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SET-Y

Ph.D./URS-EE-Jan-2022

SUBJECT : Mathematics

10229

Sr. No. ....

Time : 1¼ Hours | Max. Marks : 100 | Total Questions : 100

Roll No. (in figures)________________ (in words) ________________________

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1. Let Q is the set of all rational numbers, Z is the set of all integers and N is the set of all natural numbers. Then which one of the following statements is ***true*** ?

   (1) The set $Q \times Z$ is uncountable.

   (2) The set $\{f : f$ is a function from N to $\{0, 1\}\}$ is uncountable.

   (3) The set $\{\sqrt{p} : p$ is a prime number$\}$ is uncountable.

   (4) None of these

2. The sequence $< \dfrac{\sin\left(\frac{n\pi}{3}\right)}{\sqrt{n}} >$ converges to the limit :

   (1) 0 (2) 1 (3) 2 (4) None of these

3. Let $a_n = \dfrac{(-1)^{n+1}}{n!}, n \geq 0$ and $b_n = \sum_{k=0}^{n} a_k, n \geq 0$. Then for $|x| < 1$, the series $\sum_{n=0}^{\infty} b_n x^n$ converges to :

   (1) $\dfrac{-e^{-x}}{1+x}$ (2) $\dfrac{-e^{-x}}{1-x^2}$ (3) $\dfrac{-e^{-x}}{1-x}$ (4) $-(1+x)e^{-x}$

4. The number of limit points of the set $\left\{\dfrac{1}{m}+\dfrac{1}{n} : m, n \in N\right\}$ is :

   (1) 1 (2) 2

   (3) finitely many (4) infinitely many

5. Which of the following functions is uniformly continuous on the specified domain ?

   (1) $f_1(x) = e^{x^2}, -\infty < x < \infty$

   (2) $f_2(x) = \begin{cases} \dfrac{1}{x} , & 0 < x \leq 1 \\ 0 , & x = 0 \end{cases}$

   (3) $f_3(x) = \begin{cases} x^2 , & |x| \leq 1 \\ \dfrac{2}{1+x^2} , & |x| > 1 \end{cases}$

   (4) $f_4(x) = \begin{cases} x , & |x| \leq 1 \\ x^2 , & |x| > 1 \end{cases}$

6. For $n \in N$, let f_n, g_n : $(0, 1) \to R$ be functions defined by $f_n(x) = x^n$, $g_n(x) = x^n(1-x)$. Then :

(1) $\{f_n\}$ converges uniformly but $\{g_n\}$ does not converge uniformly.

(2) $\{g_n\}$ converges uniformly but $\{f_n\}$ does not converge uniformly.

(3) both $\{f_n\}$ and $\{g_n\}$ converges uniformly.

(4) neither $\{f_n\}$ nor $\{g_n\}$ converges uniformly.

7. For which of the following function, Lagrange's mean value theorem is ***not*** applicable ?

(1) $f(x) = x + \frac{1}{x}$ in $[1, 3]$

(2) $f(x) = \sqrt{25 - x^2}$ in $[-3, 4]$

(3) $f(x) = \frac{1}{4x-1}$ in $[1, 4]$

(4) $f(x) = x^{1/5}$ in $[-1, 1]$

8. $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{n}{n^2 + k^2}$ is equal to :

(1) $\frac{e}{3}$ (2) $\frac{5}{6}$ (3) $\frac{3}{4}$ (4) $\frac{\pi}{4}$

9. Consider the following improper integrals $I_1 = \int_0^{\pi/2} \frac{\cos x}{\sqrt{1 - \sin x}} dx$, $I_2 = \int_0^1 \frac{\log x}{\sqrt{1 - x^2}} dx$.

$I_3 = \int_2^{\infty} \frac{1}{x-1} dx$, then :

(1) All are convergent

(2) All are divergent

(3) I_1 and I_2 are convergent whereas I_3 is divergent

(4) I_1 and I_3 are convergent whereas I_2 is divergent

10. Consider the following statements :

P : There exists an unbounded subset of R whose Lebesgue measure is equal to 5.

Q : If $f : R \to R$ is continuous and $g : R \to R$ is such that $f = g$ almost everywhere on R, then g must be continuous almost everywhere on R.

Which of the above statements hold true ?

(1) only P (2) only Q

(3) both P and Q (4) neither P nor Q

11. The directional derivative of the function $f(x, y, z) = xy^2 + yz^3$ at the point (2, −1, 1) in the direction of $\hat{i} + 2\hat{j} + 2\hat{k}$ is :

(1) $-\frac{13}{3}$ (2) $-\frac{11}{3}$ (3) $-\frac{10}{3}$ (4) None of these

12. Let R be the set of all real numbers and $R^2 = \{(x, y) : x, y \in R\}$. Let $f : R^2 \to R$ be defined by $f(x, y) = \begin{cases} (x^2 + y^2)\sin\left(\dfrac{1}{x^2 + y^2}\right) & , \text{ if } (x, y) \neq (0, 0) \\ 0 & , \text{ if } (x, y) = (0, 0) \end{cases}$. Then which one of the following statements is ***true*** ?

(1) f is continuous but not differentiable at (0, 0)

(2) f is not continuous at (0, 0)

(3) f is differentiable at (0, 0)

(4) None of these

13. Consider the following statements :

P : $d_1(x, y) = \min\{2, |x - y|\}$ is a metric for R (the set of all real numbers).

Q : $d_2(x, y) = \begin{cases} |x| + |y| & , \text{ if } x \neq y \\ 0 & , \text{ if } x = y \end{cases}$ is a metric on (0, 1)

Then :

(1) both P and Q are true

(2) P is true and Q is false

(3) P is false and Q is true

(4) both P and Q are false

14. Suppose that :

$U = R^2 \setminus \{(x, y) \in R^2 : x, y \in Q\}$,

$V = R^2 \setminus \left\{(x, y) \in R^2 : x > 0, y = \frac{1}{x}\right\}$.

Then with respect to the Euclidean metric on R^2,

(1) both U and V are disconnected

(2) U is disconnected but V is connected

(3) U is connected but V is disconnected

(4) both U and V are connected

15. Let X and Y be normed linear spaces and let $T : X \to Y$ be any bijective linear map with closed graph. Then which one of the following statements is ***true*** ?

(1) The graph of T is equal to $X \times Y$

(2) T^{-1} is continuous

(3) The graph of T^{-1} is closed

(4) T is continuous

16. Let V be a vector space over R^3. Which one of the following is a subspace of V ?

(1) $\{(x, y, z) : x - 3y + 4z = 0, x, y, z \in R\}$

(2) $\{(x, y, z) : x + y \geq 0, x, y, z \in R\}$

(3) $\{(x, y, z) : x \leq 0, x, y, z \in R\}$

(4) None of these

17. The dimension of the vector space of 7×7 real symmetric matrices with trace zero and the sum of the off-diagonal elements zero, is :

(1) 47 (2) 28 (3) 27 (4) 26

18. Which of the following is ***not*** a linear transformation ?

(1) $T(x, y, z) = (x, y)$ for all $(x, y, z) \in R^3$

(2) $T(x, y, z) = (x + 1, y + z)$ for all $(x, y, z) \in R^3$

(3) $T(x, y, z) = (x - z, y)$ for all $(x, y, z) \in R^3$

(4) $T(x, y, z) = (x + y + z, 0)$ for all $(x, y, z) \in R^3$

19. Suppose that $T : R^4 \to R[x]$ is a linear transformation over R satisfying :

$T(-1, 1, 1, 1) = x^2 + 2x^4$

$T(1, 2, 3, 4) = 1 - x^2$

$T(2, -1, -1, 0) = x^3 - x^4$

Then the coefficient of x^4 in $T(-3, 5, 6, 6)$ is :

(1) 1 (2) 2 (3) 3 (4) 5

20. The rank of the matrix $\begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & 1 \\ 4 & 2 & 3 \end{bmatrix}$ is :

(1) 1 (2) 2 (3) 3 (4) None of these

21. The system of equations $x + 2y - z = 3$; $3x - y + 2z = 1$; $2x - 2y + 3z = 2$ has :

(1) no solution (2) a unique solution

(3) infinite solutions (4) None of these

22. The eigen values of the matrix $\begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$ are :

(1) −1, 1, 2 (2) −2, 3, 4 (3) 1, −1, 7 (4) 2, −3, 5

23. The matrix $\begin{bmatrix} 1 & x & z \\ 0 & 2 & y \\ 0 & 0 & 1 \end{bmatrix}$ is diagonalizable when (x, y, z) equals :

(1) (0, 0, 1) (2) (1, 1, 0) (3) $(\sqrt{2}, \sqrt{2}, 2)$ (4) $(\sqrt{2}, \sqrt{2}, \sqrt{2})$

24. Let $u = (1 + i, i, -1)$ and $v = (1 + 2i, 1 - i, 2i)$. Then $\langle u, v \rangle$ is :

(1) $2 - 2i$ (2) $-2 + 2i$ (3) $-2 - 2i$ (4) $2 + 2i$

25. The quadratic form corresponding to symmetric matrix $\begin{bmatrix} 0 & \frac{1}{2} & 1 \\ \frac{1}{2} & 0 & \frac{3}{2} \\ 1 & \frac{3}{2} & 0 \end{bmatrix}$ is :

(1) $xy + 3yz + 2zx$ (2) $xy - 3yz - 2zx$

(3) $xy + 3yz - 2zx$ (4) None of these

26. If the power series $\sum_{n=0}^{\infty} a_n(z+3-i)^n$ converges at $5i$ and diverges at $-3i$, then the power series :

(1) converges at $-2+5i$ and diverges at $2-3i$

(2) converges at $2-3i$ and diverges at $-2+5i$

(3) converges at both $2-3i$ and $-2+5i$

(4) diverges at both $2-3i$ and $-2+5i$

27. Which of the following function $f(z)$, of the complex variable z, is not analytic at all the points of the complex plane ?

(1) $f(z)=z^2$ (2) $f(z)=e^z$ (3) $f(z)=\sin z$ (4) $f(z)=\log z$

28. The function $f(z)$ of complex variable $z = x + iy$, where $i=\sqrt{-1}$, is given as $f(z)=(x^3-3xy^2)+iv(x,y)$. For this function to be analytic, $v(x, y)$ should be :

(1) $(3xy^2-y^3)$ + constant (2) $(3x^2y^2-y^3)$ + constant

(3) $(3x^2y-y^3)$ + constant (4) None of these

29. Let Γ denotes the boundary of the square region R with vertices (0, 0), (2, 0), (2, 2) and (0, 2) oriented in the counter- clockwise direction. Then value of $\oint_{\Gamma}(1-y^2)dx + x\,dy$ is :

(1) 12 (2) 15 (3) 20 (4) 25

30. Let C represent the unit circle centered at origin in the complex plane and, complex variable $z = x + iy$, where $i=\sqrt{-1}$. The value of the contour integral $\oint_C \frac{\cos h3z}{2z}dz$ (where integration is taken counter clockwise) is :

(1) 0 (2) 2 (3) πi (4) $2\pi i$

31. The coefficient of $\frac{1}{z}$ in the Laurent series expansion of the function $f(z)=\frac{1}{z^2(1-z)}$ about $z = 0$, is :

(1) 1 (2) 0 (3) -1 (4) None of these

32. The residue of the function $f(z)=\frac{z+1}{z^2(z-3)}$ at $z=0$, is :

(1) $-\frac{9}{2}$ (2) $\frac{1}{2}$ (3) $-\frac{4}{9}$ (4) None of these

33. The coefficient of magnification at $z=2+3i$ for the conformal transformation $w=z^2$ is :

(1) $\sqrt{3}$ (2) $\sqrt{5}$ (3) $2\sqrt{7}$ (4) $2\sqrt{13}$

34. Let $T(z)=\frac{az+b}{cz+d}$, $ad-bc\neq 0$, be the Mobius transformation which maps the points $z_1=0$, $z_2=-i$, $z_3=\infty$ in the z- plane onto the points $w_1=10$, $w_2=5-5i$, $w_3=5+5i$ in the w- plane respectively. Then the image of the set $S=\{z\in C : \text{Re}(z)<0\}$ under the map $w=T(z)$ is :

(1) $\{w\in C : |w|<5\}$ (2) $\{w\in C : |w|>5\}$
(3) $\{w\in C : |w-5|<5\}$ (4) $\{w\in C : |w-5|>5\}$

35. Among 100 students, 32 study Mathematics, 20 study Physics, 45 study Biology, 15 study Mathematics and Biology, 7 study Mathematics and Physics, 10 study Physics and Biology, 30 do not study any of three subjects. Then the number of students study only Mathematics, is :

(1) 8 (2) 15 (3) 25 (4) None of these

36. The no. of positive divisors of 2100 is :

(1) 50 (2) 44 (3) 40 (4) 36

37. The last two digits of 38^{2011} are :

(1) 6 (2) 2 (3) 4 (4) 8

38. Which one of the following statements is ***false*** ?

(1) If Q denotes the additive group of rational numbers and $f : Q \to Q$ is a non-trivial homomorphism, then f is an isomorphism.
(2) Any quotient group of a cyclic group is cyclic.
(3) If every subgroup of a group G is a normal subgroup, then G is abelian.
(4) Every group of order 33 is cyclic

39. Let Z_n denotes the group of integers modulo n, under the operation of addition modulo n, for any positive integer n. Then the number of elements of order 15 in the additive group $Z_{60}\times Z_{50}$ is :

(1) 48 (2) 30 (3) 25 (4) 10

40. The number of 5- Sylow subgroups in the symmetric group S_5 of degree 5, is :

(1) 2 (2) 3 (3) 5 (4) 6

41. Let G be a group of order 5^4 with center having 5^2 elements. Then the number of conjugacy classes in G is :

(1) 26 (2) 48 (3) 145 (4) None of these

42. Let I and J be the ideals generated by $\{5, \sqrt{10}\}$ and $\{4, \sqrt{10}\}$ in the ring $Z[\sqrt{10}] = \{a + b\sqrt{10} : a, b \in Z\}$, respectively. Then :

(1) both I and J are maximal ideals

(2) I is a maximal ideal but J is not a prime ideal

(3) I is not a maximal ideal but J is a prime ideal

(4) neither I nor J is a maximal ideal

43. Let Z denotes the set of all integers and Z_n denotes the set of all integers modulo n. for any positive integer n. Consider the following statements :

I. The ring $Z[\sqrt{-1}]$ is a unique factorization domain.

II. The ring $Z[\sqrt{-5}]$ is a principal ideal domain.

III. In the polynomial ring $Z_2[x]$, the ideal generated by $x^3 + x + 1$ is a maximal ideal.

IV. In the polynomial ring $Z_3[x]$, the ideal generated by $x^6 + 1$ is a prime ideal.

Which of the above statements are ***true*** ?

(1) I, II and III only (2) I and III only

(3) I, II and IV only (4) II and III only

44. Which of the following polynomial is reducible over the field Q of rational numbers ?

(1) $x^2 - 4x + 2$ (2) $x^3 - x + 1$

(3) $x^3 + 9x^2 - 3x + 6$ (4) None of these

45. Let F be the field with 4096 elements. The number of proper subfields of F is :

(1) 5 (2) 20 (3) 50 (4) 100

46. Let ω be a primitive complex cube root of unity. Then the degree of the field extension $Q(i, \sqrt{3}, \omega)$ over Q (the field of rational numbers) is :

(1) 4 (2) 3 (3) 2 (4) 1

47. For a subset S of a topological space, let Int(S) and $\bar{S}$ denote the interior and closure of S, respectively. Then which of the following statements is ***true*** ?

(1) If S is open, then $S = \text{Int}(\bar{S})$

(2) If the boundary of S is empty, then S is open.

(3) If $\bar{S} \setminus S$ is a proper subset of the boundary of S, then S is open.

(4) None of these

48. Let T_1 be the co-countable topology on R (the set of real numbers) and T_2 be the co-finite topology on R. Consider the following statements :

I. In (R, T_1), the sequence $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$ converges to 0.

II. In (R, T_2), the sequence $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$ converges to 0.

III. In (R, T_1), there is no sequence of rational numbers which converges to $\sqrt{3}$.

IV. In (R, T_2), there is no sequence of rational numbers which converges to $\sqrt{3}$.

Which of the above statements are ***true*** ?

(1) I and II only
(2) II and III only
(3) III and IV only
(4) I and IV only

49. Let R denote the set of all real numbers. Consider the following topological spaces :

$X_1 = (R, T_1)$, where T_1 is the upper limit topology having all sets $(a, b]$ as basis.

$X_2 = (R, T_2)$, where $T_2 = \{U \subset R : R \setminus U \text{ is finite}\} \cup \{\phi\}$

Then :

(1) both X_1 and X_2 are connected

(2) X_1 is connected and X_2 is not connected

(3) X_1 is not connected and X_2 is connected

(4) neither X_1 nor X_2 is connected

50. Consider the following statements :

P : Any continuous image of a compact space is compact.

Q : A topological space is compact if every basic open cover has a finite subcover.

Then :

(1) both P and Q are true
(2) P is true and Q is false
(3) P is false and Q is true
(4) both P and Q are false

51. The solution of the differential equation $\frac{dy}{dx} = 3e^x + 2y,\ y(0) = 0$ by Picard's method upto third approximation is :

(1) $21e^x - 6x^2 - 18x - 21$

(2) $21e^x + 6x^2 - 18x + 21$

(3) $21e^x + 6x^2 + 18x - 21$

(4) None of these

52. Consider the ordinary differential equation $y'' + P(x)y' + Q(x)y = 0$, where P and Q are smooth functions. Let y_1 and y_2 be any two solutions of the ODE. Let $W(x)$ be the corresponding Wronskian. Then which of the following is always true ?

(1) If y_1 and y_2 are linearly dependent, then $\exists\, x_1, x_2$ such that $W(x_1) = 0$ and $W(x_2) \neq 0$

(2) If y_1 and y_2 are linearly independent, then $W(x) = 0,\ \forall x$

(3) If y_1 and y_2 are linearly dependent, then $W(x) \neq 0,\ \forall x$

(4) If y_1 and y_2 are linearly independent, then $W(x) \neq 0,\ \forall x$

53. Consider the system of differential equations $\frac{dx}{dt} = 2x - 7y;\ \frac{dy}{dt} = 3x - 8y$. Then the critical point (0, 0) of the system is an :

(1) unstable node

(2) asymptotically stable node

(3) asymptotically stable spiral

(4) unstable spiral

54. Using method of variation of parameters, the solution of the differential equation $y'' - 6y' + 9y = e^{3x} / x^2$ is :

(1) $y = (c_1 + c_2 x)e^{3x} - e^{3x}(\log x + 1)$

(2) $y = (c_1 + c_2 x)e^{2x} + e^{2x}(\log x + 1)$

(3) $y = (c_1 + c_2 x)e^{4x} - e^{3x}(\log x - 1)$

(4) None of these

55. Consider the boundary value problem (BVP) $\frac{d^2y}{dx^2} + \alpha y(x) = 0$, $\alpha \in R$ (the set of all real numbers), with the boundary conditions $y(0) = 0$, $y(\pi) = k$ (where k is a non-zero real number). Then which one of the following statements is ***true*** ?

