)^2}=$

(1) $\sec^2\theta$ (2) $\tan^2\theta$ (3) $|\sec\theta|$ (4) $|\cot\theta|$

42. If $e^y + xy = e$, then the value of $\frac{d^2y}{dx^2}$ for $x = 0$, is :

(1) e^2 (2) $\frac{1}{e^2}$ (3) $\frac{1}{e}$ (4) $\frac{1}{e^3}$

43. If $x^y . y^x = 16$, then $\frac{dy}{dx}$ at (2, 2) is :

(1) 0 (2) 1 (3) –1 (4) –4

44. The approximate value of square root of 25.2 is :

(1) 5.01 (2) 5.02 (3) 5.03 (4) 5.04

45. The tangent at (1, 1) on the curve $y^2 = x(2-x)^2$ meets it again at the point :

(1) (–3, 7) (2) (4, 4) (3) $\left(\frac{3}{8}, \frac{9}{4}\right)$ (4) $\left(\frac{9}{4}, \frac{3}{8}\right)$

46. The distance between the origin and the normal to the curve $y = e^{2x} + x^2$ at $x = 0$ is :

(1) $4/\sqrt{5}$ (2) $3/\sqrt{5}$ (3) $2/\sqrt{5}$ (4) $2/\sqrt{7}$

47. The length of longest interval in which Rolle's theorem can be applied for the function $f(x) = |x^2 - a^2|$, $(a > 0)$, is :

(1) $2a$ (2) $3a$ (3) $4a$ (4) $a\sqrt{2}$

48. If the function $f(x) = \sqrt{3}\sin x - \cos x - 2ax + b$ decreases for all real values of x, then the value of a is given by :

(1) $a < 1$ (2) $a < \sqrt{2}$ (3) $a \geq \sqrt{2}$ (4) $a \geq 1$

49. The condition that $x^3 + ax^2 + bx + c$ may have no extremum, is :

(1) $a^2 > 3b$ (2) $a^2 < 3b$ (3) $a^2 > 2b$ (4) $a^2 < 2b$

50. $\int \frac{3+2\cos x}{(2+3\cos x)^2} dx =$

(1) $\frac{\sin x}{2+3\cos x} + c$ (2) $\frac{\cos x}{2+3\cos x} + c$ (3) $\frac{2\sin x}{2+3\cos x} + c$ (4) $\frac{2\cos x}{2+3\cos x} + c$

51. $\lim_{x \to 1} (1-x)\tan\left(\frac{\pi x}{2}\right) =$

(1) $\frac{\pi}{2}$ (2) $\frac{2}{\pi}$ (3) $\frac{\pi}{4}$ (4) 1

52. $\lim_{n \to \infty} \frac{(1-2+3-4+5-6............-2n)}{\sqrt{n^2+1}+\sqrt{4n^2-1}} =$

(1) –2 (2) $\frac{1}{2}$ (3) $\frac{1}{3}$ (4) $-\frac{1}{3}$

53. $\lim_{x \to 0} \frac{\sin(\pi \cos^2 x)}{x^2} =$

(1) π (2) $\pi/2$ (3) $-\pi$ (4) 1

54. If $f(x) = \begin{cases} \frac{x-1}{2x^2 - 7x + 5}, & x \neq 1 \\ -\frac{1}{3}, & x = 1 \end{cases}$

then the derivative of $f(x)$ at $x = 1$, is :

(1) $\frac{9}{2}$ (2) $\frac{-9}{2}$ (3) $\frac{-2}{9}$ (4) $\frac{2}{9}$

55. The mean of n terms is $\bar{x}$. If the first term is increased by 1, second by 2 and so on, then the new mean is :

(1) $\bar{x} + \frac{n+1}{2}$ (2) $\bar{x} + \frac{n}{2}$ (3) $\bar{x} + n$ (4) $\bar{x} + \frac{n-1}{2}$

56. The standard deviation of 25 numbers is 40. If each of the numbers which is greater than the standard deviation, is increased by 5, then the new standard deviation will be :

(1) 45 (2) 40 (3) 65 (4) 40.75

57. The sum of 10 items is 12 and the sum of their squares is 18, then the standard deviation is :

(1) 3/5 (2) 4/5 (3) 3/10 (4) 2/5

58. There are n persons sitting in a row. Two of them are selected at random. The probability that two selected persons are not sitting together, is :

(1) $\frac{2}{n-2}$ (2) $\frac{n}{n+2}$ (3) $\frac{2}{n}$ (4) $1 - \frac{2}{n}$

59. Seven digits from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 are written in a random order. The probability that this seven digit number is divisible by 9, is :

(1) $\frac{1}{3}$ (2) $\frac{2}{7}$ (3) $\frac{1}{9}$ (4) $\frac{2}{9}$

60. The coefficients of a quadratic equation $ax^2 + bx + c = 0$ $(a \neq b \neq c)$ are chosen from first three prime numbers, the probability that roots of the equation are real, is :

