(DO NOT OPEN THIS QUESTION BOOKLET BEFORE TIME OR UNTIL YOU ARE ASKED TO DO SO)

(MPH/PHD/URS–EE–2019)

Subject : MATHEMATICS

Sr. No. 10273

Code **A**

SET–"X"

Time : 1¼ Hours **Total Questions : 100** **Max. Marks : 100**

Roll No. ______________ (in figure)______________________ (in words)

Name : ______________________ Father's Name : ______________

Mother's Name : __________________ Date of Examination : __________

(Signature of the candidate) (Signature of the Invigilator)

CANDIDATES MUST READ THE FOLLOWING INFORMATION/ INSTRUCTIONS BEFORE STARTING THE QUESTION PAPER.

1. **Candidates are required to attempt any 75 questions out of the given 100 multiple choice questions of 4/3 marks each. No credit will be given for more than 75 correct responses.**
2. The candidates must return the Question book-let as well as OMR answer-sheet to the Invigilator concerned before leaving the Examination Hall, failing which a case of use of unfair-means / mis-behaviour will be registered against him / her, in addition to lodging of an FIR with the police. Further the answer-sheet of such a candidate will not be evaluated.
3. Keeping in view the transparency of the examination system, carbonless OMR Sheet is provided to the candidate so that a copy of OMR Sheet may be kept by the candidate.
4. Question Booklet along-with answer key of all the A,B,C and D code shall be got uploaded on the university website after the conduct of Entrance Examination. In case there is any discrepancy in the Question Booklet/Answer Key, the same may be brought to the notice of the Controller of Examination in writing/through E. Mail within 24 hours of uploading the same on the University Website. Thereafter, no complaint in any case, will be considered.
5. The candidate MUST NOT do any rough work or writing in the OMR Answer-Sheet. Rough work, if any, may be done in the question book-let itself. Answers MUST NOT be ticked in the Question book-let.
6. **There will be no negative marking. Each correct answer will be awarded 4/3 mark. Cutting, erasing, overwriting and more than one answer in OMR Answer-Sheet will be treated as incorrect answer.**
7. Use only Black or Blue <u>**BALL POINT PEN**</u> of good quality in the OMR Answer-Sheet.
8. BEFORE ANSWERING THE QUESTIONS, THE CANDIDATES SHOULD ENSURE THAT THEY HAVE BEEN SUPPLIED CORRECT AND COMPLETE BOOK-LET. COMPLAINTS, IF ANY, REGARDING MISPRINTING ETC. WILL NOT BE ENTERTAINED 30 MINUTES AFTER STARTING OF THE EXAMINATION.

Question No.	Questions
1.	Completeness of a metric space is preserved under (1) Isometry (2) Homeomorphism (3) Continuous function (4) Bijective function
2.	Given an interval (– 1, 1) and a sequence $\{a_n\}$ of elements in it. Then (1) Every limit point of $\{a_n\}$ is an (–1, 1) (2) The limit points of $\{a_n\}$ can only be in {–1, 0, 1} (3) Every limit point of $\{a_n\}$ is in [–1, 1] (4) The limit point of $\{a_n\}$ cannot be in {–1, 0, 1}
3.	If f is a function $f:\mathbb{R}\to\mathbb{R}$ s.t. $f(0)=0$ and $\lvert f'(x)\rvert \le \forall x$, then f (1) is in (1) (5, 6) (2) [– 4, 4] (3) (– ∞, – 5) ∪ (5, ∞,) (4) [– 5, 5]
4.	Let $A=\{n\in\mathbb{N} : n=1 \text{ or the only prime factors of n are 2 or 3}\}$. Let $S=\sum_{n\in A}\frac{1}{n}$, then (1) S is divergent series (2) A is finite (3) S = 3 (4) S = 6
5.	Let $x_n(t)=te^{-nt^2}, t\in\mathbb{R}, n\ge 1$. Then the sequence $\{x_n\}$ is (1) uniformly convergent on $\mathbb{R}$ (2) a sequence of unbounded functions (3) bounded and not uniformly convergent on $\mathbb{R}$ (4) uniformly convergent only on compact subsets of $\mathbb{R}$