(1) For $\alpha = 1$, the BVP has infinitely many solutions.

(2) For $\alpha = 1$, the BVP has a unique solution.

(3) For $\alpha = -1$, $k < 0$, the BVP has a solution $y(x)$ such that $y(x) > 0$ for all $x \in (0, \pi)$

(4) For $\alpha = -1$, $k > 0$, the BVP has a solution $y(x)$ such that $y(x) > 0$ for all $x \in (0, \pi)$

56. Let R be the set of all real numbers and $R^2 = \{(x, y) : x, y \in R\}$. Let the general integral of the partial differential equation $(2xy - 1)\frac{\partial z}{\partial x} + (z - 2x^2)\frac{\partial z}{\partial y} = 2(x - yz)$ be given by $F(u, v) = 0$, where $F : R^2 \to R$ is a continuously differentiable function. Then :

(1) $u = x^2 + y^2 + Z$, $v = xz + y$

(2) $u = x^2 + y^2 - Z$, $v = xz - y$

(3) $u = x^2 - y^2 + z$, $v = yz + x$

(4) $u = x^2 + y^2 - Z$, $v = yz - x$

57. The particular integral of the differential equation $(D^2 + 3DD' + 2D'^2)z = 2x + 3y$ is given by :

(1) $\frac{1}{150}(2x+3y)^3$ (2) $\frac{1}{240}(2x+3y)^3$

(3) $\frac{1}{320}(2x+3y)^3$ (4) None of these

58. If $u(x, y)$ is the solution of the Cauchy problem $x\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 1, u(x,0) = -x^2, x > 0$.

Then the value of u (2, 1) is equal to :

(1) $1-2e^{-2}$ (2) $1+4e^{-2}$ (3) $1-4e^{-2}$ (4) $1+2e^{-2}$

59. The partial differential equation $(x^2+y^2-1)\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + (x^2+y^2-1)\frac{\partial^2 u}{\partial y^2} = 0$ is :

(1) parabolic in the region $x^2 + y^2 > 2$

(2) hyperbolic in the region $x^2 + y^2 > 2$

(3) elliptic in the region $0 < x^2 + y^2 < 2$

(4) hyperbolic in the region $0 < x^2 + y^2 < 2$

60. The function $u(x, t)$ satisfies the initial value problem

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, x \in R, t > 0$$

$$u(x,0) = 0, \frac{\partial u}{\partial t}(x,0) = 4xe^{-x^2}$$

Then the value of $u(5, 5)$ is :

(1) $1-\frac{1}{e^{100}}$ (2) $1-e^{100}$ (3) $1-\frac{1}{e^{10}}$ (4) $1-e^{10}$

61. Using Newton-Raphson method, the real root of the equation $x^3 - 2x + 5 = 0$ is :

(1) 3.129 (2) 1.582 (3) −2.754 (4) −2.095

62. Using Gauss- elimination method, the solution of the following system of equations $x + y + 2z = 7$, $4x + 3y + 2z = 8$, $3x + 2y + 4z = 13$, is :

(1) $x = -1, y = 2, z = 3$
(2) $x = 1, y = 3, z = 5$
(3) $x = 1, y = 1, z = 2$
(4) None of these

63. Given that

x	10	20	30	40	50
$f(x)$	46	66	81	93	101

The value of $\nabla^2 f(50)$ is :

(1) 8 (2) 3 (3) −4 (4) −1

64. Given that

x	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
y	1	$\frac{4}{5}$	$\frac{2}{3}$	$\frac{4}{7}$	$\frac{1}{2}$

Using trapezoidal rule, the value of $\int_0^1 y\,dx$ is :

(1) 0.938 (2) 0.697 (3) 0.352 (4) 0.241

65. Given $\frac{dy}{dx} = -xy^2$ with $y(0) = 2$. Then using modified Euler's method, the value of $y(0.1)$ by taking step size $h = 0.1$, is :

(1) 1.9804 (2) 1.5636 (3) 1.2921 (4) None of these

66. Let q_j and $\dot{q}_j$ respectively are the generalized coordinates and velocity of a dynamical system and p_j are its generalized momenta. Then the relation between Hamiltonian $H(q_j, p_j, t)$ and Lagrangian $L(q_j, \dot{q}_j, t)$ is given by :

(1) $H = \sum \dot{p}_j q_j - L$
(2) $H = \sum p_j \dot{q}_j - L$
(3) $H = \sum \dot{p}_j \dot{q}_j - L$
(4) None of these

67. Let T be the kinetic energy and V be the potential energy of the dynamical system, then the integral $\int_{t_1}^{t_2}(T-V)dt$ has a stationary value, where t_1 and t_2 are fixed. This principle is known as :

(1) Hamilton's principle (2) Principle of least action

(3) D' Alembert principle (4) None of these

68. Consider the motion of a planet $P(r, \theta)$ of mass m moving around the Sun $S(0, 0)$ under the inverse square law of attraction $\mu m/r^2$. Let kinetic energy T of the system is given by :

$T=\frac{1}{2}m(\dot{r}^2+r^2\dot{\theta}^2)$, where $\dot{r}=\frac{dr}{dt}$ and $\dot{\theta}=\frac{d\theta}{dt}$ with t as time.

Then Lagrange's equations are given by :

(1) $\ddot{r}+r\dot{\theta}^2=-\mu/r^2,\ r\dot{\theta}=\text{constant}$

(2) $\ddot{r}-r\dot{\theta}=-\mu/r^2,\ r^2\ddot{\theta}=\text{constant}$

(3) $\ddot{r}-r\dot{\theta}^2=-\mu/r^2,\ r^2\dot{\theta}=\text{constant}$

(4) None of these

69. A rigid body under no forces is free to rotate about its centroid G, the principal moments of inertia at which are 7, 25, 32 units respectively. If $\omega=[\omega_1, \omega_2, \omega_3]$ be the angular velocity, then Euler's dynamical equations of motion are :

(1) $\dot{\omega}_1-\omega_2\omega_3=0,\ \dot{\omega}_2+\omega_3\omega_1=0,\ 4\dot{\omega}_3+3\omega_1\omega_2=0$

(2) $\dot{\omega}_1+\omega_2\omega_3=0,\ \dot{\omega}_2-\omega_3\omega_1=0,\ 16\dot{\omega}_3+9\omega_1\omega_2=0$

(3) $\dot{\omega}_1-2\omega_2\omega_3=0,\ \dot{\omega}_2-3\omega_3\omega_1=0,\ \dot{\omega}_3-3\omega_1\omega_2=0$

(4) None of these

70. The external of the functional $I[y(x)] = \int_{-1}^{0} (y'^2 - 2xy)\, dx$ subject to $y(-1) = 0$, $y(0) = 2$ is :

(1) $y = -\frac{x^3}{6} + \frac{13}{6}x + 2$ (2) $y = x^2 - 1$

(3) $y = \frac{1}{4}x^2$ (4) None of these

71. The external of the functional $I[y(x)] = \frac{1}{2}\int_{0}^{1} (y'')^2\, dx$ subject to $y(0) = 0$, $y(1) = \frac{1}{2}$; $y'(0) = 0$, $y'(1) = 1$, is :

(1) $y = x - 1$ (2) $y = \frac{1}{2}x^2$

(3) $y = \sin x$ (4) None of these

72. The external of the functional $I[y(x)] = \int_{0}^{1} y'^2(x)\, dx$ subject to $y(0) = 0$, $y(1) = 1$ and $\int_{0}^{1} y(x)dx = 0$, is :

(1) $3x^2 - 2x$ (2) $8x^3 - 9x^2 + 2x$

(3) $\frac{5}{4}x^4 - \frac{2}{3}x$ (4) $\frac{-21}{2}x^5 + 10x^4 + 4x^3 - \frac{5}{2}x$

73. The solution of the linear integral equation $\phi(x) = (\cos x - x - 2) + \int_{0}^{x} (\xi - x)\phi(\xi)\, d\xi$, is :

(1) $\phi(x) = \cos hx$ (2) $\phi(x) = \cos x + e^x \sin x$

(3) $\phi(x) = e^x$ (4) $\phi(x) = -\cos x - \sin x - \frac{1}{2} x \sin x$

74. The eigen values (λ) of the homogeneous integral equation $\phi(x) = \lambda \int_0^{\pi} \cos(x+\xi)\phi(\xi)\, d\xi$ are :

(1) $\lambda_1 = \frac{-4}{\pi}, \lambda_2 = \frac{4}{\pi}$ (2) $\lambda_1 = \frac{-\pi}{2}, \lambda_2 = \frac{\pi}{2}$

(3) $\lambda_1 = \frac{-2}{\pi}, \lambda_2 = \frac{2}{\pi}$ (4) None of these

75. The resolvent kernel for the integral equation $\phi(x) = x^2 + \int_0^x e^{t-x}\phi(t)\, dt$ is :

(1) e^{t-x} (2) 1 (3) e^{x-t} (4) $x^2 + e^{x-t}$

76. From the six letters A, B, C, D, E and F, three letters are chosen at random with replacement. What is the probability that either the word BAD or the word CAD can be formed from the chosen letters ?

(1) $\frac{1}{216}$ (2) $\frac{3}{216}$ (3) $\frac{6}{216}$ (4) $\frac{12}{216}$

77. Let X be a random variable which is symmetric about 0. Let F be the cumulative distribution function of X. Which of the following statements is always ***true*** ?

(1) $F(x) + F(-x) = 1$ for all $x \in R$

(2) $F(x) - F(-x) = 0$ for all $x \in R$

(3) $F(x) + F(-x) = 1 + P(X = x)$ for all $x \in R$

(4) $F(x) + F(-x) = 1 - P(X = -x)$ for all $x \in R$

78. The probability density function of the random vector (X, Y) is given by :

$$f_{X,Y}(x, y) = \begin{cases} c, & 0 < x < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

Then the value of c is equal to :

(1) 1 (2) 2 (3) 4 (4) 5

79. If X and Y are random variables such that $E[2X + Y] = 0$ and $E[X + 2Y] = 33$, then $E[X] + E[Y]$ is equal to :

(1) 26 (2) 20 (3) 11 (4) None of these

80. The moment generating function of a random variable X is given by :

$$M_X(t) = \frac{1}{6} + \frac{1}{3}e^t + \frac{1}{3}e^{2t} + \frac{1}{6}e^{3t}, \ -\infty < t < \infty$$

Then $P(X \leq 2)$ equals :

(1) $\frac{1}{3}$ (2) $\frac{1}{6}$ (3) $\frac{1}{2}$ (4) $\frac{5}{6}$

81. If X is a continuous random variable with mean μ and variance σ^2, then for any positive number k, the inequality $P\{|X - \mu| \geq k\sigma\} \leq (1/k^2)$ is known as :

(1) Lyapunov's inequality (2) Chebychev's inequality

(3) Bienayme-Chebychev's inequality (4) Khintchine's inequality

82. With the usual notations, the value of probability p for a binomial variate X, if $n = 6$ and $9P(X = 4) = P(X = 2)$, is :

(1) $\frac{1}{2}$ (2) $\frac{1}{3}$ (3) $\frac{1}{4}$ (4) None of these

83. The characteristics function of Poisson distribution with meas as m, is :

(1) $e^{m(it-1)}$ (2) $e^{m(e^{it}-1)}$ (3) e^{mit} (4) None of these

84. Which of the following do the normal distribution and the exponential density function have in common ?

(1) Both are bell- shaped.

(2) Both are symmetrical distributions.

(3) Both approach infinity as x approaches infinity.

(4) Both approach zero as x approaches infinity.

85. Let X be a random variable with uniform distribution on the interval $(-1, 1)$ and $Y = (X+1)^2$. Then the probability density function $f(y)$ of Y, over the interval $(0, 4)$, is :

(1) $\frac{3\sqrt{y}}{16}$ (2) $\frac{1}{4\sqrt{y}}$ (3) $\frac{1}{6\sqrt{y}}$ (4) $\frac{1}{\sqrt{y}}$

86. Let $\{X_n\}_{n\geq 0}$ be a homogeneous Markov chain whose state space is $\{0, 1, 2\}$ and whose one-step transition probability matrix is $P = \begin{bmatrix} 0 & 1 & 0 \\ 0.3 & 0 & 0.7 \\ 0 & 1 & 0 \end{bmatrix}$. Then $\lim_{n\to\infty} P(X_{2n} = 2 \mid X_0 = 2)$ is equal to :

(1) 0.7 (2) 0.5 (3) 0.3 (4) 0.1

87. Suppose customers arrive at an ATM facility according to a Poisson process with rate 5 customers per hour. The probability that no customer arrives at the ATM facility from 1:00 pm to 1:18 pm is :

(1) 0.94 (2) 0.75 (3) 0.45 (4) 0.22

88. In a pure birth process with birth rates $\lambda_n = 2^n$, $n \geq 0$, let the random variable T denote the time taken for the population size to grow from 0 to 5. If Var(T) denotes the variance of the random variable T, then Var(T) is equal to :

(1) $\frac{441}{256}$ (2) $\frac{341}{256}$ (3) $\frac{241}{256}$ (4) $\frac{141}{256}$

89. Let $X_1, X_2, \ldots\ldots, X_{10}$ be independent and identically distributed normal random variables with mean 0 and variance 2. Then $E\left(\frac{X_1^2}{\sum_{i=1}^{10} X_i^2}\right)$ is equal to :

(1) 0.8 (2) 0.6 (3) 0.3 (4) 0.1

90. Let $X_1, X_2, \ldots\ldots, X_n$ be a random sample drawn from a population with probability density function $f_X(x;\theta) = \frac{2x}{\theta^2}, 0 < x < \theta$. Then the method of moments estimator of θ is :

(1) $\frac{3\sum_{n=1}^{n} X_i}{2n}$ (2) $\frac{3\sum_{n=1}^{n} X_i^2}{2n}$

(3) $\frac{\sum_{i=1}^{n} X_i}{n}$ (4) $\frac{3\sum_{i=1}^{n} X_i(X_i - 1)}{2n}$

91. Let $\{0, 1, 2, 3\}$ be an observed sample of size 4 from $N(\theta, 5)$ distribution, where $\theta \in [2, \infty)$. Then the maximum likelihood estimate of θ based on the observed sample is :

(1) 16 (2) 8 (3) 4 (4) 2

92. A random sample of size 100 is classified into 10 class intervals covering all the data points. To test whether data comes from a normal population with unknown mean and unknown variance, the chi-square goodness of fit test is used. The degrees of freedom of the test statistic is equal to :

(1) 10 (2) 9 (3) 8 (4) 7

93. A random sample of 500 apples was taken from a large consignment and 60 were found to be bad. Then the 98% confidence limits for the percentage of bad apples in the consignment are :

(1) (8.82, 17.52) (2) (8.61, 15.38)

(3) (8.32, 14.63) (4) (8.19, 13.52)

94. Consider a two - way fixed effects analysis of variance model without interaction effect and one observation per cell. If there are 5 factors and 4 columns, then the degrees of freedom for the error sum of squares is :

(1) 20 (2) 19 (3) 12 (4) 11

95. The simple correlation coefficients between temperature (X_1), corn yield (X_2) and rainfall (X_3) are : $r_{12} = 0.75$, $r_{23} = 0.54$, $r_{31} = 0.43$. Then the partial correlation coefficient $r_{12.3}$ is :

(1) 0.6815 (2) 0.4223

(3) 0.2392 (4) None of these

96. Let (X_1, X_2) be a random vector following bivariate normal distribution with mean vector (0, 0), variance (X_1) = variance (X_2) = 1 and correlation coefficient ρ, where $|\rho| < 1$. Then $P(X_1 + X_2 > 0)$ is equal to :

(1) 1 (2) 0.8 (3) 0.5 (4) 0.2

97. The total number of standard 4×4 Latin squares is :

(1) 4 (2) 8 (3) 12 (4) 16

98. The minimum value of $Z = 20x + 10y$

subject to the constraints :

$x + 2y \leq 40$; $3x + y \geq 30$; $4x + 3y \geq 60$; $x, y \geq 0$, is :

(1) 240 (2) 215 (3) 272 (4) None of these

99. Consider a parallel system with two components. The lifetimes of the two components are independent and identically distributed random variables each following an exponential distribution with mean 1. The expected lifetime of the system is :

(1) 1 (2) 1/2 (3) 3/2 (4) 2

100. Consider an M/M/1 queue with interarrival time having exponential distribution with mean $\frac{1}{\lambda}$ and service time having exponential distribution with mean $\frac{1}{\mu}$. Which of the following is ***true*** ?

(1) If $0 < \lambda < \mu$, then the queue length has limiting distribution Poisson $(\mu - \lambda)$.

(2) If $0 < \mu < \lambda$, then the queue length has limiting distribution Poisson $(\lambda - \mu)$.

(3) If $0 < \lambda < \mu$, then the queue length has limiting distribution which is geometric.

(4) If $0 < \mu < \lambda$, then the queue length has limiting distribution which is geometric.

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SET-Y

Ph.D./URS-EE-Jan-2022

SUBJECT : Mathematics

10226

Sr. No. ....