(1) 2/3 (2) 1/3 (3) 1/4 (4) 3/4

61. $\int x^x (1 + \log x)\, dx =$

(1) $x^x + c$ (2) $x^x \log x + c$ (3) $x \log x + c$ (4) none of these

62. $\int \sin \sqrt{x} dx =$

(1) $(\cos\sqrt{x} - \sin\sqrt{x}) + c$

(2) $(\sqrt{x}\cos\sqrt{x} - \sin\sqrt{x}) + c$

(3) $-2(\sqrt{x}\cos\sqrt{x} - \sin\sqrt{x}) + c$

(4) $2(\sqrt{x}\cos\sqrt{x} - \sin\sqrt{x}) + c$

63. $\int \frac{1}{\sqrt{\sin^3 x \cos x}} dx =$

(1) $2(\tan x)^{-1/2} + c$

(2) $(\tan x)^{1/2} + c$

(3) $(\tan x)^{-1/2} + c$

(4) $2(\tan x)^{1/2} + c$

64. If $I_n = \int_0^1 x^n e^{-x} dx$ for $n \in N$, then $I_n - n\, I_{n-1} =$

(1) $1/e$ (2) $-1/e$ (3) e (4) $-2/e$

65. If $\int_{\pi/2}^{\theta} \sin x\, dx = \sin 2\theta$, then the value of θ satisfying $0 < \theta < \pi$, is :

(1) $\pi/6$ (2) $\pi/4$ (3) $\pi/2$ (4) $5\pi/6$

66. $\int_0^{[x]} (x - [x])\, dx =$

(1) $\frac{1}{2}[x]$ (2) $[x]$ (3) $2[x]$ (4) $-2[x]$

67. $\int_0^{\pi/4} \log(1 + \tan x) dx =$

(1) $\frac{\pi}{4}\log 2$ (2) $\frac{\pi}{8}\log 2$ (3) $\frac{\pi}{2}\log 2$ (4) $\pi \log 2$

68. The area bounded by the curve $y = x \sin x$ and x-axis between $x = 0$ and $x = 2\pi$, is :

(1) π sq. units

(2) $\frac{\pi}{2}$ sq. units

(3) 2π sq. units

(4) 4π sq. units

69. If the area bounded by the curves $y^2 = 4ax$ and $y = mx$ is $a^2/3$ sq. units, then the value of m is :

(1) 2 (2) -2 (3) $1/2$ (4) $3/2$

70. Solution of $\frac{dy}{dx} = \cos(x+y)$ is :

(1) $\sin(x+y) = x + c$

(2) $\tan\left(\frac{x+y}{2}\right) + x = c$

(3) $\cot\left(\frac{x+y}{2}\right) = x + c$

(4) $\tan\left(\frac{x+y}{2}\right) = x + c$

71. Solution of $ydx + (x - y^3)\,dy = 0$ is :

(1) $xy + \frac{y^2}{2} = c$

(2) $xy = \frac{y^2}{2} + c$

(3) $xy = \frac{y^2}{4} + c$

(4) $xy = \frac{x^2}{4} + c$

72. The differential equation $y\frac{dy}{dx} = x + a$ (a being constant) represents a set of :

(1) circles having centre on the x-axis

(2) circles having centre on the y-axis

(3) ellipses

(4) hyperbolas

73. From a bag containing 2 white, 3 red and 4 black balls, two balls are drawn one by one without replacement. The probability that at least ones ball is red, is :

(1) $\frac{5}{12}$ (2) $\frac{7}{12}$ (3) $\frac{5}{8}$ (4) $\frac{3}{7}$

74. A box contains 15 items in which 4 items are defective. The items are selected at random, one by one, and examined. The ones examined are not put back. The probability that 9th one examined is the last defective, is :

(1) $\frac{7}{195}$ (2) $\frac{8}{195}$ (3) $\frac{16}{255}$ (4) $\frac{14}{255}$

75. A person is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. The probability that it is actually a six, is :

(1) $\frac{7}{36}$ (2) $\frac{11}{36}$ (3) $\frac{3}{8}$ (4) $\frac{5}{8}$

76. Four numbers are multiplied together. The probability that the product will be divisible by 5 or 10, is :

(1) $\frac{123}{625}$ (2) $\frac{133}{625}$ (3) $\frac{357}{625}$ (4) $\frac{369}{625}$

77. The least number of times a fair coin must be tossed so that the probability of getting at least one head is at lest 0.8, is :

(1) 3 (2) 5 (3) 6 (4) 8

78. If $P(A \cup B) = \frac{3}{4}$ and $P(\overline{A}) = 2/3$, then $P(\overline{A} \cap B) =$

(1) $\frac{7}{12}$ (2) $\frac{5}{12}$ (3) $\frac{1}{12}$ (4) $\frac{1}{6}$

79. If $\vec{a} + \vec{b} + \vec{c} = \vec{0}, |\vec{a}| = 3, |\vec{b}| = 5, |\vec{c}| = 7$, then the angle between $\vec{a}$ and $\vec{b}$ is :

(1) $\pi/3$ (2) $2\pi/3$

(3) $\pi/6$ (4) $5\pi/3$

80. Let $\vec{a}$ and $\vec{b}$ be two unit vectors and α be the angle between them, then $\vec{a} + \vec{b}$ is a unit vector if $\alpha =$

(1) $\pi/2$ (2) $\pi/3$ (3) $2\pi/3$ (4) $\pi/4$

81. The value of K for which one of the roots of $x^2 - 3x + 2K = 0$ is double of one of the roots of $x^2 - x + K = 0$, is :

(1) 2 (2) –2 (3) –1 (4) 1

82. The interior angles of a regular polygon measure 160° each. The number of diagonals of the polygon are :

(1) 105 (2) 135 (3) 145 (4) 147

83. The number of ways in which 9 identical balls can be placed in three identical boxes, is :

(1) 9 (2) 12 (3) 55 (4) 27

84. In the expansion of $\left(x^2 - \frac{1}{3x}\right)^9$, the term independent of x is :

(1) 5th (2) 6th (3) 7th (4) 4th

85. If the coefficients of rth and $(r + 1)$th terms in the expansion of $(3 + 7x)^{29}$ are equal, then $r =$

(1) 14 (2) 15 (3) 18 (4) 21

86. Three numbers forms an increasing G.P. If the middle number is doubled, then the new numbers are in A. P. The common ratio of the G. P. is :