Question No.	Questions
6.	Which of the following is necessarily true for a function $f:A \to B$ (1) if f as injective, then $\exists\, g:B \to A$ s.t. $f(g(y)) = y\ \forall y \in B$ (2) if f is injective and B is countable, then A is finite (3) if f is subjective and A is countable, then B is countably infini (4) if f is subjective, then $\exists\, g:B \to A$ s.t. $f(g(y)) = y\ \forall y \in B$
7.	The difference $\log(2) - \sum_{n=1}^{100} \frac{1}{2^n \cdot n}$ is (1) less than 0 (2) less than $\frac{1}{2^{100}.101}$ (3) greater than 1 (4) greater than $\frac{1}{2^{100}.101}$
8.	Let f be function defined on the set $S=\{x \in \mathbb{R}, x \geq 0, x \neq n\pi + \frac{\pi}{2}, n \in$ and $f(x) = \tan x$. Then (1) f has a unique fixed point on S (2) there is no fixed point of f on S (3) f has infinitely many fixed points on S (4) f has finite number of fixed points on S
9.	A function $f:\mathbb{R} \to \mathbb{R}$ need not be lebesgue measurable if (1) $\{x \in \mathbb{R} : f(x) \geq \alpha\}$ is measurable for each $\alpha \in \mathbb{Q}$ (2) $\{x \in \mathbb{R} : f(x) = \alpha\}$ is measurable for each $\alpha \in \mathbb{R}$ (3) for each open set G in $\mathbb{R}$, $f^{-1}(G)$ is measurable (4) f is monotone

Question No.	Questions
10.	Let $f(x)=\left(1+\frac{1}{x}\right)^{x}$ and $g(x)=\left(1+\frac{1}{x}\right)^{x+1}$, $x>0$. then (1) f (x) and g (x) both are increasing functions (2) f (x) is increasing and g (x) is decreasing (3) f (x) is decreasing and g (x) is increasing (4) f (x) and g (x) both are decreasing functions
11.	Let $f(x,y)=\frac{x^2}{y^2}$, $(x,y)\in\left[\frac{1}{2},\frac{3}{2}\right]\times\left[\frac{1}{2},\frac{3}{2}\right]$. Then the derivative of f at (1, 1) along the direction (1, 1) is (1) 1 (2) 2 (3) 0 (4) −2
12.	Which of the following statement is not correct (1) if F is closed and K is compact, then $F\cap K$ is compact (2) if $\{K_n\}$ is a sequence of nonempty compact sets s.t. $K_{n+1}\subset K_n$ $(n=1,2,\ldots\ldots)$, then $\bigcap_{n=1}^{\infty}K_n$ is empty (3) Every closed subset of a compact set is compact (4) The set $\{x : x\in\mathbb{R} \text{ and } x(x^2-6x+8)=0\}$ is compact
13.	Let $X=\{x : x=(x_1,x_2,x_3),\ x_i\in\mathbb{R},\ x_1^2+x_2^2+x_3^2<16\}$. Then (1) X has no limit point in $\mathbb{R}^3$ (2) X has a limit point in $\mathbb{R}^3$ (3) X is compact (4) All of the above statement are true

Question No.	Questions
14.	Let A, B be n × n matrices. Then trace of $A^2 B^2$ is equal to (1) trace $\left((AB)^2\right)$ (2) $(\text{trace}\,(AB))^2$ (3) trace $\left(AB^2A\right)$ (4) trace (BABA)
15.	Let f, g, h be the functions from $\mathbb{R}^3$ to $\mathbb{R}^2$ such that $f\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}4\\x+y\end{pmatrix}, g\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}xy\\x+y\end{pmatrix}, h\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}z-x\\x+y\end{pmatrix}; \begin{pmatrix}x\\y\\z\end{pmatrix}\in\mathbb{R}^3$ Then (1) only f is a linear transformation (2) only g is a linear transformation (3) only h is a linear transformation (4) f and g are linear transformations but h is not a linear transformation
16.	Let S denote the set of all the prime numbers p such that the matrix $\begin{bmatrix}91 & 31 & 0\\29 & 31 & 0\\79 & 23 & 59\end{bmatrix}$ has an inverse in the field Z/pZ. Then (1) S = {31} (2) {31, 59} (3) S = {7, 13, 59} (4) S is infinite

Question No.	Questions
17.	Let V be the vector space of all real polynomials of degree ≤ 10. Let $TP(x) = P'(x)$, for $P \in V$, be a linear transformation from V to V. Consider the basis $\{1, x, x^2, \ldots\ldots, x^{10}\}$ of V. Let A be the matrix of T with respect to this basis. Then (1) trace (A) = 1 (2) There is no $n \in \mathbb{N}$ s.t. $A^n = 0$ (3) A has non-zero eigen value (4) det (A) = 0
18.	Given a matrix $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, $\theta = \frac{2\pi}{31}$. Then A^{2015} is equal to (1) I (2) A (3) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ (4) $\begin{pmatrix} \cos 13\theta & \sin 13\theta \\ -\sin 13\theta & \cos 13\theta \end{pmatrix}$
19.	Given a matrix $\begin{pmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{pmatrix}$. Then this form is (1) negative definite (2) positive definite (3) non-negative definite but not positive definite (4) neither negative definite nor positive definite