Time : 1¼ **Hours** Max. Marks : **100** Total Questions : **100**

Roll No. (in figures)________________ (in words) ____________________________

Name __________________________ Father's Name __________________________

Mother's Name ________________________ Date of Examination ________________________

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(Signature of the Candidate) (Signature of the Invigilator)

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3. Keeping in view the transparency of the examination system, carbonless OMR Sheet is provided to the candidate so that a copy of OMR Sheet may be kept by the candidate.
4. Question Booklet along with answer key of all the A, B, C & D code will be got uploaded on the University website after the conduct of Entrance Examination. In case there is any discrepancy in the Question Booklet/Answer Key, the same may be brought to the notice of the Controller of Examination in writing/through E.Mail within 24 hours of uploading the same on the University Website. Thereafter, no complaint in any case, will be considered.
5. The candidate ***must not*** do any rough work or writing in the OMR Answer-Sheet. Rough work, if any, may be done in the question booklet itself. Answers ***must not*** be ticked in the question booklet.
6. ***There will be no negative marking. Each correct answer will be awarded one full mark. Cutting, erasing, overwriting and more than one answer in OMR Answer-Sheet will be treated as incorrect answer.***
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8. *Before answering the questions, the candidates should ensure that they have been supplied correct and complete booklet. Complaints, if any, regarding misprinting etc. will not be entertained* **30** *minutes after starting of the examination.*

1. The directional derivative of the function $f(x, y, z) = xy^2 + yz^3$ at the point $(2, -1, 1)$ in the direction of $\hat{i} + 2\hat{j} + 2\hat{k}$ is :

(1) $-\frac{13}{3}$ (2) $-\frac{11}{3}$ (3) $-\frac{10}{3}$ (4) None of these

2. Let R be the set of all real numbers and $R^2 = \{(x, y) : x, y \in R\}$. Let $f : R^2 \to R$ be defined by $f(x, y) = \begin{cases} (x^2 + y^2)\sin\left(\frac{1}{x^2 + y^2}\right) & , \text{ if } (x, y) \neq (0, 0) \\ 0 & , \text{ if } (x, y) = (0, 0) \end{cases}$. Then which one of the following statements is ***true*** ?

(1) f is continuous but not differentiable at (0, 0)
(2) f is not continuous at (0, 0)
(3) f is differentiable at (0, 0)
(4) None of these

3. Consider the following statements :

$P : d_1(x, y) = \min\{2, |x - y|\}$ is a metric for R (the set of all real numbers).

$Q : d_2(x, y) = \begin{cases} |x| + |y| & , \text{ if } x \neq y \\ 0 & , \text{ if } x = y \end{cases}$ is a metric on (0, 1)

Then :

(1) both P and Q are true (2) P is true and Q is false
(3) P is false and Q is true (4) both P and Q are false

4. Suppose that :

$U = R^2 \setminus \{(x, y) \in R^2 : x, y \in Q\}$,

$V = R^2 \setminus \left\{(x, y) \in R^2 : x > 0, y = \frac{1}{x}\right\}$

Then with respect to the Euclidean metric on R^2,

(1) both U and V are disconnected
(2) U is disconnected but V is connected
(3) U is connected but V is disconnected
(4) both U and V are connected

5. Let X and Y be normed linear spaces and let $T : X \to Y$ be any bijective linear map with closed graph. Then which one of the following statements is ***true*** ?
   (1) The graph of T is equal to $X \times Y$
   (2) T^{-1} is continuous
   (3) The graph of T^{-1} is closed
   (4) T is continuous

6. Let V be a vector space over R^3. Which one of the following is a subspace of V ?
   (1) $\{(x, y, z) : x - 3y + 4z = 0, x, y, z \in R\}$
   (2) $\{(x, y, z) : x + y \geq 0, x, y, z \in R\}$
   (3) $\{(x, y, z) : x \leq 0, x, y, z \in R\}$
   (4) None of these

7. The dimension of the vector space of 7×7 real symmetric matrices with trace zero and the sum of the off-diagonal elements zero, is :
   (1) 47 (2) 28 (3) 27 (4) 26

8. Which of the following is ***not*** a linear transformation ?
   (1) $T(x, y, z) = (x, y)$ for all $(x, y, z) \in R^3$
   (2) $T(x, y, z) = (x + 1, y + z)$ for all $(x, y, z) \in R^3$
   (3) $T(x, y, z) = (x - z, y)$ for all $(x, y, z) \in R^3$
   (4) $T(x, y, z) = (x + y + z, 0)$ for all $(x, y, z) \in R^3$

9. Suppose that $T : R^4 \to R[x]$ is a linear transformation over R satisfying :
   $T(-1, 1, 1, 1) = x^2 + 2x^4$
   $T(1, 2, 3, 4) = 1 - x^2$
   $T(2, -1, -1, 0) = x^3 - x^4$
   Then the coefficient of x^4 in $T(-3, 5, 6, 6)$ is :
   (1) 1 (2) 2 (3) 3 (4) 5

10. The rank of the matrix $\begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & 1 \\ 4 & 2 & 3 \end{bmatrix}$ is :
   (1) 1 (2) 2 (3) 3 (4) None of these

with

11. Let {0, 1, 2, 3} be an observed sample of size 4 from $N(\theta, 5)$ distribution, where $\theta \in [2, \infty)$. Then the maximum likelihood estimate of θ based on the observed sample is :

(1) 16 (2) 8 (3) 4 (4) 2

12. A random sample of size 100 is classified into 10 class intervals covering all the data points. To test whether data comes from a normal population with unknown mean and unknown variance, the chi-square goodness of fit test is used. The degrees of freedom of the test statistic is equal to :

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(1) 1 (2) 0.8 (3) 0.5 (4) 0.2

17. The total number of standard 4×4 Latin squares is :

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subject to the constraints :

$x + 2y \le 40$; $3x + y \ge 30$; $4x + 3y \ge 60$; $x, y \ge 0$, is :

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(1) If $0 < \lambda < \mu$, then the queue length has limiting distribution Poisson $(\mu - \lambda)$.

(2) If $0 < \mu < \lambda$, then the queue length has limiting distribution Poisson $(\lambda - \mu)$.

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21. The external of the functional $I[y(x)] = \frac{1}{2}\int_0^1 (y'')^2\, dx$ subject to $y(0) = 0$, $y(1) = \frac{1}{2}$; $y'(0) = 0$, $y'(1) = 1$, is :

(1) $y = x - 1$ (2) $y = \frac{1}{2}x^2$

(3) $y = \sin x$ (4) None of these

22. The external of the functional $I[y(x)] = \int_0^1 y'^2(x)\, dx$ subject to $y(0) = 0$, $y(1) = 1$ and $\int_0^1 y(x)dx = 0$, is :

(1) $3x^2 - 2x$ (2) $8x^3 - 9x^2 + 2x$

(3) $\frac{5}{4}x^4 - \frac{2}{3}x$ (4) $\frac{-21}{2}x^5 + 10x^4 + 4x^3 - \frac{5}{2}x$

23. The solution of the linear integral equation $\phi(x) = (\cos x - x - 2) + \int_0^x (\xi - x)\phi(\xi)\, d\xi$, is :

(1) $\phi(x) = \cos hx$

(2) $\phi(x) = \cos x + e^x \sin x$

(3) $\phi(x) = e^x$

(4) $\phi(x) = -\cos x - \sin x - \frac{1}{2} x \sin x$

24. The eigen values (λ) of the homogeneous integral equation $\phi(x) = \lambda \int_0^\pi \cos(x + \xi)\phi(\xi)\, d\xi$ are :

(1) $\lambda_1 = \frac{-4}{\pi}, \lambda_2 = \frac{4}{\pi}$

(2) $\lambda_1 = \frac{-\pi}{2}, \lambda_2 = \frac{\pi}{2}$

(3) $\lambda_1 = \frac{-2}{\pi}, \lambda_2 = \frac{2}{\pi}$

(4) None of these

25. The resolvent kernel for the integral equation $\phi(x) = x^2 + \int_0^x e^{t-x}\phi(t)\, dt$ is :

(1) e^{t-x} (2) 1 (3) e^{x-t} (4) $x^2 + e^{x-t}$

26. From the six letters A, B, C, D, E and F, three letters are chosen at random with replacement. What is the probability that either the word BAD or the word CAD can be formed from the chosen letters ?

(1) $\frac{1}{216}$ (2) $\frac{3}{216}$ (3) $\frac{6}{216}$ (4) $\frac{12}{216}$

27. Let X be a random variable which is symmetric about 0. Let F be the cumulative distribution function of X. Which of the following statements is always ***true*** ?

(1) $F(x) + F(-x) = 1$ for all $x \in R$

(2) $F(x) - F(-x) = 0$ for all $x \in R$

(3) $F(x) + F(-x) = 1 + P(X = x)$ for all $x \in R$

(4) $F(x) + F(-x) = 1 - P(X = -x)$ for all $x \in R$

4

28. The probability density function of the random vector (X, Y) is given by :

$$f_{X,Y}(x,y)=\begin{cases} c &, \quad 0<x<y<1 \\ 0 &, \quad \text{otherwise} \end{cases}$$

Then the value of c is equal to :

(1) 1 (2) 2 (3) 4 (4) 5

29. If X and Y are random variables such that $E[2X+Y]=0$ and $E[X+2Y]=33$, then $E[X]+E[Y]$ is equal to :

(1) 26 (2) 20 (3) 11 (4) None of these

30. The moment generating function of a random variable X is given by :

$$M_X(t)=\frac{1}{6}+\frac{1}{3}e^t+\frac{1}{3}e^{2t}+\frac{1}{6}e^{3t}, \ -\infty<t<\infty$$

Then $P(X \le 2)$ equals :

(1) $\frac{1}{3}$ (2) $\frac{1}{6}$ (3) $\frac{1}{2}$ (4) $\frac{5}{6}$

31. The solution of the differential equation $\frac{dy}{dx}=3e^x+2y,\ y(0)=0$ by Picard's method upto third approximation is :

(1) $21e^x-6x^2-18x-21$

(2) $21e^x+6x^2-18x+21$

(3) $21e^x+6x^2+18x-21$

(4) None of these

32. Consider the ordinary differential equation $y''+P(x)y'+Q(x)y=0$, where P and Q are smooth functions. Let y_1 and y_2 be any two solutions of the ODE. Let $W(x)$ be the corresponding Wronskian. Then which of the following is always true ?

(1) If y_1 and y_2 are linearly dependent, then $\exists\, x_1, x_2$ such that $W(x_1)=0$ and $W(x_2)\neq 0$

(2) If y_1 and y_2 are linearly independent, then $W(x)=0,\ \forall x$

(3) If y_1 and y_2 are linearly dependent, then $W(x)\neq 0,\ \forall x$

(4) If y_1 and y_2 are linearly independent, then $W(x)\neq 0,\ \forall x$

33. Consider the system of differential equations $\frac{dx}{dt} = 2x - 7y; \frac{dy}{dt} = 3x - 8y$. Then the critical point (0, 0) of the system is an :

(1) unstable node

(2) asymptotically stable node

(3) asymptotically stable spiral

(4) unstable spiral

34. Using method of variation of parameters, the solution of the differential equation $y'' - 6y' + 9y = e^{3x} / x^2$ is :

(1) $y = (c_1 + c_2 x)e^{3x} - e^{3x}(\log x + 1)$

(2) $y = (c_1 + c_2 x)e^{2x} + e^{2x}(\log x + 1)$

(3) $y = (c_1 + c_2 x)e^{4x} - e^{3x}(\log x - 1)$

(4) None of these

35. Consider the boundary value problem (BVP) $\frac{d^2 y}{dx^2} + \alpha y(x) = 0$, $\alpha \in R$ (the set of all real numbers), with the boundary conditions $y(0) = 0$, $y(\pi) = k$ (where k is a non-zero real number). Then which one of the following statements is ***true*** ?

(1) For $\alpha = 1$, the BVP has infinitely many solutions.

(2) For $\alpha = 1$, the BVP has a unique solution.

(3) For $\alpha = -1$, $k < 0$, the BVP has a solution $y(x)$ such that $y(x) > 0$ for all $x \in (0, \pi)$

(4) For $\alpha = -1$, $k > 0$, the BVP has a solution $y(x)$ such that $y(x) > 0$ for all $x \in (0, \pi)$

36. Let R be the set of all real numbers and $R^2 = \{(x, y) : x, y \in R\}$. Let the general integral of the partial differential equation $(2xy-1)\frac{\partial z}{\partial x}+(z-2x^2)\frac{\partial z}{\partial y}=2(x-yz)$ be given by $F(u, v) = 0$, where $F : R^2 \to R$ is a continuously differentiable function. Then :

(1) $u = x^2 + y^2 + Z, v = xz + y$

(2) $u = x^2 + y^2 - Z, v = xz - y$

(3) $u = x^2 - y^2 + z, v = yz + x$

(4) $u = x^2 + y^2 - Z, v = yz - x$

37. The particular integral of the differential equation $(D^2 + 3DD' + 2D'^2)z = 2x + 3y$ is given by :

(1) $\frac{1}{150}(2x+3y)^3$

(2) $\frac{1}{240}(2x+3y)^3$

(3) $\frac{1}{320}(2x+3y)^3$

(4) None of these

38. If u (x, y) is the solution of the Cauchy problem $x\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=1, u(x,0)=-x^2, x>0$.

Then the value of u (2, 1) is equal to :

(1) $1-2e^{-2}$ (2) $1+4e^{-2}$ (3) $1-4e^{-2}$ (4) $1+2e^{-2}$

39. The partial differential equation $(x^2+y^2-1)\frac{\partial^2 u}{\partial x^2}+2\frac{\partial^2 u}{\partial x \partial y}+(x^2+y^2-1)\frac{\partial^2 u}{\partial y^2}=0$ is :

(1) parabolic in the region $x^2 + y^2 > 2$

(2) hyperbolic in the region $x^2 + y^2 > 2$

(3) elliptic in the region $0 < x^2 + y^2 < 2$

(4) hyperbolic in the region $0 < x^2 + y^2 < 2$

40. The function $u(x, t)$ satisfies the initial value problem

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, x \in R, t > 0$$

$$u(x,0) = 0, \frac{\partial u}{\partial t}(x,0) = 4xe^{-x^2}$$

Then the value of $u(5, 5)$ is :

(1) $1 - \frac{1}{e^{100}}$ (2) $1 - e^{100}$ (3) $1 - \frac{1}{e^{10}}$ (4) $1 - e^{10}$

41. The coefficient of $\frac{1}{z}$ in the Laurent series expansion of the function $f(z) = \frac{1}{z^2(1-z)}$ about $z = 0$, is :

(1) 1 (2) 0 (3) -1 (4) None of these

42. The residue of the function $f(z) = \frac{z+1}{z^2(z-3)}$ at $z = 0$, is :

(1) $-\frac{9}{2}$ (2) $\frac{1}{2}$ (3) $-\frac{4}{9}$ (4) None of these

43. The coefficient of magnification at $z = 2 + 3i$ for the conformal transformation $w = z^2$ is :

(1) $\sqrt{3}$ (2) $\sqrt{5}$ (3) $2\sqrt{7}$ (4) $2\sqrt{13}$

44. Let $T(z) = \frac{az + b}{cz + d}$, $ad - bc \neq 0$, be the Mobius transformation which maps the points $z_1 = 0$, $z_2 = -i$, $z_3 = \infty$ in the z- plane onto the points $w_1 = 10$, $w_2 = 5 - 5i$, $w_3 = 5 + 5i$ in the w- plane respectively. Then the image of the set $S = \{z \in C : \text{Re}(z) < 0\}$ under the map $w = T(z)$ is :

(1) $\{w \in C : |w| < 5\}$ (2) $\{w \in C : |w| > 5\}$
(3) $\{w \in C : |w - 5| < 5\}$ (4) $\{w \in C : |w - 5| > 5\}$

45. Among 100 students, 32 study Mathematics, 20 study Physics, 45 study Biology, 15 study Mathematics and Biology, 7 study Mathematics and Physics, 10 study Physics and Biology, 30 do not study any of three subjects. Then the number of students study only Mathematics, is :

(1) 8 (2) 15 (3) 25 (4) None of these

46. The no. of positive divisors of 2100 is :

(1) 50 (2) 44 (3) 40 (4) 36

47. The last two digits of 38^{2011} are :

(1) 6 (2) 2 (3) 4 (4) 8

48. Which one of the following statements is ***false*** ?

(1) If Q denotes the additive group of rational numbers and $f : Q \to Q$ is a non-trivial homomorphism, then f is an isomorphism.

(2) Any quotient group of a cyclic group is cyclic.

(3) If every subgroup of a group G is a normal subgroup, then G is abelian.

(4) Every group of order 33 is cyclic

49. Let Z_n denotes the group of integers modulo n, under the operation of addition modulo n, for any positive integer n. Then the number of elements of order 15 in the additive group $Z_{60} \times Z_{50}$ is :

(1) 48 (2) 30 (3) 25 (4) 10

50. The number of 5- Sylow subgroups in the symmetric group S_5 of degree 5, is :

(1) 2 (2) 3 (3) 5 (4) 6

51. The system of equations $x + 2y - z = 3; 3x - y + 2z = 1; 2x - 2y + 3z = 2$ has :

(1) no solution (2) a unique solution

(3) infinite solutions (4) None of these

52. The eigen values of the matrix $\begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$ are :

(1) −1, 1, 2 (2) −2, 3, 4 (3) 1, −1, 7 (4) 2, −3, 5

53. The matrix $\begin{bmatrix} 1 & x & z \\ 0 & 2 & y \\ 0 & 0 & 1 \end{bmatrix}$ is diagonalizable when (x, y, z) equals :

(1) (0, 0, 1) (2) (1, 1, 0) (3) $(\sqrt{2}, \sqrt{2}, 2)$ (4) $(\sqrt{2}, \sqrt{2}, \sqrt{2})$

54. Let $u = (1 + i, i, -1)$ and $v = (1 + 2i, 1 - i, 2i)$. Then $\langle u, v \rangle$ is :

(1) $2 - 2i$ (2) $-2 + 2i$ (3) $-2 - 2i$ (4) $2 + 2i$

55. The quadratic form corresponding to symmetric matrix $\begin{bmatrix} 0 & \frac{1}{2} & 1 \\ \frac{1}{2} & 0 & \frac{3}{2} \\ 1 & \frac{3}{2} & 0 \end{bmatrix}$ is :

(1) $xy + 3yz + 2zx$
(2) $xy - 3yz - 2zx$
(3) $xy + 3yz - 2zx$
(4) None of these

56. If the power series $\sum_{n=0}^{\infty} a_n (z + 3 - i)^n$ converges at $5i$ and diverges at $-3i$, then the power series :

(1) converges at $-2 + 5i$ and diverges at $2 - 3i$
(2) converges at $2 - 3i$ and diverges at $-2 + 5i$
(3) converges at both $2 - 3i$ and $-2 + 5i$
(4) diverges at both $2 - 3i$ and $-2 + 5i$

57. Which of the following function $f(z)$, of the complex variable z, is not analytic at all the points of the complex plane ?