(1) $2 + \sqrt{3}$ (2) $3 + \sqrt{2}$

(3) $\sqrt{3} + 1$ (4) $3 - \sqrt{2}$

87. If $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\ldots\ldots\ldots$ to $\infty = \frac{\pi^2}{6}$, then $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\ldots\ldots =$

(1) $\frac{\pi^2}{3}$ (2) $\frac{\pi^2}{4}$ (3) $\frac{\pi^2}{8}$ (4) $\frac{\pi^2}{12}$

88. If a, b, c are in A.P. as well as G.P., then which of the following is ***true*** ?

(1) $a = b = c$ (2) $a = b \neq c$

(3) $a \neq b = c$ (4) $a \neq b \neq c$

89. If the AM of the roots of a quadratic equation in x is A and their GM is G, then the quadratic equation is :

(1) $x^2 - Ax + G^2 = 0$ (2) $x^2 - Ax + G = 0$

(3) $x^2 - 2Ax + G = 0$ (4) $x^2 - 2Ax + G^2 = 0$

90. A line passes through the point (2, 2) and is perpendicular to the line $3x + y = 3$, then its y-intercept is :

(1) 2/3 (2) 4/3 (3) 4/5 (4) 3/4

91. If A an orthogonal matrix, then which of the following is ***true*** ?

(1) $|A| = 0$ (2) $|A| = \pm 1$

(3) $|A| = \pm 2$ (4) $|A| = \pi/2$

92. If $A(\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then $A(\alpha)\,A(\beta) =$

(1) $A(\alpha) + A(\beta)$ (2) $A(\alpha) - A(\beta)$

(3) $A(\alpha + \beta)$ (4) $A(\alpha - \beta)$

93. If A and B are two matrices such that $AB = B$ and $BA = A$, then $A^2 + B^2 =$

(1) $A + B$ (2) AB (3) $2AB$ (4) I

94. If K is a real cube root of -2, then the value of $\begin{vmatrix} 1 & 2K & 1 \\ K^2 & 1 & 3K^2 \\ 2 & 2K & 1 \end{vmatrix}$ is equal to :

(1) -10 (2) -12 (3) -13 (4) -15

95. The equations $Kx - y = 2$, $2x - 3y = -K$, $3x - 2y = -1$ are consistent if $K =$

(1) 2, –3 (2) –2, 3 (3) 1, –4 (4) –1, 4

96. If $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, then $f(2x) - f(x) =$

(1) $ax(3a + 2x)$ (2) $ax(2a + 3x)$ (3) $a(2a + 3x)$ (4) $x(3a + 2x)$

97. Let $f(x) = \dfrac{1 - \tan x}{4x - \pi}, x \neq \pi/4$ and $x \in [0, \pi/2] = a, x = \pi/4$

If $f(x)$ is continuous in $[0, \pi/2]$, then $a =$

(1) 1/2 (2) –1/2 (3) 1 (4) 0

98. Let $f(x) = 1 + x (\sin x) [\cos x]$, $0 < x \leq \pi/2$, where [.] denotes the greatest integer function. Then which of the following is ***true*** ?

(1) $f(x)$ is continuous in $(0, \pi/2)$ (2) $f(x)$ is strictly increasing in $(0, \pi/2)$

(3) $f(x)$ is strictly decreasing in $(0, \pi/2)$ (4) $f(x)$ has global maximum value 2

99. If $y = \tan^{-1} \sqrt{\dfrac{1 + \sin x}{1 - \sin x}}$, $\pi/2 < x < \pi$, then $\dfrac{dy}{dx} =$

(1) –1 (2) 1 (3) 1/2 (4) –1/2

100. If $x = e^{y + e^{y + e^{y + \ldots\ldots\ldots\infty}}}$, then $\dfrac{dy}{dx} =$

(1) $\dfrac{1 - x}{x}$ (2) $\dfrac{x}{1 - x}$ (3) $\dfrac{1 + x}{x}$ (4) $\dfrac{x}{1 + x}$

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PG-EE-2019

SUBJECT : Mathematics Hons. (Five Year)-(SET-X)

D

Sr. No. 10760

Time : **1¼ Hours (75 minutes)** Total Questions : **100** Max. Marks : **100**

Roll No. (in figures)__________ (in words) __________

Name __________ Date of Birth __________

Father's Name __________ Mother's Name __________

Date of Exam __________

(Signature of the Candidate) (Signature of the Invigilator)

SAMPLE

CANDIDATES MUST READ THE FOLLOWING INFORMATION/INSTRUCTIONS BEFORE STARTING THE QUESTION PAPER.

1. ***All questions are compulsory and carry equal marks. The candidates are required to attempt all questions.***
2. The candidate ***must return*** this question booklet and the OMR Answer-Sheet to the Invigilator concerned before leaving the Examination Hall, failing which a case of use of unfair-means / misbehaviour will be registered against him / her, in addition to lodging of an FIR with the police. Further the answer-sheet of such a candidate will not be evaluated.
3. Keeping in view the transparency of the examination system, carbonless OMR Sheet is provided to the candidate so that a copy of OMR Sheet may be kept by the candidate.
4. Question Booklet along-with answer key of all the A, B, C and D code shall be got uploaded on the University Website immediately after the conduct of Entrance Examination. Candidates may raise valid objection/complaint if any, with regard to discrepancy in the question booklet/answer key within 24 hours of uploading the same on the University website. The complaint be sent by the students to the Controller of Examinations by hand or through email. Thereafter, no complaint in any case will be considered.
5. The candidate ***must not*** do any rough work or writing in the OMR Answer-Sheet. Rough work, if any, may be done in the question booklet itself. Answers ***must not*** be ticked in the question booklet.
6. ***Use only black or blue ball point pen of good quality in the OMR Answer-Sheet.***
7. There will be ***no negative*** marking. Each correct answer will be awarded ***one*** full mark. Cutting, erasing, overwriting and more than one answer in OMR Answer-Sheet will be treated as incorrect answer.
8. *Before answering the questions, the candidates should ensure that they have been supplied correct and complete question booklet. Complaints, if any, regarding misprinting etc. will not be entertained* **30** *minutes after starting of the examination.*