(1) $f(z) = z^2$
(2) $f(z) = e^z$
(3) $f(z) = \sin z$
(4) $f(z) = \log z$

58. The function $f(z)$ of complex variable $z = x + iy$, where $i = \sqrt{-1}$, is given as $f(z) = (x^3 - 3xy^2) + iv(x, y)$. For this function to be analytic, $v(x, y)$ should be :

(1) $(3xy^2 - y^3)$ + constant
(2) $(3x^2y^2 - y^3)$ + constant
(3) $(3x^2y - y^3)$ + constant
(4) None of these

59. Let Γ denotes the boundary of the square region R with vertices (0, 0), (2, 0), (2, 2) and (0, 2) oriented in the counter- clockwise direction. Then value of $\oint_{\Gamma} (1 - y^2)dx + x\,dy$ is :

(1) 12
(2) 15
(3) 20
(4) 25

60. Let C represent the unit circle centered at origin in the complex plane and, complex variable $z = x + iy$, where $i = \sqrt{-1}$. The value of the contour integral $\oint_C \frac{\cos h3z}{2z} dz$ (where integration is taken counter clockwise) is :

(1) 0
(2) 2
(3) πi
(4) $2\pi i$

61. Let G be a group of order 5^4 with center having 5^2 elements. Then the number of conjugacy classes in G is :

(1) 26 (2) 48 (3) 145 (4) None of these

62. Let I and J be the ideals generated by $\{5, \sqrt{10}\}$ and $\{4, \sqrt{10}\}$ in the ring $Z[\sqrt{10}] = \{a + b\sqrt{10} : a, b \in Z\}$, respectively. Then :

(1) both I and J are maximal ideals

(2) I is a maximal ideal but J is not a prime ideal

(3) I is not a maximal ideal but J is a prime ideal

(4) neither I nor J is a maximal ideal

63. Let Z denotes the set of all integers and Z_n denotes the set of all integers modulo n, for any positive integer n. Consider the following statements :

I. The ring $Z[\sqrt{-1}]$ is a unique factorization domain.

II. The ring $Z[\sqrt{-5}]$ is a principal ideal domain.

III. In the polynomial ring $Z_2[x]$, the ideal generated by $x^3 + x + 1$ is a maximal ideal.

IV. In the polynomial ring $Z_3[x]$, the ideal generated by $x^6 + 1$ is a prime ideal.

Which of the above statements are ***true*** ?

(1) I, II and III only (2) I and III only

(3) I, II and IV only (4) II and III only

64. Which of the following polynomial is reducible over the field Q of rational numbers ?

(1) $x^2 - 4x + 2$ (2) $x^3 - x + 1$

(3) $x^3 + 9x^2 - 3x + 6$ (4) None of these

65. Let F be the field with 4096 elements. The number of proper subfields of F is :

(1) 5 (2) 20 (3) 50 (4) 100

66. Let ω be a primitive complex cube root of unity. Then the degree of the field extension $Q(i, \sqrt{3}, \omega)$ over Q (the field of rational numbers) is :

(1) 4 (2) 3 (3) 2 (4) 1

67. For a subset S of a topological space, let Int(S) and $\overline{S}$ denote the interior and closure of S, respectively. Then which of the following statements is ***true*** ?

(1) If S is open, then $S = \text{Int}(\overline{S})$

(2) If the boundary of S is empty, then S is open.

(3) If $\overline{S} \setminus S$ is a proper subset of the boundary of S, then S is open.

(4) None of these

68. Let T_1 be the co-countable topology on R (the set of real numbers) and T_2 be the co-finite topology on R. Consider the following statements :

I. In (R, T_1), the sequence $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$ converges to 0.

II. In (R, T_2), the sequence $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$ converges to 0.

III. In (R, T_1), there is no sequence of rational numbers which converges to $\sqrt{3}$.

IV. In (R, T_2), there is no sequence of rational numbers which converges to $\sqrt{3}$.

Which of the above statements are ***true*** ?

(1) I and II only
(2) II and III only
(3) III and IV only
(4) I and IV only

69. Let R denote the set of all real numbers. Consider the following topological spaces :

$X_1 = (R, T_1)$, where T_1 is the upper limit topology having all sets $(a, b]$ as basis.

$X_2 = (R, T_2)$, where $T_2 = \{U \subset R : R \setminus U \text{ is finite}\} \cup \{\phi\}$

Then :

(1) both X_1 and X_2 are connected

(2) X_1 is connected and X_2 is not connected

(3) X_1 is not connected and X_2 is connected

(4) neither X_1 nor X_2 is connected

70. Consider the following statements :

P : Any continuous image of a compact space is compact.

Q : A topological space is compact if every basic open cover has a finite subcover.

Then :

(1) both P and Q are true (2) P is true and Q is false

(3) P is false and Q is true (4) both P and Q are false

71. Using Newton-Raphson method, the real root of the equation $x^3 - 2x + 5 = 0$ is :

(1) 3.129 (2) 1.582 (3) –2.754 (4) –2.095

72. Using Gauss- elimination method, the solution of the following system of equations $x + y + 2z = 7, 4x + 3y + 2z = 8, 3x + 2y + 4z = 13$, is :

(1) $x = -1, y = 2, z = 3$ (2) $x = 1, y = 3, z = 5$

(3) $x = 1, y = 1, z = 2$ (4) None of these

73. Given that

x	10	20	30	40	50
$f(x)$	46	66	81	93	101

The value of $\nabla^2 f(50)$ is :

(1) 8 (2) 3 (3) –4 (4) –1

74. Given that

x	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
y	1	$\frac{4}{5}$	$\frac{2}{3}$	$\frac{4}{7}$	$\frac{1}{2}$

Using trapezoidal rule, the value of $\int_0^1 y\,dx$ is :

(1) 0.938 (2) 0.697 (3) 0.352 (4) 0.241

75. Given $\frac{dy}{dx} = -xy^2$ with $y(0) = 2$. Then using modified Euler's method, the value of $y(0.1)$ by taking step size $h = 0.1$, is :

(1) 1.9804 (2) 1.5636 (3) 1.2921 (4) None of these

76. Let q_j and $\dot{q}_j$ respectively are the generalized coordinates and velocity of a dynamical system and p_j are its generalized momenta. Then the relation between Hamiltonian $H(q_j, p_j, t)$ and Lagrangian $L(q_j, \dot{q}_j, t)$ is given by :

(1) $H = \sum \dot{p}_j q_j - L$ (2) $H = \sum p_j \dot{q}_j - L$

(3) $H = \sum \dot{p}_j \dot{q}_j - L$ (4) None of these

77. Let T be the kinetic energy and V be the potential energy of the dynamical system, then the integral $\int_{t_1}^{t_2} (T - V)dt$ has a stationary value, where t_1 and t_2 are fixed. This principle is known as :

(1) Hamilton's principle (2) Principle of least action

(3) D' Alembert principle (4) None of these

78. Consider the motion of a planet $P(r, \theta)$ of mass m moving around the Sun $S(0, 0)$ under the inverse square law of attraction $\mu m / r^2$. Let kinetic energy T of the system is given by :

$T = \frac{1}{2} m(\dot{r}^2 + r^2 \dot{\theta}^2)$, where $\dot{r} = \frac{dr}{dt}$ and $\dot{\theta} = \frac{d\theta}{dt}$ with t as time.

Then Lagrange's equations are given by :

(1) $\ddot{r} + r\dot{\theta}^2 = -\mu / r^2$, $r\dot{\theta} =$ constant

(2) $\ddot{r} - r\dot{\theta} = -\mu / r^2$, $r^2 \ddot{\theta} =$ constant

(3) $\ddot{r} - r\dot{\theta}^2 = -\mu / r^2$, $r^2 \dot{\theta} =$ constant

(4) None of these

79. A rigid body under no forces is free to rotate about its centroid G, the principal moments of inertia at which are 7, 25, 32 units respectively. If $\omega = [\omega_1, \omega_2, \omega_3]$ be the angular velocity, then Euler's dynamical equations of motion are :

(1) $\dot{\omega}_1 - \omega_2\omega_3 = 0,\ \dot{\omega}_2 + \omega_3\omega_1 = 0,\ 4\dot{\omega}_3 + 3\omega_1\omega_2 = 0$

(2) $\dot{\omega}_1 + \omega_2\omega_3 = 0,\ \dot{\omega}_2 - \omega_3\omega_1 = 0,\ 16\dot{\omega}_3 + 9\omega_1\omega_2 = 0$

(3) $\dot{\omega}_1 - 2\omega_2\omega_3 = 0,\ \dot{\omega}_2 - 3\omega_3\omega_1 = 0,\ \dot{\omega}_3 - 3\omega_1\omega_2 = 0$

(4) None of these

80. The external of the functional $I[y(x)] = \int_{-1}^{0}(y'^2 - 2xy)\,dx$ subject to $y(-1) = 0,\ y(0) = 2$ is :

(1) $y = -\frac{x^3}{6} + \frac{13}{6}x + 2$

(2) $y = x^2 - 1$

(3) $y = \frac{1}{4}x^2$

(4) None of these

81. Let Q is the set of all rational numbers, Z is the set of all integers and N is the set of all natural numbers. Then which one of the following statements is ***true*** ?

(1) The set $Q \times Z$ is uncountable.

(2) The set $\{f : f$ is a function from N to $\{0, 1\}\}$ is uncountable.

(3) The set $\{\sqrt{p} : p$ is a prime number$\}$ is uncountable.

(4) None of these

82. The sequence $\left< \frac{\sin\left(\frac{n\pi}{3}\right)}{\sqrt{n}} \right>$ converges to the limit :

(1) 0 (2) 1 (3) 2 (4) None of these

83. Let $a_n = \frac{(-1)^{n+1}}{n!}, n \geq 0$ and $b_n = \sum_{k=0}^{n} a_k, n \geq 0$. Then for $|x| < 1$, the series $\sum_{n=0}^{\infty} b_n x^n$ converges to :

(1) $\frac{-e^{-x}}{1+x}$ (2) $\frac{-e^{-x}}{1-x^2}$ (3) $\frac{-e^{-x}}{1-x}$ (4) $-(1+x)e^{-x}$

84. The number of limit points of the set $\left\{\frac{1}{m}+\frac{1}{n} : m, n \in N\right\}$ is :

(1) 1
(2) 2
(3) finitely many
(4) infinitely many

85. Which of the following functions is uniformly continuous on the specified domain ?

(1) $f_1(x) = e^{x^2}, -\infty < x < \infty$

(2) $f_2(x) = \begin{cases} \frac{1}{x} & , 0 < x \leq 1 \\ 0 & , x = 0 \end{cases}$

(3) $f_3(x) = \begin{cases} \frac{x^2}{2} & , |x| \leq 1 \\ \frac{1}{1+x^2} & , |x| > 1 \end{cases}$

(4) $f_4(x) = \begin{cases} x & , |x| \leq 1 \\ x^2 & , |x| > 1 \end{cases}$

86. For $n \in N$, let $f_n, g_n : (0, 1) \to R$ be functions defined by $f_n(x) = x^n, g_n(x) = x^n(1-x)$. Then :

(1) $\{f_n\}$ converges uniformly but $\{g_n\}$ does not converge uniformly.

(2) $\{g_n\}$ converges uniformly but $\{f_n\}$ does not converge uniformly.

(3) both $\{f_n\}$ and $\{g_n\}$ converges uniformly.

(4) neither $\{f_n\}$ nor $\{g_n\}$ converges uniformly.

87. For which of the following function, Lagrange's mean value theorem is ***not*** applicable ?

(1) $f(x) = x + \frac{1}{x}$ in [1, 3]
(2) $f(x) = \sqrt{25 - x^2}$ in [–3, 4]

(3) $f(x) = \frac{1}{4x-1}$ in [1, 4]
(4) $f(x) = x^{1/5}$ in [–1, 1]

88. $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{n}{n^2+k^2}$ is equal to :

(1) $\frac{e}{3}$ (2) $\frac{5}{6}$ (3) $\frac{3}{4}$ (4) $\frac{\pi}{4}$

89. Consider the following improper integrals $I_1=\int_0^{\pi/2}\frac{\cos x}{\sqrt{1-\sin x}}dx$, $I_2=\int_0^1\frac{\log x}{\sqrt{1-x^2}}dx$,

$I_3=\int_2^{\infty}\frac{1}{x-1}dx$, then :

(1) All are convergent

(2) All are divergent

(3) I_1 and I_2 are convergent whereas I_3 is divergent

(4) I_1 and I_3 are convergent whereas I_2 is divergent

90. Consider the following statements :

P : There exists an unbounded subset of R whose Lebesgue measure is equal to 5.

Q : If $f : R \to R$ is continuous and $g : R \to R$ is such that $f = g$ almost everywhere on R, then g must be continuous almost everywhere on R.

Which of the above statements hold true ?

(1) only P (2) only Q

(3) both P and Q (4) neither P nor Q

91. If X is a continuous random variable with mean μ and variance σ^2, then for any positive number k, the inequality $P\{|X-\mu|\geq k\sigma\}\leq(1/k^2)$ is known as :

(1) Lyapunov's inequality (2) Chebychev's inequality

(3) Bienayme-Chebychev's inequality (4) Khintchine's inequality

92. With the usual notations, the value of probability p for a binomial variate X, if $n=6$ and $9P(X=4)=P(X=2)$, is :

(1) $\frac{1}{2}$ (2) $\frac{1}{3}$ (3) $\frac{1}{4}$ (4) None of these

93. The characteristics function of Poisson distribution with meas as m, is :

(1) $e^{m(it-1)}$ (2) $e^{m(e^{it}-1)}$ (3) e^{mit} (4) None of these

94. Which of the following do the normal distribution and the exponential density function have in common ?

(1) Both are bell- shaped.

(2) Both are symmetrical distributions.

(3) Both approach infinity as x approaches infinity.

(4) Both approach zero as x approaches infinity.

95. Let X be a random variable with uniform distribution on the interval (–1, 1) and $Y=(X+1)^2$. Then the probability density function $f(y)$ of Y, over the interval (0, 4), is :

(1) $\frac{3\sqrt{y}}{16}$ (2) $\frac{1}{4\sqrt{y}}$ (3) $\frac{1}{6\sqrt{y}}$ (4) $\frac{1}{\sqrt{y}}$

96. Let $\{X_n\}_{n\geq 0}$ be a homogeneous Markov chain whose state space is {0, 1, 2} and whose one-step transition probability matrix is $P=\begin{bmatrix} 0 & 1 & 0 \\ 0.3 & 0 & 0.7 \\ 0 & 1 & 0 \end{bmatrix}$. Then $\lim_{n\to\infty} P(X_{2n}=2 \mid X_0=2)$ is equal to :

(1) 0.7 (2) 0.5 (3) 0.3 (4) 0.1

97. Suppose customers arrive at an ATM facility according to a Poisson process with rate 5 customers per hour. The probability that no customer arrives at the ATM facility from 1:00 pm to 1:18 pm is :

(1) 0.94 (2) 0.75 (3) 0.45 (4) 0.22

98. In a pure birth process with birth rates $\lambda_n = 2^n$, $n \geq 0$, let the random variable T denote the time taken for the population size to grow from 0 to 5. If Var(T) denotes the variance of the random variable T, then Var(T) is equal to :

(1) $\frac{441}{256}$ (2) $\frac{341}{256}$ (3) $\frac{241}{256}$ (4) $\frac{141}{256}$

99. Let $X_1, X_2, \ldots\ldots, X_{10}$ be independent and identically distributed normal random variables with mean 0 and variance 2. Then $E\left(\dfrac{X_1^2}{\sum_{i=1}^{10} X_i^2}\right)$ is equal to :

(1) 0.8 (2) 0.6 (3) 0.3 (4) 0.1

100. Let $X_1, X_2, \ldots\ldots, X_n$ be a random sample drawn from a population with probability density function $f_x(x;\theta) = \dfrac{2x}{\theta^2}, 0 < x < \theta$. Then the method of moments estimator of θ is :

(1) $\dfrac{3\sum_{n=1}^{n} X_i}{2n}$

(2) $\dfrac{3\sum_{n=1}^{n} X_i^2}{2n}$

(3) $\dfrac{\sum_{i=1}^{n} X_i}{n}$

(4) $\dfrac{3\sum_{i=1}^{n} X_i(X_i - 1)}{2n}$

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Ph.D./URS-EE-Jan-2022

SUBJECT : Mathematics

10227

Sr. No. ....

Time : 1¼ **Hours** Max. Marks : **100** Total Questions : **100**

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1. Let G be a group of order 5^4 with center having 5^2 elements. Then the number of conjugacy classes in G is :

(1) 26 (2) 48 (3) 145 (4) None of these

2. Let I and J be the ideals generated by $\{5, \sqrt{10}\}$ and $\{4, \sqrt{10}\}$ in the ring $Z[\sqrt{10}] = \{a + b\sqrt{10} : a, b \in Z\}$, respectively. Then :

(1) both I and J are maximal ideals

(2) I is a maximal ideal but J is not a prime ideal

(3) I is not a maximal ideal but J is a prime ideal

(4) neither I nor J is a maximal ideal

3. Let Z denotes the set of all integers and Z_n denotes the set of all integers modulo n, for any positive integer n. Consider the following statements :

I. The ring $Z[\sqrt{-1}]$ is a unique factorization domain.

II. The ring $Z[\sqrt{-5}]$ is a principal ideal domain.

III. In the polynomial ring $Z_2[x]$, the ideal generated by $x^3 + x + 1$ is a maximal ideal.

IV. In the polynomial ring $Z_3[x]$, the ideal generated by $x^6 + 1$ is a prime ideal.

Which of the above statements are ***true*** ?

(1) I, II and III only (2) I and III only

(3) I, II and IV only (4) II and III only

4. Which of the following polynomial is reducible over the field Q of rational numbers ?

(1) $x^2 - 4x + 2$ (2) $x^3 - x + 1$

(3) $x^3 + 9x^2 - 3x + 6$ (4) None of these

5. Let F be the field with 4096 elements. The number of proper subfields of F is :

(1) 5 (2) 20 (3) 50 (4) 100

6. Let ω be a primitive complex cube root of unity. Then the degree of the field extension $Q(i, \sqrt{3}, \omega)$ over Q (the field of rational numbers) is :

(1) 4 (2) 3 (3) 2 (4) 1

7. For a subset S of a topological space, let Int(S) and $\overline{S}$ denote the interior and closure of S, respectively. Then which of the following statements is ***true*** ?

(1) If S is open, then $S = \text{Int}(\overline{S})$

(2) If the boundary of S is empty, then S is open.

(3) If $\overline{S} \setminus S$ is a proper subset of the boundary of S, then S is open.