Sumeet Poonam

1. The value of K for which one of the roots of $x^2 - 3x + 2K = 0$ is double of one of the roots of $x^2 - x + K = 0$, is :
(1) 2 (2) –2 (3) –1 (4) 1

2. The interior angles of a regular polygon measure 160° each. The number of diagonals of the polygon are :
(1) 105 (2) 135 (3) 145 (4) 147

3. The number of ways in which 9 identical balls can be placed in three identical boxes, is :
(1) 9 (2) 12 (3) 55 (4) 27

4. In the expansion of $\left(x^2 - \frac{1}{3x}\right)^9$, the term independent of x is :
(1) 5th (2) 6th (3) 7th (4) 4th

5. If the coefficients of rth and $(r + 1)$th terms in the expansion of $(3 + 7x)^{29}$ are equal, then $r =$
(1) 14 (2) 15 (3) 18 (4) 21

6. Three numbers forms an increasing G.P. If the middle number is doubled, then the new numbers are in A. P. The common ratio of the G. P. is :
(1) $2 + \sqrt{3}$ (2) $3 + \sqrt{2}$
(3) $\sqrt{3} + 1$ (4) $3 - \sqrt{2}$

7. If $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots\ldots\ldots$ to $\infty = \frac{\pi^2}{6}$, then $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots\ldots =$
(1) $\frac{\pi^2}{3}$ (2) $\frac{\pi^2}{4}$ (3) $\frac{\pi^2}{8}$ (4) $\frac{\pi^2}{12}$

8. If a, b, c are in A.P. as well as G.P., then which of the following is ***true*** ?
(1) $a = b = c$ (2) $a = b \neq c$ (3) $a \neq b = c$ (4) $a \neq b \neq c$

9. If the AM of the roots of a quadratic equation in x is A and their GM is G, then the quadratic equation is :
(1) $x^2 - Ax + G^2 = 0$ (2) $x^2 - Ax + G = 0$
(3) $x^2 - 2Ax + G = 0$ (4) $x^2 - 2Ax + G^2 = 0$

10. A line passes through the point (2, 2) and is perpendicular to the line $3x + y = 3$, then its y-intercept is :

(1) 2/3 (2) 4/3 (3) 4/5 (4) 3/4

11. The unit vector perpendicular to the vectors $\hat{i} + \hat{j}$ and $\hat{j} + \hat{k}$ is/are :

(1) $\pm\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} - \hat{k})$ (2) $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} - \hat{k})$

(3) $\pm\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} + \hat{k})$ (4) $\frac{1}{\sqrt{2}}(\hat{i} + \hat{k})$

12. The vector $\hat{i} + x\hat{j} + 3\hat{k}$ is rotated through an angle θ and doubled in magnitude, then it becomes $4\hat{i} + (4x-2)\hat{j} + 2\hat{k}$. The value of x is :

(1) $\frac{1}{3}$ (2) -3 (3) $\frac{2}{3}$ (4) $-\frac{2}{3}$

13. The vectors $\overrightarrow{AB} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\overrightarrow{BC} = -\hat{i} - 2\hat{k}$ are the adjacent sides of a parallelogram. The angle between its diagonals is :

(1) π/6 (2) π/3 (3) π/2 (4) π/4

14. Consider a LPP : min $Z = 6x + 10y$
subjected to $x \geq 6, y \geq 2, 2x + y \geq 10; x, y \geq 0$.
Redundand constraints in this LPP are :

(1) $x \geq 0, y \geq 0$ (2) $2x + y \geq 10$

(3) $x \geq 6, 2x + y \geq 10$ (4) None of these

15. The angle between the lines having direction ratios 4, –3, 5 and 3, 4, 5 is :

(1) π/3 (2) π/4 (3) π/6 (4) 2π/3

16. If a plane meets the coordinate axes at A, B and C in such a way that the centroid of triangle ABC it at the point (1, 2, 3), then the equation of the plane is :

(1) $\frac{x}{1} + \frac{y}{2} + \frac{z}{3} = 1$ (2) $\frac{x}{3} + \frac{y}{6} + \frac{z}{9} = 1$ (3) $\frac{x}{1} + \frac{y}{2} + \frac{z}{3} = \frac{1}{3}$ (4) $\frac{x}{3} + \frac{y}{6} + \frac{z}{9} = 3$

17. The image of the point (1, 3, 4) in the plane $2x - y + z + 3 = 0$ is :

(1) (3, 5, 2) (2) (3, 5, –2)

(3) (–3, 5, 2) (4) (3, –5, 2)

18. The lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x-1}{-2}=\frac{y-2}{-4}=\frac{z-3}{-6}$ are :

(1) intersecting (2) parallel (3) coincidental (4) skew

19. The distance between the line $\vec{r}=2\hat{i}-2\hat{j}+3\hat{k}+\lambda(\hat{i}-\hat{j}+4\hat{k})$ and the plane $\vec{r}.(\hat{i}+5\hat{j}+\hat{k})=5$ is :

(1) $\frac{10}{3\sqrt{3}}$ (2) $\frac{10}{\sqrt{3}}$ (3) $\frac{10}{3}$ (4) $\frac{5}{3\sqrt{3}}$