(4) None of these

8. Let T_1 be the co-countable topology on R (the set of real numbers) and T_2 be the co-finite topology on R. Consider the following statements :

I. In (R, T_1), the sequence $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$ converges to 0.

II. In (R, T_2), the sequence $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$ converges to 0.

III. In (R, T_1), there is no sequence of rational numbers which converges to $\sqrt{3}$.

IV. In (R, T_2), there is no sequence of rational numbers which converges to $\sqrt{3}$.

Which of the above statements are ***true*** ?

(1) I and II only

(2) II and III only

(3) III and IV only

(4) I and IV only

9. Let R denote the set of all real numbers. Consider the following topological spaces :

$X_1 = (R, T_1)$, where T_1 is the upper limit topology having all sets $(a, b]$ as basis.

$X_2 = (R, T_2)$, where $T_2 = \{U \subset R : R \setminus U \text{ is finite}\} \cup \{\phi\}$

Then :

(1) both X_1 and X_2 are connected

(2) X_1 is connected and X_2 is not connected

(3) X_1 is not connected and X_2 is connected

(4) neither X_1 nor X_2 is connected

10. Consider the following statements :

P : Any continuous image of a compact space is compact.

Q : A topological space is compact if every basic open cover has a finite subcover.

Then :

(1) both P and Q are true
(2) P is true and Q is false
(3) P is false and Q is true
(4) both P and Q are false

11. The system of equations $x + 2y - z = 3$; $3x - y + 2z = 1$; $2x - 2y + 3z = 2$ has :

(1) no solution
(2) a unique solution
(3) infinite solutions
(4) None of these

12. The eigen values of the matrix $\begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$ are :

(1) $-1, 1, 2$
(2) $-2, 3, 4$
(3) $1, -1, 7$
(4) $2, -3, 5$

13. The matrix $\begin{bmatrix} 1 & x & z \\ 0 & 2 & y \\ 0 & 0 & 1 \end{bmatrix}$ is diagonalizable when (x, y, z) equals :

(1) $(0, 0, 1)$
(2) $(1, 1, 0)$
(3) $(\sqrt{2}, \sqrt{2}, 2)$
(4) $(\sqrt{2}, \sqrt{2}, \sqrt{2})$

14. Let $u = (1 + i, i, -1)$ and $v = (1 + 2i, 1 - i, 2i)$. Then $\langle u, v \rangle$ is :

(1) $2 - 2i$
(2) $-2 + 2i$
(3) $-2 - 2i$
(4) $2 + 2i$

15. The quadratic form corresponding to symmetric matrix $\begin{bmatrix} 0 & \frac{1}{2} & 1 \\ \frac{1}{2} & 0 & \frac{3}{2} \\ 1 & \frac{3}{2} & 0 \end{bmatrix}$ is :

(1) $xy + 3yz + 2zx$
(2) $xy - 3yz - 2zx$
(3) $xy + 3yz - 2zx$
(4) None of these

16. If the power series $\sum_{n=0}^{\infty} a_n (z + 3 - i)^n$ converges at $5i$ and diverges at $-3i$, then the power series :

(1) converges at $-2 + 5i$ and diverges at $2 - 3i$
(2) converges at $2 - 3i$ and diverges at $-2 + 5i$
(3) converges at both $2 - 3i$ and $-2 + 5i$
(4) diverges at both $2 - 3i$ and $-2 + 5i$

17. Which of the following function $f(z)$, of the complex variable z, is not analytic at all the points of the complex plane ?

(1) $f(z)=z^2$ (2) $f(z)=e^z$ (3) $f(z) = \sin z$ (4) $f(z) = \log z$

18. The function $f(z)$ of complex variable $z = x + iy$, where $i = \sqrt{-1}$, is given as $f(z)=(x^3-3xy^2)+iv(x,y)$. For this function to be analytic, $v(x, y)$ should be :

(1) $(3xy^2 - y^3)$ + constant

(2) $(3x^2y^2 - y^3)$ + constant

(3) $(3x^2y - y^3)$ + constant

(4) None of these

19. Let Γ denotes the boundary of the square region R with vertices (0, 0), (2, 0), (2, 2) and (0, 2) oriented in the counter- clockwise direction. Then value of $\oint_{\Gamma}(1-y^2)dx + x\,dy$ is :

(1) 12 (2) 15 (3) 20 (4) 25

20. Let C represent the unit circle centered at origin in the complex plane and, complex variable $z = x + iy$, where $i = \sqrt{-1}$. The value of the contour integral $\oint_C \frac{\cos h3z}{2z}\,dz$ (where integration is taken counter clockwise) is :

(1) 0 (2) 2 (3) πi (4) $2\pi i$

21. Let Q is the set of all rational numbers, Z is the set of all integers and N is the set of all natural numbers. Then which one of the following statements is ***true*** ?

(1) The set $Q \times Z$ is uncountable.

(2) The set $\{f : f$ is a function from N to $\{0, 1\}\}$ is uncountable.

(3) The set $\{\sqrt{p} : p$ is a prime number$\}$ is uncountable.

(4) None of these

22. The sequence $< \frac{\sin\left(\frac{n\pi}{3}\right)}{\sqrt{n}} >$ converges to the limit :

(1) 0 (2) 1 (3) 2 (4) None of these

23. Let $a_n = \frac{(-1)^{n+1}}{n!}, n \geq 0$ and $b_n = \sum_{k=0}^{n} a_k, n \geq 0$. Then for $|x| < 1$, the series $\sum_{n=0}^{\infty} b_n x^n$ converges to :

(1) $\frac{-e^{-x}}{1+x}$ (2) $\frac{-e^{-x}}{1-x^2}$ (3) $\frac{-e^{-x}}{1-x}$ (4) $-(1+x)e^{-x}$

24. The number of limit points of the set $\left\{\frac{1}{m}+\frac{1}{n} : m, n \in N\right\}$ is :

(1) 1 (2) 2

(3) finitely many (4) infinitely many

25. Which of the following functions is uniformly continuous on the specified domain ?

(1) $f_1(x) = e^{x^2}, -\infty < x < \infty$

(2) $f_2(x) = \begin{cases} \frac{1}{x} & , 0 < x \leq 1 \\ 0 & , x = 0 \end{cases}$

(3) $f_3(x) = \begin{cases} x^2 & , |x| \leq 1 \\ \frac{2}{1+x^2} & , |x| > 1 \end{cases}$

(4) $f_4(x) = \begin{cases} x & , |x| \leq 1 \\ x^2 & , |x| > 1 \end{cases}$

26. For $n \in N$, let $f_n, g_n : (0, 1) \to R$ be functions defined by $f_n(x) = x^n, g_n(x) = x^n(1-x)$. Then :

(1) $\{f_n\}$ converges uniformly but $\{g_n\}$ does not converge uniformly.

(2) $\{g_n\}$ converges uniformly but $\{f_n\}$ does not converge uniformly.

(3) both $\{f_n\}$ and $\{g_n\}$ converges uniformly.

(4) neither $\{f_n\}$ nor $\{g_n\}$ converges uniformly.

27. For which of the following function, Lagrange's mean value theorem is ***not*** applicable ?

(1) $f(x) = x + \frac{1}{x}$ in $[1, 3]$

(2) $f(x) = \sqrt{25 - x^2}$ in $[-3, 4]$

(3) $f(x) = \frac{1}{4x-1}$ in $[1, 4]$

(4) $f(x) = x^{1/5}$ in $[-1, 1]$

28. $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{n}{n^2 + k^2}$ is equal to :

(1) $\frac{e}{3}$ (2) $\frac{5}{6}$ (3) $\frac{3}{4}$ (4) $\frac{\pi}{4}$

29. Consider the following improper integrals $I_1 = \int_0^{\pi/2} \frac{\cos x}{\sqrt{1-\sin x}}\, dx$, $I_2 = \int_0^1 \frac{\log x}{\sqrt{1-x^2}}\, dx$,

$I_3 = \int_2^{\infty} \frac{1}{x-1}\, dx$, then :

(1) All are convergent

(2) All are divergent

(3) I_1 and I_2 are convergent whereas I_3 is divergent

(4) I_1 and I_3 are convergent whereas I_2 is divergent

30. Consider the following statements :

P : There exists an unbounded subset of R whose Lebesgue measure is equal to 5.

Q : If $f : R \to R$ is continuous and $g : R \to R$ is such that $f = g$ almost everywhere on R, then g must be continuous almost everywhere on R.

Which of the above statements hold true ?

(1) only P (2) only Q

(3) both P and Q (4) neither P nor Q

31. Let {0, 1, 2, 3} be an observed sample of size 4 from $N(\theta, 5)$ distribution, where $\theta \in [2, \infty)$. Then the maximum likelihood estimate of θ based on the observed sample is :

(1) 16 (2) 8 (3) 4 (4) 2

32. A random sample of size 100 is classified into 10 class intervals covering all the data points. To test whether data comes from a normal population with unknown mean and unknown variance, the chi-square goodness of fit test is used. The degrees of freedom of the test statistic is equal to :

(1) 10 (2) 9 (3) 8 (4) 7

33. A random sample of 500 apples was taken from a large consignment and 60 were found to be bad. Then the 98% confidence limits for the percentage of bad apples in the consignment are :

(1) (8.82, 17.52) (2) (8.61, 15.38)

(3) (8.32, 14.63) (4) (8.19, 13.52)

34. Consider a two - way fixed effects analysis of variance model without interaction effect and one observation per cell. If there are 5 factors and 4 columns, then the degrees of freedom for the error sum of squares is :

(1) 20 (2) 19 (3) 12 (4) 11

35. The simple correlation coefficients between temperature (X_1), corn yield (X_2) and rainfall (X_3) are : $r_{12} = 0.75$, $r_{23} = 0.54$, $r_{31} = 0.43$. Then the partial correlation coefficient $r_{12.3}$ is :

(1) 0.6815 (2) 0.4223

(3) 0.2392 (4) None of these

36. Let (X_1, X_2) be a random vector following bivariate normal distribution with mean vector (0, 0), variance (X_1) = variance $(X_2) = 1$ and correlation coefficient ρ, where $|\rho| < 1$. Then $P(X_1 + X_2 > 0)$ is equal to :

(1) 1 (2) 0.8 (3) 0.5 (4) 0.2

37. The total number of standard 4×4 Latin squares is :

(1) 4 (2) 8 (3) 12 (4) 16

38. The minimum value of $Z = 20x + 10y$

subject to the constraints :

$x + 2y \leq 40$; $3x + y \geq 30$; $4x + 3y \geq 60$; $x, y \geq 0$, is :

(1) 240 (2) 215 (3) 272 (4) None of these

39. Consider a parallel system with two components. The lifetimes of the two components are independent and identically distributed random variables each following an exponential distribution with mean 1. The expected lifetime of the system is :

(1) 1 (2) 1/2 (3) 3/2 (4) 2

40. Consider an M/M/1 queue with interarrival time having exponential distribution with mean $\frac{1}{\lambda}$ and service time having exponential distribution with mean $\frac{1}{\mu}$. Which of the following is ***true*** ?

(1) If $0 < \lambda < \mu$, then the queue length has limiting distribution Poisson $(\mu - \lambda)$.

(2) If $0 < \mu < \lambda$, then the queue length has limiting distribution Poisson $(\lambda - \mu)$.

(3) If $0 < \lambda < \mu$, then the queue length has limiting distribution which is geometric.

(4) If $0 < \mu < \lambda$, then the queue length has limiting distribution which is geometric.

41. Using Newton-Raphson method, the real root of the equation $x^3 - 2x + 5 = 0$ is :

(1) 3.129 (2) 1.582 (3) −2.754 (4) −2.095

42. Using Gauss- elimination method, the solution of the following system of equations $x + y + 2z = 7, 4x + 3y + 2z = 8, 3x + 2y + 4z = 13$, is :

(1) $x = -1, y = 2, z = 3$ (2) $x = 1, y = 3, z = 5$

(3) $x = 1, y = 1, z = 2$ (4) None of these

43. Given that

x	10	20	30	40	50
$f(x)$	46	66	81	93	101

The value of $\nabla^2 f(50)$ is :

(1) 8 (2) 3 (3) −4 (4) −1

44. Given that

x	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
y	1	$\frac{4}{5}$	$\frac{2}{3}$	$\frac{4}{7}$	$\frac{1}{2}$

Using trapezoidal rule, the value of $\int_0^1 y\,dx$ is :

(1) 0.938 (2) 0.697 (3) 0.352 (4) 0.241

45. Given $\frac{dy}{dx} = -xy^2$ with $y(0) = 2$. Then using modified Euler's method, the value of $y(0.1)$ by taking step size $h = 0.1$, is :

(1) 1.9804 (2) 1.5636 (3) 1.2921 (4) None of these

46. Let q_j and $\dot{q}_j$ respectively are the generalized coordinates and velocity of a dynamical system and p_j are its generalized momenta. Then the relation between Hamiltonian $H(q_j, p_j, t)$ and Lagrangian $L(q_j, \dot{q}_j, t)$ is given by :

(1) $H = \sum \dot{p}_j q_j - L$

(2) $H = \sum p_j \dot{q}_j - L$

(3) $H = \sum \dot{p}_j \dot{q}_j - L$

(4) None of these

47. Let T be the kinetic energy and V be the potential energy of the dynamical system, then the integral $\int_{t_1}^{t_2} (T - V)dt$ has a stationary value, where t_1 and t_2 are fixed. This principle is known as :

(1) Hamilton's principle

(2) Principle of least action

(3) D' Alembert principle

(4) None of these

48. Consider the motion of a planet $P(r, \theta)$ of mass m moving around the Sun $S(0, 0)$ under the inverse square law of attraction $\mu m/r^2$. Let kinetic energy T of the system is given by :

$$T = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2), \text{ where } \dot{r} = \frac{dr}{dt} \text{ and } \dot{\theta} = \frac{d\theta}{dt} \text{ with } t \text{ as time.}$$

Then Lagrange's equations are given by :

(1) $\ddot{r} + r\dot{\theta}^2 = -\mu/r^2, \; r\dot{\theta} = \text{constant}$

(2) $\ddot{r} - r\dot{\theta} = -\mu/r^2, \; r^2\ddot{\theta} = \text{constant}$

(3) $\ddot{r} - r\dot{\theta}^2 = -\mu/r^2, \; r^2\dot{\theta} = \text{constant}$

(4) None of these

49. A rigid body under no forces is free to rotate about its centroid G, the principal moments of inertia at which are 7, 25, 32 units respectively. If $\omega = [\omega_1, \omega_2, \omega_3]$ be the angular velocity, then Euler's dynamical equations of motion are :

(1) $\dot{\omega}_1 - \omega_2\omega_3 = 0, \; \dot{\omega}_2 + \omega_3\omega_1 = 0, \; 4\dot{\omega}_3 + 3\omega_1\omega_2 = 0$

(2) $\dot{\omega}_1 + \omega_2\omega_3 = 0, \; \dot{\omega}_2 - \omega_3\omega_1 = 0, \; 16\dot{\omega}_3 + 9\omega_1\omega_2 = 0$

(3) $\dot{\omega}_1 - 2\omega_2\omega_3 = 0, \; \dot{\omega}_2 - 3\omega_3\omega_1 = 0, \; \dot{\omega}_3 - 3\omega_1\omega_2 = 0$

(4) None of these

50. The external of the functional $I[y(x)] = \int_{-1}^{0}(y'^2 - 2xy)\,dx$ subject to $y(-1) = 0, y(0) = 2$ is :

(1) $y = -\frac{x^3}{6} + \frac{13}{6}x + 2$

(2) $y = x^2 - 1$

(3) $y = \frac{1}{4}x^2$

(4) None of these

51. The coefficient of $\frac{1}{z}$ in the Laurent series expansion of the function $f(z)=\frac{1}{z^2(1-z)}$ about $z=0$, is :

(1) 1 (2) 0 (3) −1 (4) None of these

52. The residue of the function $f(z)=\frac{z+1}{z^2(z-3)}$ at $z=0$, is :

(1) $-\frac{9}{2}$ (2) $\frac{1}{2}$ (3) $-\frac{4}{9}$ (4) None of these

53. The coefficient of magnification at $z=2+3i$ for the conformal transformation $w=z^2$ is :

(1) $\sqrt{3}$ (2) $\sqrt{5}$ (3) $2\sqrt{7}$ (4) $2\sqrt{13}$

54. Let $T(z)=\frac{az+b}{cz+d}$, $ad-bc\neq 0$, be the Mobius transformation which maps the points $z_1=0$, $z_2=-i$, $z_3=\infty$ in the z- plane onto the points $w_1=10$, $w_2=5-5i$, $w_3=5+5i$ in the w- plane respectively. Then the image of the set $S=\{z\in C:\mathrm{Re}(z)<0\}$ under the map $w=T(z)$ is :

(1) $\{w\in C:|w|<5\}$ (2) $\{w\in C:|w|>5\}$
(3) $\{w\in C:|w-5|<5\}$ (4) $\{w\in C:|w-5|>5\}$

55. Among 100 students, 32 study Mathematics, 20 study Physics, 45 study Biology, 15 study Mathematics and Biology, 7 study Mathematics and Physics, 10 study Physics and Biology, 30 do not study any of three subjects. Then the number of students study only Mathematics, is :

(1) 8 (2) 15 (3) 25 (4) None of these

56. The no. of positive divisors of 2100 is :

(1) 50 (2) 44 (3) 40 (4) 36

57. The last two digits of 38^{2011} are :

(1) 6 (2) 2 (3) 4 (4) 8

58. Which one of the following statements is ***false*** ?

(1) If Q denotes the additive group of rational numbers and $f:Q\to Q$ is a non-trivial homomorphism, then f is an isomorphism.
(2) Any quotient group of a cyclic group is cyclic.
(3) If every subgroup of a group G is a normal subgroup, then G is abelian.
(4) Every group of order 33 is cyclic

59. Let Z_n denotes the group of integers modulo n, under the operation of addition modulo n, for any positive integer n. Then the number of elements of order 15 in the additive group $Z_{60} \times Z_{50}$ is :

(1) 48 (2) 30 (3) 25 (4) 10

60. The number of 5- Sylow subgroups in the symmetric group S_5 of degree 5, is :

(1) 2 (2) 3 (3) 5 (4) 6

61. The external of the functional $I[y(x)] = \frac{1}{2}\int_0^1 (y'')^2\, dx$ subject to $y(0) = 0$, $y(1) = \frac{1}{2}$; $y'(0) = 0$, $y'(1) = 1$, is :

(1) $y = x - 1$ (2) $y = \frac{1}{2}x^2$

(3) $y = \sin x$ (4) None of these

62. The external of the functional $I[y(x)] = \int_0^1 y'^2(x)\, dx$ subject to $y(0) = 0$, $y(1) = 1$ and $\int_0^1 y(x)dx = 0$, is :

(1) $3x^2 - 2x$ (2) $8x^3 - 9x^2 + 2x$

(3) $\frac{5}{4}x^4 - \frac{2}{3}x$ (4) $\frac{-21}{2}x^5 + 10x^4 + 4x^3 - \frac{5}{2}x$

63. The solution of the linear integral equation $\phi(x) = (\cos x - x - 2) + \int_0^x (\xi - x)\phi(\xi)\, d\xi$, is :

(1) $\phi(x) = \cos hx$ (2) $\phi(x) = \cos x + e^x \sin x$

(3) $\phi(x) = e^x$ (4) $\phi(x) = -\cos x - \sin x - \frac{1}{2} x \sin x$

64. The eigen values (λ) of the homogeneous integral equation $\phi(x) = \lambda \int_0^{\pi} \cos(x+\xi)\phi(\xi)\, d\xi$ are :

(1) $\lambda_1 = \frac{-4}{\pi}, \lambda_2 = \frac{4}{\pi}$
(2) $\lambda_1 = \frac{-\pi}{2}, \lambda_2 = \frac{\pi}{2}$
(3) $\lambda_1 = \frac{-2}{\pi}, \lambda_2 = \frac{2}{\pi}$
(4) None of these

65. The resolvent kernel for the integral equation $\phi(x) = x^2 + \int_0^x e^{t-x}\phi(t)\, dt$ is :

(1) e^{t-x}
(2) 1
(3) e^{x-t}
(4) $x^2 + e^{x-t}$

66. From the six letters A, B, C, D, E and F, three letters are chosen at random with replacement. What is the probability that either the word BAD or the word CAD can be formed from the chosen letters ?