20. The lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar if $k=$

(1) 4 (2) 3 (3) –1 (4) –3

21. $\int x^x (1+\log x)\, dx=$

(1) x^x+c (2) $x^x \log x+c$ (3) $x \log x+c$ (4) none of these

22. $\int \sin \sqrt{x} dx=$

(1) $(\cos\sqrt{x}-\sin\sqrt{x})+c$
(2) $(\sqrt{x}\cos\sqrt{x}-\sin\sqrt{x})+c$
(3) $-2(\sqrt{x}\cos\sqrt{x}-\sin\sqrt{x})+c$
(4) $2(\sqrt{x}\cos\sqrt{x}-\sin\sqrt{x})+c$

23. $\int \frac{1}{\sqrt{\sin^3 x \cos x}} dx=$

(1) $2(\tan x)^{-1/2}+c$ (2) $(\tan x)^{1/2}+c$ (3) $(\tan x)^{-1/2}+c$ (4) $2(\tan x)^{1/2}+c$

24. If $I_n=\int_0^1 x^n e^{-x} dx$ for $n \in N$, then $I_n - n\, I_{n-1}=$

(1) $1/e$ (2) $-1/e$ (3) e (4) $-2/e$

25. If $\int_{\pi/2}^{\theta} \sin x\, dx=\sin 2\theta$, then the value of θ satisfying $0<\theta<\pi$, is :

(1) $\pi/6$ (2) $\pi/4$ (3) $\pi/2$ (4) $5\pi/6$

26. $\int_0^{[x]} (x-[x])\, dx=$

(1) $\frac{1}{2}[x]$ (2) $[x]$ (3) $2[x]$ (4) $-2[x]$

27. $\int_0^{\pi/4} \log(1+\tan x)dx =$

(1) $\frac{\pi}{4}\log 2$ (2) $\frac{\pi}{8}\log 2$ (3) $\frac{\pi}{2}\log 2$ (4) $\pi \log 2$

28. The area bounded by the curve $y = x \sin x$ and x-axis between $x = 0$ and $x = 2\pi$, is :

(1) π sq. units (2) $\frac{\pi}{2}$ sq. units (3) 2π sq. units (4) 4π sq. units

29. If the area bounded by the curves $y^2 = 4ax$ and $y = mx$ is $a^2/3$ sq. units, then the value of m is :

(1) 2 (2) –2 (3) 1/2 (4) 3/2

30. Solution of $\frac{dy}{dx} = \cos(x+y)$ is :

(1) $\sin(x+y) = x + c$

(2) $\tan\left(\frac{x+y}{2}\right) + x = c$

(3) $\cot\left(\frac{x+y}{2}\right) = x + c$

(4) $\tan\left(\frac{x+y}{2}\right) = x + c$

31. If A an orthogonal matrix, then which of the following is ***true*** ?

(1) $|A| = 0$ (2) $|A| = \pm 1$ (3) $|A| = \pm 2$ (4) $|A| = \pi/2$

32. If $A(\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then $A(\alpha)\,A(\beta) =$

(1) $A(\alpha) + A(\beta)$

(2) $A(\alpha) - A(\beta)$

(3) $A(\alpha + \beta)$

(4) $A(\alpha - \beta)$

33. If A and B are two matrices such that $AB = B$ and $BA = A$, then $A^2 + B^2 =$

(1) $A + B$ (2) AB (3) $2AB$ (4) I

34. If K is a real cube root of –2, then the value of $\begin{vmatrix} 1 & 2K & 1 \\ K^2 & 1 & 3K^2 \\ 2 & 2K & 1 \end{vmatrix}$ is equal to :

(1) –10 (2) –12 (3) –13 (4) –15

35. The equations $Kx - y = 2$, $2x - 3y = -K$, $3x - 2y = -1$ are consistent if $K =$

(1) 2, –3 (2) –2, 3 (3) 1, –4 (4) –1, 4

36. If $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, then $f(2x) - f(x) =$

(1) $ax(3a + 2x)$ (2) $ax(2a + 3x)$ (3) $a(2a + 3x)$ (4) $x(3a + 2x)$

37. Let $f(x) = \dfrac{1 - \tan x}{4x - \pi}, x \neq \pi/4$ and $x \in [0, \pi/2] = a, x = \pi/4$

If $f(x)$ is continuous in $[0, \pi/2]$, then $a =$

(1) 1/2 (2) –1/2 (3) 1 (4) 0

38. Let $f(x) = 1 + x (\sin x) [\cos x]$, $0 < x \leq \pi/2$, where [.] denotes the greatest integer function. Then which of the following is ***true*** ?

(1) $f(x)$ is continuous in $(0, \pi/2)$ (2) $f(x)$ is strictly increasing in $(0, \pi/2)$

(3) $f(x)$ is strictly decreasing in $(0, \pi/2)$ (4) $f(x)$ has global maximum value 2

39. If $y = \tan^{-1} \sqrt{\dfrac{1 + \sin x}{1 - \sin x}}$, $\pi/2 < x < \pi$, then $\dfrac{dy}{dx} =$

(1) –1 (2) 1 (3) 1/2 (4) –1/2

40. If $x = e^{y + e^{y + e^{y + \ldots\ldots\ldots\ldots \infty}}}$, then $\dfrac{dy}{dx} =$

(1) $\dfrac{1 - x}{x}$ (2) $\dfrac{x}{1 - x}$ (3) $\dfrac{1 + x}{x}$ (4) $\dfrac{x}{1 + x}$

41. $\lim\limits_{x \to 1} (1 - x) \tan\left(\dfrac{\pi x}{2}\right) =$

(1) $\dfrac{\pi}{2}$ (2) $\dfrac{2}{\pi}$ (3) $\dfrac{\pi}{4}$ (4) 1

42. $\lim\limits_{n \to \infty} \dfrac{(1 - 2 + 3 - 4 + 5 - 6 \ldots\ldots\ldots\ldots - 2n)}{\sqrt{n^2 + 1} + \sqrt{4n^2 - 1}} =$

(1) –2 (2) $\dfrac{1}{2}$ (3) $\dfrac{1}{3}$ (4) $-\dfrac{1}{3}$

43. $\lim_{x \to 0} \frac{\sin(\pi \cos^2 x)}{x^2} =$

(1) π (2) $\pi/2$ (3) $-\pi$ (4) 1

44. If $f(x) = \begin{cases} \frac{x-1}{2x^2 - 7x + 5}, & x \neq 1 \\ -\frac{1}{3}, & x = 1 \end{cases}$

then the derivative of $f(x)$ at $x = 1$, is :

(1) $\frac{9}{2}$ (2) $\frac{-9}{2}$ (3) $\frac{-2}{9}$ (4) $\frac{2}{9}$

45. The mean of n terms is $\bar{x}$. If the first term is increased by 1, second by 2 and so on, then the new mean is :

(1) $\bar{x} + \frac{n+1}{2}$ (2) $\bar{x} + \frac{n}{2}$ (3) $\bar{x} + n$ (4) $\bar{x} + \frac{n-1}{2}$

46. The standard deviation of 25 numbers is 40. If each of the numbers which is greater than the standard deviation, is increased by 5, then the new standard deviation will be :

(1) 45 (2) 40 (3) 65 (4) 40.75

47. The sum of 10 items is 12 and the sum of their squares is 18, then the standard deviation is :

(1) 3/5 (2) 4/5 (3) 3/10 (4) 2/5

48. There are n persons sitting in a row. Two of them are selected at random. The probability that two selected persons are not sitting together, is :

(1) $\frac{2}{n-2}$ (2) $\frac{n}{n+2}$ (3) $\frac{2}{n}$ (4) $1 - \frac{2}{n}$

49. Seven digits from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 are written in a random order. The probability that this seven digit number is divisible by 9, is :

(1) $\frac{1}{3}$ (2) $\frac{2}{7}$ (3) $\frac{1}{9}$ (4) $\frac{2}{9}$

50. The coefficients of a quadratic equation $ax^2 + bx + c = 0$ $(a \neq b \neq c)$ are chosen from first three prime numbers, the probability that roots of the equation are real, is :

(1) 2/3 (2) 1/3 (3) 1/4 (4) 3/4

51. The line $x + y = 4$ divides the line joining $(-1, 1)$ and $(5, 7)$ in the ratio $K : 1$, then the value of K is :

(1) 1/4 (2) 4/3 (3) 1/2 (4) 2

52. If the foot of the perpendicular from the origin to a straight line is at the point $(3, -4)$. Then the equation of the line is :

(1) $3x - 4y = 25$
(2) $4x - 3y = 25$
(3) $4x + 3y = 25$
(4) $3x + 4y = 25$

53. The distance between the parallel lines $6x - 3y - 5 = 0$ and $2x - y + 4 = 0$ is :

(1) $3/\sqrt{5}$
(2) $\sqrt{5}/3$
(3) $17/3\sqrt{5}$
(4) $17/\sqrt{3}$

54. The points $(K + 1, 1)$, $(2K + 1, 3)$ and $(2K + 2, 2K)$ are collinear, then $K =$

(1) -1 (2) $\frac{1}{3}$ (3) $\frac{1}{2}$ (4) $-\frac{1}{2}$

55. The equation of the circle of radius 5 whose centre lies on x-axis and passing through $(2, 3)$ is :

(1) $x^2 + y^2 - 4x - 21 = 0$
(2) $x^2 + y^2 + 4x - 21 = 0$
(3) $x^2 + y^2 + 4x - 17 = 0$
(4) $x^2 + y^2 - 4x + 21 = 0$

56. If the parabola $y^2 = 4\,ax$ passes through $(3, 2)$, then the length of its latus-rectum is :

(1) 2/3 (2) 3/4 (3) 4 (4) 4/3

57. The eccentricity of the hyperbola $16x^2 - 3y^2 - 32x + 12y - 44 = 0$ is :

(1) $\sqrt{13}$ (2) $\sqrt{7}$ (3) $\sqrt{\frac{17}{3}}$ (4) $\sqrt{\frac{19}{3}}$

58. The eccentricity of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose latus-rectum is half of its major axis, is :

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{\sqrt{3}}{4}$ (3) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{2}$

59. The ratio in which the yz-plane divides the segment joining the points $(-2, 4, 7)$ and $(3, -5, 8)$ is :

(1) 7 : 8 (2) –7 : 8 (3) 2 : 3 (4) –3 : 2

60. If α, β, γ are the angles which a directed line makes with the positive directions of the co-ordinate axes, then $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma =$

(1) 0 (2) 1 (3) 2 (4) 3

61. The graph of the function $y = f(x)$ is symmetrical about the line $x = a$, then which of the following is ***true*** ?