(1) $\frac{1}{216}$
(2) $\frac{3}{216}$
(3) $\frac{6}{216}$
(4) $\frac{12}{216}$

67. Let X be a random variable which is symmetric about 0. Let F be the cumulative distribution function of X. Which of the following statements is always ***true*** ?

(1) $F(x) + F(-x) = 1$ for all $x \in R$
(2) $F(x) - F(-x) = 0$ for all $x \in R$
(3) $F(x) + F(-x) = 1 + P(X = x)$ for all $x \in R$
(4) $F(x) + F(-x) = 1 - P(X = -x)$ for all $x \in R$

68. The probability density function of the random vector (X, Y) is given by :

$$f_{X,Y}(x, y) = \begin{cases} c, & 0 < x < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

Then the value of c is equal to :

(1) 1
(2) 2
(3) 4
(4) 5

69. If X and Y are random variables such that $E[2X + Y] = 0$ and $E[X + 2Y] = 33$, then $E[X] + E[Y]$ is equal to :

(1) 26
(2) 20
(3) 11
(4) None of these

70. The moment generating function of a random variable X is given by :

$$M_X(t) = \frac{1}{6} + \frac{1}{3}e^t + \frac{1}{3}e^{2t} + \frac{1}{6}e^{3t}, \; -\infty < t < \infty$$

Then $P(X \leq 2)$ equals :

(1) $\frac{1}{3}$ (2) $\frac{1}{6}$ (3) $\frac{1}{2}$ (4) $\frac{5}{6}$

71. If X is a continuous random variable with mean μ and variance σ^2, then for any positive number k, the inequality $P\{|X - \mu| \geq k\sigma\} \leq (1/k^2)$ is known as :

(1) Lyapunov's inequality (2) Chebychev's inequality

(3) Bienayme-Chebychev's inequality (4) Khintchine's inequality

72. With the usual notations, the value of probability p for a binomial variate X, if $n = 6$ and $9P(X = 4) = P(X = 2)$, is :

(1) $\frac{1}{2}$ (2) $\frac{1}{3}$ (3) $\frac{1}{4}$ (4) None of these

73. The characteristics function of Poisson distribution with meas as m, is :

(1) $e^{m(it-1)}$ (2) $e^{m(e^{it}-1)}$ (3) e^{mit} (4) None of these

74. Which of the following do the normal distribution and the exponential density function have in common ?

(1) Both are bell- shaped.

(2) Both are symmetrical distributions.

(3) Both approach infinity as x approaches infinity.

(4) Both approach zero as x approaches infinity.

75. Let X be a random variable with uniform distribution on the interval $(-1, 1)$ and $Y = (X+1)^2$. Then the probability density function $f(y)$ of Y, over the interval $(0, 4)$, is :

(1) $\frac{3\sqrt{y}}{16}$ (2) $\frac{1}{4\sqrt{y}}$ (3) $\frac{1}{6\sqrt{y}}$ (4) $\frac{1}{\sqrt{y}}$

76. Let $\{X_n\}_{n\geq 0}$ be a homogeneous Markov chain whose state space is $\{0, 1, 2\}$ and whose one-step transition probability matrix is $P = \begin{bmatrix} 0 & 1 & 0 \\ 0.3 & 0 & 0.7 \\ 0 & 1 & 0 \end{bmatrix}$. Then $\lim_{n\to\infty} P(X_{2n} = 2 \mid X_0 = 2)$ is equal to :

(1) 0.7 (2) 0.5 (3) 0.3 (4) 0.1

77. Suppose customers arrive at an ATM facility according to a Poisson process with rate 5 customers per hour. The probability that no customer arrives at the ATM facility from 1:00 pm to 1:18 pm is :

(1) 0.94 (2) 0.75 (3) 0.45 (4) 0.22

78. In a pure birth process with birth rates $\lambda_n = 2^n$, $n \geq 0$, let the random variable T denote the time taken for the population size to grow from 0 to 5. If Var(T) denotes the variance of the random variable T, then Var(T) is equal to :

(1) $\frac{441}{256}$ (2) $\frac{341}{256}$ (3) $\frac{241}{256}$ (4) $\frac{141}{256}$

79. Let $X_1, X_2, \ldots\ldots, X_{10}$ be independent and identically distributed normal random variables with mean 0 and variance 2. Then $E\left(\frac{X_1^2}{\sum_{i=1}^{10} X_i^2}\right)$ is equal to :

(1) 0.8 (2) 0.6 (3) 0.3 (4) 0.1

80. Let $X_1, X_2, \ldots\ldots, X_n$ be a random sample drawn from a population with probability density function $f_X(x;\theta) = \frac{2x}{\theta^2}, 0 < x < \theta$. Then the method of moments estimator of θ is :

(1) $\frac{3\sum_{n=1}^{n} X_i}{2n}$

(2) $\frac{3\sum_{n=1}^{n} X_i^2}{2n}$

(3) $\frac{\sum_{i=1}^{n} X_i}{n}$

(4) $\frac{3\sum_{i=1}^{n} X_i(X_i - 1)}{2n}$

81. The directional derivative of the function $f(x, y, z) = xy^2 + yz^3$ at the point (2, −1, 1) in the direction of $\hat{i} + 2\hat{j} + 2\hat{k}$ is :

(1) $-\frac{13}{3}$ (2) $-\frac{11}{3}$ (3) $-\frac{10}{3}$ (4) None of these

82. Let R be the set of all real numbers and $R^2 = \{(x, y) : x, y \in R\}$. Let $f : R^2 \to R$ be defined by $f(x, y) = \begin{cases} (x^2 + y^2)\sin\left(\frac{1}{x^2 + y^2}\right) & , \text{ if } (x, y) \neq (0, 0) \\ 0 & , \text{ if } (x, y) = (0, 0) \end{cases}$. Then which one of the following statements is ***true*** ?

(1) f is continuous but not differentiable at (0, 0)
(2) f is not continuous at (0, 0)
(3) f is differentiable at (0, 0)
(4) None of these

83. Consider the following statements :

$P : d_1(x, y) = \min\{2, |x - y|\}$ is a metric for R (the set of all real numbers).

$Q : d_2(x, y) = \begin{cases} |x| + |y| & , \text{ if } x \neq y \\ 0 & , \text{ if } x = y \end{cases}$ is a metric on (0, 1)

Then :

(1) both P and Q are true (2) P is true and Q is false
(3) P is false and Q is true (4) both P and Q are false

84. Suppose that :

$U = R^2 \setminus \{(x, y) \in R^2 : x, y \in Q\}$,

$V = R^2 \setminus \left\{(x, y) \in R^2 : x > 0, y = \frac{1}{x}\right\}$

Then with respect to the Euclidean metric on R^2,

(1) both U and V are disconnected
(2) U is disconnected but V is connected
(3) U is connected but V is disconnected
(4) both U and V are connected

85. Let X and Y be normed linear spaces and let $T : X \to Y$ be any bijective linear map with closed graph. Then which one of the following statements is ***true*** ?

(1) The graph of T is equal to $X \times Y$

(2) T^{-1} is continuous

(3) The graph of T^{-1} is closed

(4) T is continuous

86. Let V be a vector space over R^3. Which one of the following is a subspace of V ?

(1) $\{(x, y, z) : x - 3y + 4z = 0, x, y, z \in R\}$

(2) $\{(x, y, z) : x + y \geq 0, x, y, z \in R\}$

(3) $\{(x, y, z) : x \leq 0, x, y, z \in R\}$

(4) None of these

87. The dimension of the vector space of 7×7 real symmetric matrices with trace zero and the sum of the off-diagonal elements zero, is :

(1) 47 (2) 28 (3) 27 (4) 26

88. Which of the following is ***not*** a linear transformation ?

(1) $T(x, y, z) = (x, y)$ for all $(x, y, z) \in R^3$

(2) $T(x, y, z) = (x + 1, y + z)$ for all $(x, y, z) \in R^3$

(3) $T(x, y, z) = (x - z, y)$ for all $(x, y, z) \in R^3$

(4) $T(x, y, z) = (x + y + z, 0)$ for all $(x, y, z) \in R^3$

89. Suppose that $T : R^4 \to R[x]$ is a linear transformation over R satisfying :

$T(-1, 1, 1, 1) = x^2 + 2x^4$

$T(1, 2, 3, 4) = 1 - x^2$

$T(2, -1, -1, 0) = x^3 - x^4$

Then the coefficient of x^4 in $T(-3, 5, 6, 6)$ is :

(1) 1 (2) 2 (3) 3 (4) 5

90. The rank of the matrix $\begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & 1 \\ 4 & 2 & 3 \end{bmatrix}$ is :

(1) 1 (2) 2 (3) 3 (4) None of these

91. The solution of the differential equation $\frac{dy}{dx} = 3e^x + 2y,\ y(0) = 0$ by Picard's method upto third approximation is :

(1) $21e^x - 6x^2 - 18x - 21$

(2) $21e^x + 6x^2 - 18x + 21$

(3) $21e^x + 6x^2 + 18x - 21$

(4) None of these

92. Consider the ordinary differential equation $y'' + P(x)y' + Q(x)y = 0$, where P and Q are smooth functions. Let y_1 and y_2 be any two solutions of the ODE. Let $W(x)$ be the corresponding Wronskian. Then which of the following is always true ?

(1) If y_1 and y_2 are linearly dependent, then $\exists\, x_1,\ x_2$ such that $W(x_1) = 0$ and $W(x_2) \neq 0$

(2) If y_1 and y_2 are linearly independent, then $W(x) = 0,\ \forall x$

(3) If y_1 and y_2 are linearly dependent, then $W(x) \neq 0,\ \forall x$

(4) If y_1 and y_2 are linearly independent, then $W(x) \neq 0,\ \forall x$

93. Consider the system of differential equations $\frac{dx}{dt} = 2x - 7y;\ \frac{dy}{dt} = 3x - 8y$. Then the critical point (0, 0) of the system is an :

(1) unstable node

(2) asymptotically stable node

(3) asymptotically stable spiral

(4) unstable spiral

94. Using method of variation of parameters, the solution of the differential equation $y'' - 6y' + 9y = e^{3x}/x^2$ is :

(1) $y = (c_1 + c_2 x)e^{3x} - e^{3x}(\log x + 1)$

(2) $y = (c_1 + c_2 x)e^{2x} + e^{2x}(\log x + 1)$

(3) $y = (c_1 + c_2 x)e^{4x} - e^{3x}(\log x - 1)$

(4) None of these

95. Consider the boundary value problem (BVP) $\frac{d^2y}{dx^2} + \alpha y(x) = 0$, $\alpha \in R$ (the set of all real numbers), with the boundary conditions $y(0) = 0$, $y(\pi) = k$ (where k is a non-zero real number). Then which one of the following statements is ***true*** ?

(1) For $\alpha = 1$, the BVP has infinitely many solutions.

(2) For $\alpha = 1$, the BVP has a unique solution.

(3) For $\alpha = -1$, k < 0, the BVP has a solution $y(x)$ such that $y(x) > 0$ for all $x \in (0, \pi)$

(4) For $\alpha = -1$, k > 0, the BVP has a solution $y(x)$ such that $y(x) > 0$ for all $x \in (0, \pi)$

96. Let R be the set of all real numbers and $R^2 = \{(x, y) : x, y \in R\}$. Let the general integral of the partial differential equation $(2xy-1)\frac{\partial z}{\partial x} + (z - 2x^2)\frac{\partial z}{\partial y} = 2(x - yz)$ be given by $F(u, v) = 0$, where $F : R^2 \to R$ is a continuously differentiable function. Then :

(1) $u = x^2 + y^2 + Z$, $v = xz + y$

(2) $u = x^2 + y^2 - Z$, $v = xz - y$

(3) $u = x^2 - y^2 + z$, $v = yz + x$

(4) $u = x^2 + y^2 - Z$, $v = yz - x$

97. The particular integral of the differential equation $(D^2 + 3DD' + 2D'^2)z = 2x + 3y$ is given by :

(1) $\frac{1}{150}(2x+3y)^3$ (2) $\frac{1}{240}(2x+3y)^3$

(3) $\frac{1}{320}(2x+3y)^3$ (4) None of these

98. If $u(x, y)$ is the solution of the Cauchy problem $x\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 1,\ u(x,0) = -x^2, x > 0$.

Then the value of u (2, 1) is equal to :

(1) $1-2e^{-2}$ (2) $1+4e^{-2}$ (3) $1-4e^{-2}$ (4) $1+2e^{-2}$

99. The partial differential equation $(x^2 + y^2 - 1)\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + (x^2 + y^2 - 1)\frac{\partial^2 u}{\partial y^2} = 0$ is :

(1) parabolic in the region $x^2 + y^2 > 2$

(2) hyperbolic in the region $x^2 + y^2 > 2$

(3) elliptic in the region $0 < x^2 + y^2 < 2$

(4) hyperbolic in the region $0 < x^2 + y^2 < 2$

100. The function $u(x, t)$ satisfies the initial value problem

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, x \in R, t > 0$$

$$u(x,0) = 0, \frac{\partial u}{\partial t}(x,0) = 4xe^{-x^2}$$

Then the value of $u(5, 5)$ is :

(1) $1-\frac{1}{e^{100}}$ (2) $1-e^{100}$ (3) $1-\frac{1}{e^{10}}$ (4) $1-e^{10}$

Total No. of Printed Pages : 21

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D **SET-Y**

Ph.D./URS-EE-Jan-2022

SUBJECT : Mathematics

10228

Sr. No. ....

Time : 1¼ Hours Max. Marks : 100 Total Questions : 100

Roll No. (in figures)________________ (in words) ______________________________

Name ______________________________ Father's Name______________________________

Mother's Name______________________________ Date of Examination______________________________

______________________________ ______________________________

(Signature of the Candidate) (Signature of the Invigilator)

CANDIDATES MUST READ THE FOLLOWING INFORMATION/INSTRUCTIONS BEFORE STARTING THE QUESTION PAPER.

1. ***All questions are compulsory.***
2. The candidates ***must return*** the question booklet as well as OMR Answer-Sheet to the Invigilator concerned before leaving the Examination Hall, failing which a case of use of unfair-means / mis-behaviour will be registered against him / her, in addition to lodging of an FIR with the police. Further the answer-sheet of such a candidate will not be evaluated.
3. Keeping in view the transparency of the examination system, carbonless OMR Sheet is provided to the candidate so that a copy of OMR Sheet may be kept by the candidate.
4. Question Booklet along with answer key of all the A, B, C & D code will be got uploaded on the University website after the conduct of Entrance Examination. In case there is any discrepancy in the Question Booklet/Answer Key, the same may be brought to the notice of the Controller of Examination in writing/through E.Mail within 24 hours of uploading the same on the University Website. Thereafter, no complaint in any case, will be considered.
5. The candidate ***must not*** do any rough work or writing in the OMR Answer-Sheet. Rough work, if any, may be done in the question booklet itself. Answers ***must not*** be ticked in the question booklet.
6. ***There will be no negative marking. Each correct answer will be awarded one full mark. Cutting, erasing, overwriting and more than one answer in OMR Answer-Sheet will be treated as incorrect answer.***
7. Use only **Black** or **Blue Ball Point Pen** of good quality in the OMR Answer-Sheet.
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1. The external of the functional $I[y(x)] = \frac{1}{2}\int_0^1 (y'')^2\, dx$ subject to $y(0) = 0,\ y(1) = \frac{1}{2};$ $y'(0) = 0,\ y'(1) = 1$, is :

(1) $y = x - 1$ (2) $y = \frac{1}{2}x^2$

(3) $y = \sin x$ (4) None of these

2. The external of the functional $I[y(x)] = \int_0^1 y'^2(x)\, dx$ subject to $y(0) = 0,\ y(1) = 1$ and $\int_0^1 y(x)dx = 0$, is :

(1) $3x^2 - 2x$ (2) $8x^3 - 9x^2 + 2x$

(3) $\frac{5}{4}x^4 - \frac{2}{3}x$ (4) $\frac{-21}{2}x^5 + 10x^4 + 4x^3 - \frac{5}{2}x$

3. The solution of the linear integral equation $\phi(x) = (\cos x - x - 2) + \int_0^x (\xi - x)\phi(\xi)\, d\xi$, is :

(1) $\phi(x) = \cos hx$ (2) $\phi(x) = \cos x + e^x \sin x$

(3) $\phi(x) = e^x$ (4) $\phi(x) = -\cos x - \sin x - \frac{1}{2} x \sin x$

4. The eigen values (λ) of the homogeneous integral equation $\phi(x) = \lambda \int_0^\pi \cos(x + \xi)\phi(\xi)\, d\xi$ are :

(1) $\lambda_1 = \frac{-4}{\pi},\ \lambda_2 = \frac{4}{\pi}$ (2) $\lambda_1 = \frac{-\pi}{2},\ \lambda_2 = \frac{\pi}{2}$

(3) $\lambda_1 = \frac{-2}{\pi},\ \lambda_2 = \frac{2}{\pi}$ (4) None of these

5. The resolvent kernel for the integral equation $\phi(x) = x^2 + \int_0^x e^{t-x}\phi(t)\,dt$ is :

(1) e^{t-x} (2) 1 (3) e^{x-t} (4) $x^2 + e^{x-t}$

6. From the six letters A, B, C, D, E and F, three letters are chosen at random with replacement. What is the probability that either the word BAD or the word CAD can be formed from the chosen letters ?