(1) $f(x+a) = f(x-a)$ (2) $f(x) = f(-x)$

(3) $f(a+x) = f(a-x)$ (4) $f(x) = -f(-x)$

62. Let $f : R \to R$ be a function defined by $f(x) = \dfrac{x^2+2x+5}{x^2+x+1}$, then f is :

(1) one-one and onto (2) one-one and into

(3) many one and onto (4) many one and into

63. $\sin \lambda x + \cos \lambda x$ and $|\sin x| + |\cos x|$ are periodic of same fundamental period, if $\lambda =$

(1) 4 (2) 0 (3) 2 (4) 1

64. Let R be a relation on the set N of natural numbers defined by $nRm \Leftrightarrow n$ is a factor of m (i.e. n/m). Then R is :

(1) equivalence (2) transitive and symmetric

(3) reflexive and symmetric (4) reflexive, transitive but not symmetric

65. If $\sin^{-1}\left(\dfrac{x}{5}\right) + \operatorname{cosec}^{-1}\left(\dfrac{5}{4}\right) = \dfrac{\pi}{2}$, then $x =$

(1) 4 (2) 3 (3) 5 (4) 2

66. A solution of the equation :

$\tan^{-1}(1+x) + \tan^{-1}(1-x) = \dfrac{\pi}{2}$, is :

(1) $x = 0$ (2) $x = 1$ (3) $x = -1$ (4) $x = \pi$

67. The value of $\tan\left(\dfrac{1}{2}\cos^{-1}\left(\dfrac{\sqrt{5}}{3}\right)\right)$ is :

(1) $3+\sqrt{5}$ (2) $3-\sqrt{5}$ (3) $\dfrac{1}{2}(3-\sqrt{5})$ (4) $\dfrac{1}{2}(\sqrt{5}+3)$

68. Solution of $\sin^{-1} x - \cos^{-1} x = \cos^{-1} \frac{\sqrt{3}}{2}$ is :

(1) $x = \frac{1}{2}$ (2) $x = \frac{1}{\sqrt{3}}$ (3) $x = \frac{\sqrt{3}}{2}$ (4) $x = 1$

69. If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$ and $A^2 = 8A + KI_2$, then the value of K is :

(1) –1 (2) 1 (3) 7 (4) –7

70. If 1, w, w^2 are cube roots of unity, inverse of which of the following matrices exists ?

(1) $\begin{bmatrix} w & w^2 \\ w^2 & 1 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & w \\ w & w^2 \end{bmatrix}$

(3) $\begin{bmatrix} w^2 & 1 \\ 1 & w \end{bmatrix}$ (4) None of these

71. If $x = \text{a} \cos^3 \theta$, $y = a \sin^3 \theta$, then $\sqrt{1 + \left(\frac{dy}{dx}\right)^2} =$

(1) $\sec^2\theta$ (2) $\tan^2\theta$ (3) $|\sec \theta|$ (4) $|\cot \theta|$

72. If $e^y + xy = e$, then the value of $\frac{d^2y}{dx^2}$ for $x = 0$, is :

(1) e^2 (2) $\frac{1}{e^2}$ (3) $\frac{1}{e}$ (4) $\frac{1}{e^3}$

73. If $x^y . y^x = 16$, then $\frac{dy}{dx}$ at (2, 2) is :

(1) 0 (2) 1 (3) –1 (4) –4

74. The approximate value of square root of 25.2 is :

(1) 5.01 (2) 5.02 (3) 5.03 (4) 5.04

75. The tangent at (1, 1) on the curve $y^2 = x(2 - x)^2$ meets it again at the point :

(1) (–3, 7) (2) (4, 4)

(3) $\left(\frac{3}{8}, \frac{9}{4}\right)$ (4) $\left(\frac{9}{4}, \frac{3}{8}\right)$

76. The distance between the origin and the normal to the curve $y = e^{2x} + x^2$ at $x = 0$ is :

(1) $4/\sqrt{5}$ (2) $3/\sqrt{5}$ (3) $2/\sqrt{5}$ (4) $2/\sqrt{7}$

77. The length of longest interval in which Rolle's theorem can be applied for the function $f(x) = |x^2 - a^2|$, $(a > 0)$, is :

(1) $2a$ (2) $3a$ (3) $4a$ (4) $a\sqrt{2}$

78. If the function $f(x) = \sqrt{3}\,\sin x - \cos x - 2ax + b$ decreases for all real values of x, then the value of a is given by :

(1) $a < 1$ (2) $a < \sqrt{2}$ (3) $a \geq \sqrt{2}$ (4) $a \geq 1$

79. The condition that $x^3 + ax^2 + bx + c$ may have no extremum, is :

(1) $a^2 > 3b$ (2) $a^2 < 3b$ (3) $a^2 > 2b$ (4) $a^2 < 2b$

80. $\int \frac{3 + 2\cos x}{(2 + 3\cos x)^2} dx =$

(1) $\frac{\sin x}{2 + 3\cos x} + c$ (2) $\frac{\cos x}{2 + 3\cos x} + c$ (3) $\frac{2\sin x}{2 + 3\cos x} + c$ (4) $\frac{2\cos x}{2 + 3\cos x} + c$