(1) $\frac{1}{216}$ (2) $\frac{3}{216}$ (3) $\frac{6}{216}$ (4) $\frac{12}{216}$

7. Let X be a random variable which is symmetric about 0. Let F be the cumulative distribution function of X. Which of the following statements is always ***true*** ?

(1) $F(x) + F(-x) = 1$ for all $x \in R$

(2) $F(x) - F(-x) = 0$ for all $x \in R$

(3) $F(x) + F(-x) = 1 + P(X = x)$ for all $x \in R$

(4) $F(x) + F(-x) = 1 - P(X = -x)$ for all $x \in R$

8. The probability density function of the random vector (X, Y) is given by :

$$f_{X,Y}(x, y) = \begin{cases} c & , \quad 0 < x < y < 1 \\ 0 & , \quad \text{otherwise} \end{cases}$$

Then the value of c is equal to :

(1) 1 (2) 2 (3) 4 (4) 5

9. If X and Y are random variables such that $E[2X + Y] = 0$ and $E[X + 2Y] = 33$, then $E[X] + E[Y]$ is equal to :

(1) 26 (2) 20 (3) 11 (4) None of these

10. The moment generating function of a random variable X is given by :

$$M_X(t) = \frac{1}{6} + \frac{1}{3}e^t + \frac{1}{3}e^{2t} + \frac{1}{6}e^{3t}, \; -\infty < t < \infty$$

Then $P(X \le 2)$ equals :

(1) $\frac{1}{3}$ (2) $\frac{1}{6}$ (3) $\frac{1}{2}$ (4) $\frac{5}{6}$

11. The solution of the differential equation $\frac{dy}{dx} = 3e^x + 2y,\ y(0) = 0$ by Picard's method upto third approximation is :

(1) $21e^x - 6x^2 - 18x - 21$

(2) $21e^x + 6x^2 - 18x + 21$

(3) $21e^x + 6x^2 + 18x - 21$

(4) None of these

12. Consider the ordinary differential equation $y'' + P(x)y' + Q(x)y = 0$, where P and Q are smooth functions. Let y_1 and y_2 be any two solutions of the ODE. Let $W(x)$ be the corresponding Wronskian. Then which of the following is always true ?

(1) If y_1 and y_2 are linearly dependent, then $\exists\, x_1,\ x_2$ such that $W(x_1) = 0$ and $W(x_2) \neq 0$

(2) If y_1 and y_2 are linearly independent, then $W(x) = 0,\ \forall x$

(3) If y_1 and y_2 are linearly dependent, then $W(x) \neq 0,\ \forall x$

(4) If y_1 and y_2 are linearly independent, then $W(x) \neq 0,\ \forall x$

13. Consider the system of differential equations $\frac{dx}{dt} = 2x - 7y;\ \frac{dy}{dt} = 3x - 8y$. Then the critical point (0, 0) of the system is an :

(1) unstable node

(2) asymptotically stable node

(3) asymptotically stable spiral

(4) unstable spiral

14. Using method of variation of parameters, the solution of the differential equation $y'' - 6y' + 9y = e^{3x}/x^2$ is :

(1) $y = (c_1 + c_2x)e^{3x} - e^{3x}(\log x + 1)$

(2) $y = (c_1 + c_2x)e^{2x} + e^{2x}(\log x + 1)$

(3) $y = (c_1 + c_2x)e^{4x} - e^{3x}(\log x - 1)$

(4) None of these

15. Consider the boundary value problem (BVP) $\frac{d^2y}{dx^2} + \alpha y(x) = 0$, $\alpha \in R$ (the set of all real numbers), with the boundary conditions $y(0) = 0$, $y(\pi) = k$ (where k is a non-zero real number). Then which one of the following statements is ***true*** ?

(1) For $\alpha = 1$, the BVP has infinitely many solutions.

(2) For $\alpha = 1$, the BVP has a unique solution.

(3) For $\alpha = -1$, $k < 0$, the BVP has a solution $y(x)$ such that $y(x) > 0$ for all $x \in (0, \pi)$

(4) For $\alpha = -1$, $k > 0$, the BVP has a solution $y(x)$ such that $y(x) > 0$ for all $x \in (0, \pi)$

16. Let R be the set of all real numbers and $R^2 = \{(x, y) : x, y \in R\}$. Let the general integral of the partial differential equation $(2xy - 1)\frac{\partial z}{\partial x} + (z - 2x^2)\frac{\partial z}{\partial y} = 2(x - yz)$ be given by $F(u, v) = 0$, where $F : R^2 \to R$ is a continuously differentiable function. Then :

(1) $u = x^2 + y^2 + Z$, $v = xz + y$

(2) $u = x^2 + y^2 - Z$, $v = xz - y$

(3) $u = x^2 - y^2 + z$, $v = yz + x$

(4) $u = x^2 + y^2 - Z$, $v = yz - x$

17. The particular integral of the differential equation $(D^2 + 3DD' + 2D'^2)z = 2x + 3y$ is given by :

(1) $\frac{1}{150}(2x + 3y)^3$

(2) $\frac{1}{240}(2x + 3y)^3$

(3) $\frac{1}{320}(2x + 3y)^3$

(4) None of these

18. If $u(x, y)$ is the solution of the Cauchy problem $x\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 1$, $u(x,0) = -x^2, x > 0$.

Then the value of u (2, 1) is equal to :

(1) $1 - 2e^{-2}$ (2) $1 + 4e^{-2}$ (3) $1 - 4e^{-2}$ (4) $1 + 2e^{-2}$

19. The partial differential equation $(x^2+y^2-1)\frac{\partial^2 u}{\partial x^2}+2\frac{\partial^2 u}{\partial x \partial y}+(x^2+y^2-1)\frac{\partial^2 u}{\partial y^2}=0$ is :

(1) parabolic in the region $x^2+y^2>2$

(2) hyperbolic in the region $x^2+y^2>2$

(3) elliptic in the region $0<x^2+y^2<2$

(4) hyperbolic in the region $0<x^2+y^2<2$

20. The function $u(x, t)$ satisfies the initial value problem

$$\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}, x \in R, t>0$$

$$u(x,0)=0, \frac{\partial u}{\partial t}(x,0)=4xe^{-x^2}$$

Then the value of $u(5, 5)$ is :

(1) $1-\frac{1}{e^{100}}$ (2) $1-e^{100}$ (3) $1-\frac{1}{e^{10}}$ (4) $1-e^{10}$

21. The coefficient of $\frac{1}{z}$ in the Laurent series expansion of the function $f(z)=\frac{1}{z^2(1-z)}$ about $z=0$, is :

(1) 1 (2) 0 (3) -1 (4) None of these

22. The residue of the function $f(z)=\frac{z+1}{z^2(z-3)}$ at $z=0$, is :

(1) $-\frac{9}{2}$ (2) $\frac{1}{2}$ (3) $-\frac{4}{9}$ (4) None of these

23. The coefficient of magnification at $z=2+3i$ for the conformal transformation $w=z^2$ is :

(1) $\sqrt{3}$ (2) $\sqrt{5}$ (3) $2\sqrt{7}$ (4) $2\sqrt{13}$

24. Let $T(z) = \frac{az+b}{cz+d}$, $ad - bc \neq 0$, be the Mobius transformation which maps the points $z_1 = 0$, $z_2 = -i$, $z_3 = \infty$ in the z- plane onto the points $w_1 = 10$, $w_2 = 5 - 5i$, $w_3 = 5 + 5i$ in the w- plane respectively. Then the image of the set $S = \{z \in C : \text{Re}(z) < 0\}$ under the map $w = T(z)$ is :

(1) $\{w \in C : |w| < 5\}$ (2) $\{w \in C : |w| > 5\}$

(3) $\{w \in C : |w - 5| < 5\}$ (4) $\{w \in C : |w - 5| > 5\}$

25. Among 100 students, 32 study Mathematics, 20 study Physics, 45 study Biology, 15 study Mathematics and Biology, 7 study Mathematics and Physics, 10 study Physics and Biology, 30 do not study any of three subjects. Then the number of students study only Mathematics, is :

(1) 8 (2) 15 (3) 25 (4) None of these

26. The no. of positive divisors of 2100 is :

(1) 50 (2) 44 (3) 40 (4) 36

27. The last two digits of 38^{2011} are :

(1) 6 (2) 2 (3) 4 (4) 8

28. Which one of the following statements is ***false*** ?

(1) If Q denotes the additive group of rational numbers and $f : Q \to Q$ is a non-trivial homomorphism, then f is an isomorphism.

(2) Any quotient group of a cyclic group is cyclic.

(3) If every subgroup of a group G is a normal subgroup, then G is abelian.

(4) Every group of order 33 is cyclic

29. Let Z_n denotes the group of integers modulo n, under the operation of addition modulo n, for any positive integer n. Then the number of elements of order 15 in the additive group $Z_{60} \times Z_{50}$ is :

(1) 48 (2) 30 (3) 25 (4) 10

30. The number of 5- Sylow subgroups in the symmetric group S_5 of degree 5, is :

(1) 2 (2) 3 (3) 5 (4) 6

31. The directional derivative of the function $f(x, y, z) = xy^2 + yz^3$ at the point (2, −1, 1) in the direction of $\hat{i} + 2\hat{j} + 2\hat{k}$ is :

(1) $-\frac{13}{3}$ (2) $-\frac{11}{3}$ (3) $-\frac{10}{3}$ (4) None of these

32. Let R be the set of all real numbers and $R^2 = \{(x, y) : x, y \in R\}$. Let $f : R^2 \to R$ be defined by $f(x,y) = \begin{cases} (x^2+y^2)\sin\left(\dfrac{1}{x^2+y^2}\right) & , \text{ if } (x,y) \neq (0,0) \\ 0 & , \text{ if } (x,y) = (0,0) \end{cases}$. Then which one of the following statements is ***true*** ?

(1) f is continuous but not differentiable at (0, 0)

(2) f is not continuous at (0, 0)

(3) f is differentiable at (0, 0)

(4) None of these

33. Consider the following statements :

$P : d_1(x, y) = \min\{2, |x - y|\}$ is a metric for R (the set of all real numbers).

$Q : d_2(x, y) = \begin{cases} |x|+|y| & , \text{ if } x \neq y \\ 0 & , \text{ if } x = y \end{cases}$ is a metric on (0, 1)

Then :

(1) both P and Q are true

(2) P is true and Q is false

(3) P is false and Q is true

(4) both P and Q are false

34. Suppose that :

$U = R^2 \setminus \{(x, y) \in R^2 : x, y \in Q\}$,

$V = R^2 \setminus \left\{(x, y) \in R^2 : x > 0, y = \dfrac{1}{x}\right\}$

Then with respect to the Euclidean metric on R^2,

(1) both U and V are disconnected

(2) U is disconnected but V is connected

(3) U is connected but V is disconnected

(4) both U and V are connected

35. Let X and Y be normed linear spaces and let $T : X \to Y$ be any bijective linear ma closed graph. Then which one of the following statements is ***true*** ?

(1) The graph of T is equal to $X \times Y$

(2) T^{-1} is continuous

(3) The graph of T^{-1} is closed

(4) T is continuous

36. Let V be a vector space over R^3. Which one of the following is a subspace of V ?

(1) $\{(x, y, z) : x - 3y + 4z = 0, x, y, z \in R\}$

(2) $\{(x, y, z) : x + y \geq 0, x, y, z \in R\}$

(3) $\{(x, y, z) : x \leq 0, x, y, z \in R\}$

(4) None of these

37. The dimension of the vector space of 7×7 real symmetric matrices with trace zero the sum of the off-diagonal elements zero, is :

(1) 47 (2) 28 (3) 27 (4) 26

38. Which of the following is ***not*** a linear transformation ?

(1) $T(x, y, z) = (x, y)$ for all $(x, y, z) \in R^3$

(2) $T(x, y, z) = (x + 1, y + z)$ for all $(x, y, z) \in R^3$

(3) $T(x, y, z) = (x - z, y)$ for all $(x, y, z) \in R^3$

(4) $T(x, y, z) = (x + y + z, 0)$ for all $(x, y, z) \in R^3$

39. Suppose that $T : R^4 \to R[x]$ is a linear transformation over R satisfying :

$T(-1, 1, 1, 1) = x^2 + 2x^4$

$T(1, 2, 3, 4) = 1 - x^2$

$T(2, -1, -1, 0) = x^3 - x^4$

Then the coefficient of x^4 in $T(-3, 5, 6, 6)$ is :

(1) 1 (2) 2 (3) 3 (4) 5

40. The rank of the matrix $\begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & 1 \\ 4 & 2 & 3 \end{bmatrix}$ is :

(1) 1 (2) 2 (3) 3 (4) None of these

41. Let {0, 1, 2, 3} be an observed sample of size 4 from $N(\theta, 5)$ distribution, where $\theta \in [2, \infty)$. Then the maximum likelihood estimate of θ based on the observed sample is :

(1) 16 (2) 8 (3) 4 (4) 2

42. A random sample of size 100 is classified into 10 class intervals covering all the data points. To test whether data comes from a normal population with unknown mean and unknown variance, the chi-square goodness of fit test is used. The degrees of freedom of the test statistic is equal to :

(1) 10 (2) 9 (3) 8 (4) 7

43. A random sample of 500 apples was taken from a large consignment and 60 were found to be bad. Then the 98% confidence limits for the percentage of bad apples in the consignment are :

(1) (8.82, 17.52) (2) (8.61, 15.38)

(3) (8.32, 14.63) (4) (8.19, 13.52)

44. Consider a two - way fixed effects analysis of variance model without interaction effect and one observation per cell. If there are 5 factors and 4 columns, then the degrees of freedom for the error sum of squares is :

(1) 20 (2) 19 (3) 12 (4) 11

45. The simple correlation coefficients between temperature (X_1), corn yield (X_2) and rainfall (X_3) are : $r_{12} = 0.75$, $r_{23} = 0.54$, $r_{31} = 0.43$. Then the partial correlation coefficient $r_{12.3}$ is :

(1) 0.6815 (2) 0.4223

(3) 0.2392 (4) None of these

46. Let (X_1, X_2) be a random vector following bivariate normal distribution with mean vector (0, 0), variance (X_1) = variance $(X_2) = 1$ and correlation coefficient ρ, where $|\rho| < 1$. Then $P(X_1 + X_2 > 0)$ is equal to :

(1) 1 (2) 0.8 (3) 0.5 (4) 0.2

47. The total number of standard 4×4 Latin squares is :

(1) 4 (2) 8 (3) 12 (4) 16

48. The minimum value of $Z = 20x + 10y$

subject to the constraints :

$x + 2y \leq 40$; $3x + y \geq 30$; $4x + 3y \geq 60$; $x, y \geq 0$, is :

(1) 240 (2) 215 (3) 272 (4) None of these

49. Consider a parallel system with two components. The lifetimes of the two components are independent and identically distributed random variables each following an exponential distribution with mean 1. The expected lifetime of the system is :

(1) 1 (2) 1/2 (3) 3/2 (4) 2

50. Consider an M/M/1 queue with interarrival time having exponential distribution with mean $\frac{1}{\lambda}$ and service time having exponential distribution with mean $\frac{1}{\mu}$. Which of the following is ***true*** ?

(1) If $0 < \lambda < \mu$, then the queue length has limiting distribution Poisson $(\mu - \lambda)$.

(2) If $0 < \mu < \lambda$, then the queue length has limiting distribution Poisson $(\lambda - \mu)$.

(3) If $0 < \lambda < \mu$, then the queue length has limiting distribution which is geometric.

(4) If $0 < \mu < \lambda$, then the queue length has limiting distribution which is geometric.