81. The set A has 3 elements and the Set B has 7 elements. The minimum number of elements in the set A ∪ B is :

(1) 21 (2) 10 (3) 7 (4) Can not say

82. If A and B are two sets, then $A \cap (A \cup B)^C$ (where 'C' denotes complement) is equal to :

(1) ϕ (2) A (3) B (4) A – B

83. Let $A = \{0, 1, 2, 3, 4, 5\}$ and a relation R is defined by xRy such that $2x + y = 10$. Then R^{-1} is :

(1) {(4, 3), (2, 4), (5, 0)} (2) {(4, 3), (2, 4), (0, 5)}

(3) {(3, 4), (4, 2), (5, 0)} (4) {(3, 4), (4, 2), (0, 5)}

84. If A + C = B, then tan A tan B tan C =

(1) tan A + tan B + tan C (2) tan A + tan B – tan C

(3) tan B – tan C – tan A (4) tan B + tan C – tan A

85. If $\sin x + \sin^2 x = 1$, then $\cos^8 x + 2\cos^6 x + \cos^4 x =$

(1) 0 (2) 1 (3) –1 (4) 2

86. If $4 \sin^2 x = 1$, then the values of x are :

(1) $n\pi \pm \frac{\pi}{3}$ (2) $n\pi \pm \frac{\pi}{4}$

(3) $2n\pi \pm \frac{\pi}{6}$ (4) $n\pi \pm \frac{\pi}{6}$

87. If $n \in N$, then $3^{3n} - 26n - 1$ is divisible by :

(1) 4 (2) 3 (3) 9 (4) 15

88. If $z = (K + 3) + i\sqrt{5-k^2}$, then the locus of z is :

(1) a straight line (2) a parabola

(3) an ellipse (4) a circle

89. If 1, w and w^2 are the three cube roots of unity, then the roots of the equation $(x-1)^3 - 8 = 0$ are :

(1) $2, 2w, 2w^2$ (2) $3, 2w, 2w^2$

(3) $3, 1+2w, 1+2w^2$ (4) $2, 1-2w, 1-2w^2$

90. The smallest positive integer n for which $\left(\frac{1+i}{1-i}\right)^n = 1$, is :

(1) 4 (2) 3 (3) 2 (4) 1

91. Solution of $ydx + (x - y^3)\,dy = 0$ is :

(1) $xy + \frac{y^2}{2} = c$ (2) $xy = \frac{y^2}{2} + c$

(3) $xy = \frac{y^2}{4} + c$ (4) $xy = \frac{x^2}{4} + c$

92. The differential equation $y\frac{dy}{dx} = x + a$ (a being constant) represents a set of :

(1) circles having centre on the x-axis (2) circles having centre on the y-axis

(3) ellipses (4) hyperbolas

93. From a bag containing 2 white, 3 red and 4 black balls, two balls are drawn one by one without replacement. The probability that at least ones ball is red, is :

(1) $\frac{5}{12}$ (2) $\frac{7}{12}$ (3) $\frac{5}{8}$ (4) $\frac{3}{7}$

94. A box contains 15 items in which 4 items are defective. The items are selected at random, one by one, and examined. The ones examined are not put back. The probability that 9th one examined is the last defective, is :

(1) $\frac{7}{195}$ (2) $\frac{8}{195}$ (3) $\frac{16}{255}$ (4) $\frac{14}{255}$

95. A person is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. The probability that it is actually a six, is :

(1) $\frac{7}{36}$ (2) $\frac{11}{36}$ (3) $\frac{3}{8}$ (4) $\frac{5}{8}$

96. Four numbers are multiplied together. The probability that the product will be divisible by 5 or 10, is :

(1) $\frac{123}{625}$ (2) $\frac{133}{625}$ (3) $\frac{357}{625}$ (4) $\frac{369}{625}$

97. The least number of times a fair coin must be tossed so that the probability of getting at least one head is at lest 0.8, is :

(1) 3 (2) 5 (3) 6 (4) 8

98. If $P(A \cup B) = \frac{3}{4}$ and $P(\overline{A}) = 2/3$, then $P(\overline{A} \cap B) =$

(1) $\frac{7}{12}$ (2) $\frac{5}{12}$ (3) $\frac{1}{12}$ (4) $\frac{1}{6}$

99. If $\vec{a}+\vec{b}+\vec{c}=\vec{0}, |\vec{a}|=3, |\vec{b}|=5, |\vec{c}|=7$, then the angle between $\vec{a}$ and $\vec{b}$ is :

(1) $\pi/3$ (2) $2\pi/3$ (3) $\pi/6$ (4) $5\pi/3$

100. Let $\vec{a}$ and $\vec{b}$ be two unit vectors and α be the angle between them, then $\vec{a}+\vec{b}$ is a unit vector if $\alpha =$

(1) $\pi/2$ (2) $\pi/3$ (3) $2\pi/3$ (4) $\pi/4$

Answer Key : Mathematics (Hons) Five Year Integrated Course

Sr. No.	Code A	Code B	Code C	Code D
1	3	1	3	2
2	1	3	4	2
3	2	4	1	2
4	3	2	4	3
5	2	3	2	4
6	4	1	1	1
7	1	2	3	3
8	4	4	3	1
9	3	1	4	4
10	1	4	4	2
11	2	2	3	3
12	2	3	1	4
13	2	1	3	4
14	3	3	4	2
15	4	4	2	1
16	1	2	4	2
17	3	2	4	3
18	1	1	1	3
19	4	4	2	1
20	2	1	3	4
21	3	3	3	1
22	1	1	1	3
23	3	2	2	4
24	4	2	3	2
25	2	3	2	3
26	4	4	4	1
27	4	1	1	2
28	1	2	4	4
29	2	1	3	1
30	3	3	1	4
31	2	2	3	2
32	4	2	4	3
33	1	2	4	1
34	3	3	2	3
35	1	4	1	4
36	2	1	2	2
37	1	3	3	2
38	4	1	3	1
39	3	4	1	4
40	2	2	4	1
41	3	3	3	2
42	4	4	2	4
43	1	4	3	1
44	4	2	2	3
45	2	1	4	1
46	1	2	3	2
47	3	3	1	1
48	3	3	4	4
49	4	1	2	3
50	4	4	1	2

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51	2	3	2	3
52	3	2	4	1
53	1	3	1	3
54	3	2	3	4
55	4	4	1	2
56	2	3	2	4
57	2	1	1	4
58	1	4	4	1
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61	3	2	1	3
62	2	4	3	4
63	3	1	4	1
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66	3	2	1	1
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77	2	3	1	1
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92	4	1	3	1
93	4	2	1	2
94	2	3	3	2
95	1	2	4	3
96	2	4	2	4
97	3	1	2	1
98	3	4	1	2
99	1	3	4	1
100	4	1	1	3

08/07/2019