51. Using Newton-Raphson method, the real root of the equation $x^3 - 2x + 5 = 0$ is :

(1) 3.129 (2) 1.582 (3) −2.754 (4) −2.095

52. Using Gauss- elimination method, the solution of the following system of equations $x + y + 2z = 7$, $4x + 3y + 2z = 8$, $3x + 2y + 4z = 13$, is :

(1) $x = -1, y = 2, z = 3$ (2) $x = 1, y = 3, z = 5$

(3) $x = 1, y = 1, z = 2$ (4) None of these

53. Given that

x	10	20	30	40	50
$f(x)$	46	66	81	93	101

The value of $\nabla^2 f(50)$ is :

(1) 8 (2) 3 (3) −4 (4) −1

54. Given that

x	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
y	1	$\frac{4}{5}$	$\frac{2}{3}$	$\frac{4}{7}$	$\frac{1}{2}$

Using trapezoidal rule, the value of $\int_0^1 y\,dx$ is :

(1) 0.938 (2) 0.697 (3) 0.352 (4) 0.241

55. Given $\frac{dy}{dx} = -xy^2$ with $y(0) = 2$. Then using modified Euler's method, the value of $y(0.1)$ by taking step size $h = 0.1$, is :

(1) 1.9804 (2) 1.5636 (3) 1.2921 (4) None of these

56. Let q_j and $\dot{q}_j$ respectively are the generalized coordinates and velocity of a dynamical system and p_j are its generalized momenta. Then the relation between Hamiltonian $H(q_j, p_j, t)$ and Lagrangian $L(q_j, \dot{q}_j, t)$ is given by :

(1) $H = \sum \dot{p}_j q_j - L$

(2) $H = \sum p_j \dot{q}_j - L$

(3) $H = \sum \dot{p}_j \dot{q}_j - L$

(4) None of these

57. Let T be the kinetic energy and V be the potential energy of the dynamical system. the the integral $\int_{t_1}^{t_2}(T-V)dt$ has a stationary value, where t_1 and t_2 are fixed. This principle is known as :

(1) Hamilton's principle

(2) Principle of least action

(3) D' Alembert principle

(4) None of these

58. Consider the motion of a planet $P(r, \theta)$ of mass m moving around the Sun $S(0, 0)$ under the inverse square law of attraction $\mu m/r^2$. Let kinetic energy T of the system is given by :

$$T=\frac{1}{2}m(\dot{r}^2+r^2\dot{\theta}^2), \text{ where } \dot{r}=\frac{dr}{dt} \text{ and } \dot{\theta}=\frac{d\theta}{dt} \text{ with } t \text{ as time.}$$

Then Lagrange's equations are given by :

(1) $\ddot{r}+r\dot{\theta}^2=-\mu/r^2,\ r\dot{\theta}=\text{constant}$

(2) $\ddot{r}-r\dot{\theta}=-\mu/r^2,\ r^2\ddot{\theta}=\text{constant}$

(3) $\ddot{r}-r\dot{\theta}^2=-\mu/r^2,\ r^2\dot{\theta}=\text{constant}$

(4) None of these

59. A rigid body under no forces is free to rotate about its centroid G, the principal moments of inertia at which are 7, 25, 32 units respectively. If $\omega=[\omega_1, \omega_2, \omega_3]$ be the angular velocity, then Euler's dynamical equations of motion are :

(1) $\dot{\omega}_1-\omega_2\omega_3=0,\ \dot{\omega}_2+\omega_3\omega_1=0,\ 4\dot{\omega}_3+3\omega_1\omega_2=0$

(2) $\dot{\omega}_1+\omega_2\omega_3=0,\ \dot{\omega}_2-\omega_3\omega_1=0,\ 16\dot{\omega}_3+9\omega_1\omega_2=0$

(3) $\dot{\omega}_1-2\omega_2\omega_3=0,\ \dot{\omega}_2-3\omega_3\omega_1=0,\ \dot{\omega}_3-3\omega_1\omega_2=0$

(4) None of these

60. The external of the functional $I[y(x)] = \int_{-1}^{0} (y'^2 - 2xy)\, dx$ subject to $y(-1) = 0, y(0) = 2$ is :

(1) $y = -\frac{x^3}{6} + \frac{13}{6}x + 2$ (2) $y = x^2 - 1$

(3) $y = \frac{1}{4}x^2$ (4) None of these

61. If X is a continuous random variable with mean μ and variance σ^2, then for any positive number k, the inequality $P\{|X - \mu| \geq k\sigma\} \leq (1/k^2)$ is known as :

(1) Lyapunov's inequality (2) Chebychev's inequality

(3) Bienayme-Chebychev's inequality (4) Khintchine's inequality

62. With the usual notations, the value of probability p for a binomial variate X, if $n = 6$ and $9P(X = 4) = P(X = 2)$, is :

(1) $\frac{1}{2}$ (2) $\frac{1}{3}$ (3) $\frac{1}{4}$ (4) None of these

63. The characteristics function of Poisson distribution with meas as m, is :

(1) $e^{m(it-1)}$ (2) $e^{m(e^{it}-1)}$ (3) e^{mit} (4) None of these

64. Which of the following do the normal distribution and the exponential density function have in common ?

(1) Both are bell- shaped.
(2) Both are symmetrical distributions.
(3) Both approach infinity as x approaches infinity.
(4) Both approach zero as x approaches infinity.

65. Let X be a random variable with uniform distribution on the interval $(-1, 1)$ and $Y = (X + 1)^2$. Then the probability density function $f(y)$ of Y, over the interval $(0, 4)$, is :

(1) $\frac{3\sqrt{y}}{16}$ (2) $\frac{1}{4\sqrt{y}}$ (3) $\frac{1}{6\sqrt{y}}$ (4) $\frac{1}{\sqrt{y}}$

66. Let $\{X_n\}_{n\geq 0}$ be a homogeneous Markov chain whose state space is {0, 1, 2} whose one-step transition probability matrix is $P = \begin{bmatrix} 0 & 1 & 0 \\ 0.3 & 0 & 0.7 \\ 0 & 1 & 0 \end{bmatrix}$. T

$\lim_{n\to\infty} P(X_{2n} = 2 \mid X_0 = 2)$ is equal to :

(1) 0.7 (2) 0.5 (3) 0.3 (4) 0.1

67. Suppose customers arrive at an ATM facility according to a Poisson process with rat customers per hour. The probability that no customer arrives at the ATM facility fr 1:00 pm to 1:18 pm is :

(1) 0.94 (2) 0.75 (3) 0.45 (4) 0.22

68. In a pure birth process with birth rates $\lambda_n = 2^n$, $n \geq 0$, let the random variable T den the time taken for the population size to grow from 0 to 5. If Var(T) denotes variance of the random variable T, then Var(T) is equal to :

(1) $\frac{441}{256}$ (2) $\frac{341}{256}$ (3) $\frac{241}{256}$ (4) $\frac{141}{256}$

69. Let $X_1, X_2, \ldots\ldots, X_{10}$ be independent and identically distributed normal rando variables with mean 0 and variance 2. Then $E\left(\frac{X_1^2}{\sum_{i=1}^{10} X_i^2}\right)$ is equal to :

(1) 0.8 (2) 0.6 (3) 0.3 (4) 0.1

70. Let $X_1, X_2, \ldots\ldots, X_n$ be a random sample drawn from a population with probabili density function $f_X(x;\theta) = \frac{2x}{\theta^2}, 0 < x < \theta$. Then the method of moments estimator θ is :

(1) $\frac{3\sum_{n=1}^{n} X_i}{2n}$ (2) $\frac{3\sum_{n=1}^{n} X_i^2}{2n}$

(3) $\frac{\sum_{i=1}^{n} X_i}{n}$ (4) $\frac{3\sum_{i=1}^{n} X_i(X_i - 1)}{2n}$

71. Let G be a group of order 5^4 with center having 5^2 elements. Then the number of conjugacy classes in G is :

(1) 26 (2) 48 (3) 145 (4) None of these

72. Let I and J be the ideals generated by $\{5, \sqrt{10}\}$ and $\{4, \sqrt{10}\}$ in the ring $Z[\sqrt{10}] = \{a + b\sqrt{10} : a, b \in Z\}$, respectively. Then :

(1) both I and J are maximal ideals

(2) I is a maximal ideal but J is not a prime ideal

(3) I is not a maximal ideal but J is a prime ideal

(4) neither I nor J is a maximal ideal

73. Let Z denotes the set of all integers and Z_n denotes the set of all integers modulo n, for any positive integer n. Consider the following statements :

I. The ring $Z[\sqrt{-1}]$ is a unique factorization domain.

II. The ring $Z[\sqrt{-5}]$ is a principal ideal domain.

III. In the polynomial ring $Z_2[x]$, the ideal generated by $x^3 + x + 1$ is a maximal ideal.

IV. In the polynomial ring $Z_3[x]$, the ideal generated by $x^6 + 1$ is a prime ideal.

Which of the above statements are ***true*** ?

(1) I, II and III only (2) I and III only

(3) I, II and IV only (4) II and III only

74. Which of the following polynomial is reducible over the field Q of rational numbers ?

(1) $x^2 - 4x + 2$ (2) $x^3 - x + 1$

(3) $x^3 + 9x^2 - 3x + 6$ (4) None of these

75. Let F be the field with 4096 elements. The number of proper subfields of F is :

(1) 5 (2) 20 (3) 50 (4) 100

76. Let ω be a primitive complex cube root of unity. Then the degree of the field extension $Q(i, \sqrt{3}, \omega)$ over Q (the field of rational numbers) is :

(1) 4 (2) 3 (3) 2 (4) 1

77. For a subset S of a topological space, let Int(S) and $\bar{S}$ denote the interior and closure S, respectively. Then which of the following statements is ***true*** ?

(1) If S is open, then $S = \text{Int}(\bar{S})$

(2) If the boundary of S is empty, then S is open.

(3) If $\bar{S} \setminus S$ is a proper subset of the boundary of S, then S is open.

(4) None of these

78. Let T_1 be the co-countable topology on R (the set of real numbers) and T_2 be the c finite topology on R. Consider the following statements :

I. In (R, T_1), the sequence $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$ converges to 0.

II. In (R, T_2), the sequence $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$ converges to 0.

III. In (R, T_1), there is no sequence of rational numbers which converges to $\sqrt{3}$.

IV. In (R, T_2), there is no sequence of rational numbers which converges to $\sqrt{3}$.

Which of the above statements are ***true*** ?

(1) I and II only

(2) II and III only

(3) III and IV only

(4) I and IV only

79. Let R denote the set of all real numbers. Consider the following topological spaces :

$X_1 = (R, T_1)$, where T_1 is the upper limit topology having all sets $(a, b]$ as basis.

$X_2 = (R, T_2)$, where $T_2 = \{U \subset R : R \setminus U \text{ is finite}\} \cup \{\phi\}$

Then :

(1) both X_1 and X_2 are connected

(2) X_1 is connected and X_2 is not connected

(3) X_1 is not connected and X_2 is connected

(4) neither X_1 nor X_2 is connected

80. Consider the following statements :

P : Any continuous image of a compact space is compact.

Q : A topological space is compact if every basic open cover has a finite subcover.

Then :

(1) both P and Q are true (2) P is true and Q is false

(3) P is false and Q is true (4) both P and Q are false

81. The system of equations $x + 2y - z = 3; 3x - y + 2z = 1; 2x - 2y + 3z = 2$ has :

(1) no solution (2) a unique solution

(3) infinite solutions (4) None of these

82. The eigen values of the matrix $\begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$ are :

(1) $-1, 1, 2$ (2) $-2, 3, 4$ (3) $1, -1, 7$ (4) $2, -3, 5$

83. The matrix $\begin{bmatrix} 1 & x & z \\ 0 & 2 & y \\ 0 & 0 & 1 \end{bmatrix}$ is diagonalizable when (x, y, z) equals :

(1) $(0, 0, 1)$ (2) $(1, 1, 0)$ (3) $(\sqrt{2}, \sqrt{2}, 2)$ (4) $(\sqrt{2}, \sqrt{2}, \sqrt{2})$

84. Let $u = (1 + i, i, -1)$ and $v = (1 + 2i, 1 - i, 2i)$. Then $\langle u, v \rangle$ is :

(1) $2 - 2i$ (2) $-2 + 2i$ (3) $-2 - 2i$ (4) $2 + 2i$

85. The quadratic form corresponding to symmetric matrix $\begin{bmatrix} 0 & \frac{1}{2} & 1 \\ \frac{1}{2} & 0 & \frac{3}{2} \\ 1 & \frac{3}{2} & 0 \end{bmatrix}$ is :

(1) $xy + 3yz + 2zx$ (2) $xy - 3yz - 2zx$

(3) $xy + 3yz - 2zx$ (4) None of these

86. If the power series $\sum_{n=0}^{\infty} a_n(z+3-i)^n$ converges at $5i$ and diverges at $-3i$, then the power series :

(1) converges at $-2+5i$ and diverges at $2-3i$

(2) converges at $2-3i$ and diverges at $-2+5i$

(3) converges at both $2-3i$ and $-2+5i$

(4) diverges at both $2-3i$ and $-2+5i$

87. Which of the following function $f(z)$, of the complex variable z, is not analytic at all the points of the complex plane ?

(1) $f(z)=z^2$ (2) $f(z)=e^z$ (3) $f(z)=\sin z$ (4) $f(z)=\log z$

88. The function $f(z)$ of complex variable $z = x + iy$, where $i=\sqrt{-1}$, is given as $f(z)=(x^3-3xy^2)+iv(x,y)$. For this function to be analytic, $v(x, y)$ should be :

(1) $(3xy^2-y^3)$ + constant (2) $(3x^2y^2-y^3)$ + constant

(3) $(3x^2y-y^3)$ + constant (4) None of these

89. Let Γ denotes the boundary of the square region R with vertices (0, 0), (2, 0), (2, 2) and (0, 2) oriented in the counter- clockwise direction. Then value of $\oint_{\Gamma}(1-y^2)dx + x\,dy$ is :

(1) 12 (2) 15 (3) 20 (4) 25

90. Let C represent the unit circle centered at origin in the complex plane and, complex variable $z = x + iy$, where $i=\sqrt{-1}$. The value of the contour integral $\oint_C \frac{\cos h3z}{2z}dz$ (where integration is taken counter clockwise) is :

(1) 0 (2) 2 (3) πi (4) $2\pi i$

91. Let Q is the set of all rational numbers, Z is the set of all integers and N is the set of all natural numbers. Then which one of the following statements is ***true*** ?

(1) The set $Q \times Z$ is uncountable.

(2) The set $\{f : f$ is a function from N to $\{0, 1\}\}$ is uncountable.

(3) The set $\{\sqrt{p} : p$ is a prime number$\}$ is uncountable.

(4) None of these

92. The sequence $< \dfrac{\sin\left(\frac{n\pi}{3}\right)}{\sqrt{n}} >$ converges to the limit :

(1) 0 (2) 1 (3) 2 (4) None of these

93. Let $a_n = \dfrac{(-1)^{n+1}}{n!}, n \geq 0$ and $b_n = \sum_{k=0}^{n} a_k, n \geq 0$. Then for $|x| < 1$, the series $\sum_{n=0}^{\infty} b_n x^n$ converges to :

(1) $\dfrac{-e^{-x}}{1+x}$ (2) $\dfrac{-e^{-x}}{1-x^2}$ (3) $\dfrac{-e^{-x}}{1-x}$ (4) $-(1+x)e^{-x}$

94. The number of limit points of the set $\left\{\dfrac{1}{m}+\dfrac{1}{n} : m, n \in N\right\}$ is :

(1) 1 (2) 2

(3) finitely many (4) infinitely many

95. Which of the following functions is uniformly continuous on the specified domain ?

(1) $f_1(x) = e^{x^2}, -\infty < x < \infty$

(2) $f_2(x) = \begin{cases} \dfrac{1}{x} & , \quad 0 < x \leq 1 \\ 0 & , \quad x = 0 \end{cases}$

(3) $f_3(x) = \begin{cases} x^2 & , \quad |x| \leq 1 \\ \dfrac{2}{1+x^2} & , \quad |x| > 1 \end{cases}$

(4) $f_4(x) = \begin{cases} x & , \quad |x| \leq 1 \\ x^2 & , \quad |x| > 1 \end{cases}$

96. For $n \in N$, let $f_n, g_n : (0, 1) \to R$ be functions defined by $f_n(x) = x^n, g_n(x) = x^n(1-x)$. Then :

(1) $\{f_n\}$ converges uniformly but $\{g_n\}$ does not converge uniformly.

(2) $\{g_n\}$ converges uniformly but $\{f_n\}$ does not converge uniformly.

(3) both $\{f_n\}$ and $\{g_n\}$ converges uniformly.

(4) neither $\{f_n\}$ nor $\{g_n\}$ converges uniformly.

97. For which of the following function, Lagrange's mean value theorem is ***not*** applicable ?

(1) $f(x) = x + \frac{1}{x}$ in $[1, 3]$

(2) $f(x) = \sqrt{25 - x^2}$ in $[-3, 4]$

(3) $f(x) = \frac{1}{4x-1}$ in $[1, 4]$

(4) $f(x) = x^{1/5}$ in $[-1, 1]$

98. $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{n}{n^2 + k^2}$ is equal to :

(1) $\frac{e}{3}$ (2) $\frac{5}{6}$ (3) $\frac{3}{4}$ (4) $\frac{\pi}{4}$

99. Consider the following improper integrals $I_1 = \int_0^{\pi/2} \frac{\cos x}{\sqrt{1-\sin x}}\,dx$, $I_2 = \int_0^1 \frac{\log x}{\sqrt{1-x^2}}\,dx$,

$I_3 = \int_2^{\infty} \frac{1}{x-1}\,dx$, then :

(1) All are convergent

(2) All are divergent

(3) I_1 and I_2 are convergent whereas I_3 is divergent

(4) I_1 and I_3 are convergent whereas I_2 is divergent

100. Consider the following statements :

P : There exists an unbounded subset of R whose Lebesgue measure is equal to 5.

Q : If $f : R \to R$ is continuous and $g : R \to R$ is such that $f = g$ almost everywhere on R, then g must be continuous almost everywhere on R.

Which of the above statements hold true ?

(1) only P (2) only Q

(3) both P and Q (4) neither P nor Q

Answerkey of Entrance test of PHD/URS Mathematics 2021-22				
Sr. No.	A	B	C	D
1	2	2	3	2
2	1	3	2	1
3	3	1	2	4
4	4	3	4	3
5	3	3	1	2
6	2	1	1	4
7	4	4	2	3
8	4	2	2	2
9	3	4	3	3
10	1	2	1	4
11	2	4	2	1
12	3	4	1	4
13	1	2	3	2
14	3	3	4	1
15	3	1	1	4
16	1	3	1	1
17	4	1	4	2
18	2	1	3	3
19	4	3	1	4
20	2	3	3	1
21	2	2	2	1
22	1	1	1	3
23	3	4	3	4
24	4	3	4	3
25	1	2	3	2
26	1	4	2	4
27	4	3	4	2
28	3	2	4	3
29	1	3	3	1
30	3	4	1	4
31	1	1	4	2
32	3	4	4	3
33	4	2	2	1
34	3	1	3	3
35	2	4	1	3
36	4	1	3	1
37	2	2	1	4
38	3	3	1	2
39	1	4	3	4
40	4	1	3	2
41	3	1	4	4
42	2	3	1	4
43	2	4	3	2
44	4	3	2	3
45	1	2	1	1
46	1	4	2	3
47	2	2	1	1
48	2	3	3	1
49	3	1	2	3
50	1	4	1	3
51	1	2	1	4

Sumeet
11/2/22

11/02/2022

Ekta
11/02/2022

52	4	1	3	1
53	2	3	4	3
54	1	4	3	2
55	4	1	2	1
56	1	1	4	2
57	2	4	2	1
58	3	3	3	3
59	4	1	1	2
60	1	3	4	1
61	4	3	2	2
62	1	2	1	3
63	3	2	4	2
64	2	4	3	4
65	1	1	2	2
66	2	1	4	1
67	1	2	3	4
68	3	2	2	2
69	2	3	3	4
70	1	1	4	1
71	2	4	2	3
72	1	1	3	2
73	4	3	2	2
74	3	2	4	4
75	2	1	2	1
76	4	2	1	1
77	3	1	4	2
78	2	3	2	2
79	3	2	4	3
80	4	1	1	1
81	2	2	2	2
82	3	1	3	1
83	2	3	1	3
84	4	4	3	4
85	2	3	3	1
86	1	2	1	1
87	4	4	4	4
88	2	4	2	3
89	4	3	4	1
90	1	1	2	3
91	4	2	1	2
92	4	3	4	1
93	2	2	2	3
94	3	4	1	4
95	1	2	4	3
96	3	1	1	2
97	1	4	2	4
98	1	2	3	4
99	3	4	4	3
100	3	1	1	1

11/02/2022 11/02/2022

Sumeet 11/2/22