Question No.	Questions
20.	Let W_1 and W_2 be subspaces of $\mathbb{R}^3$ given by $W_1 = \{(x, y, z) \in \mathbb{R}^3 / x + y + z = 0\}$ $W_2 = \{(x, y, z) \in \mathbb{R}^3 / x - y + z = 0\}$ If W is a subspace of $\mathbb{R}^3$ such that (i) $W \cap W_2 =$ space {0, 1, 1} (ii) $W \cap W_1 =$ orthogonal to $W \cap W_2$ with respect to the usual inner product of $\mathbb{R}^3$. Then (1) $W = \text{Span}\{(0,1,-1), (0,1,1)\}$ (2) $W = \text{Span}\{(1,0,-1), (0,1,-1)\}$ (3) $W = \text{Span}\{(1,0,-1), (0,1,1)\}$ (4) $W = \text{Span}\{(1,0,-1), (1,0,1)\}$
21.	Which of the following matrices is not diagonalizable over $\mathbb{R}$? (1) $\begin{pmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ (2) $\begin{pmatrix} 2 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ (3) $\begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix}$ (4) $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$
22.	The rank of the matrix $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 & 3 \\ 1 & 2 & 3 & 4 & 4 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}$ is (1) 2 (2) 3 (3) 4 (4) 5

Question No.	Questions
23.	Let $A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 2 & 3 & \alpha \end{pmatrix}$, $b = \begin{pmatrix} 1 \\ 3 \\ \beta \end{pmatrix}$ Then the system $AX = b$ over the field of real numbers has (1) no solution wherever $\beta \neq 7$ (2) an infinite number of solutions if $\alpha = 2$ and $\beta \neq 7$ (3) unique solution if $\alpha \neq 2$ (4) an infinite number of solutions whenever $\alpha \neq 2$
24.	Which of the following subset of $\mathbb{R}^4$ is basis of $\mathbb{R}^4$? $B_1 = \{(1,0,0,0), (1,1,0,0), (1,1,1,0), (1,1,1,1)\}$ $B_2 = \{(1,0,0,0), (1,2,0,0), (1,2,3,0), (1,2,3,4)\}$ $B_3 \{(1,2,0,0), (0,0,1,1), (2,1,0,0), (-5,5,0,0)\}$ (1) B_1 and B_3 but not B_2 (2) B_1, B_2 and B_3 (3) only B_1 (4) B_1 and B_2 but not B_3
25.	Let A be an invertible real n×n matrix. Let $F : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ defined by F (x, y) = (Ax, y), where (x, y) denote the inner product of x and y. Let DF (x, y) denote the derivative of F at (x, y) which is a linear transformation from $\mathbb{R}^n \times \mathbb{R}^n \to R$. The which of the following is incorrect ? (1) if $x \neq 0$, then DF (x, 0) $\neq 0$ (2) if $x \neq 0$, then DF (0, y) $\neq 0$ (3) if x = 0 or y = 0, then DF (x, y) = 0 (4) if (x, y) $\neq$ (0, 0), then DF (x, y) $\neq 0$

Question No.	Questions
26.	Let P (n) be a polynomial of degree $d \geq 2$. The radius of convergence of the power series $\sum_{n=0}^{\infty} P(n) z^n$ is (1) 0 (2) 1 (3) ∞ (4) depends upon d
27.	The residue of the function $f(z) = e^{-e^{\frac{1}{z}}}$ at $z = 0$ is (1) $1 + e^{-1}$ (2) e^{-1} (3) $-e^{-1}$ (4) $1 - e^{-1}$
28.	Let f be a holomorphic function on $0 < \|z\| < \varepsilon$, $\varepsilon > 0$, given by a convergent Laurent series $\sum_{n=-\infty}^{\infty} a_n z^n$. Also given that $\lim_{z \to 0} \|f(z)\| = \infty$. We can conclude that (1) $a_{-1} \neq 0$, and $a_{-n} = 0 \ \forall n \geq 2$ (2) $a_{-n} = 0 \ \forall n \geq 1$ (3) $a_{-n} \neq 0 \ \forall n \geq 1$ (4) $a_{-m} \neq 0$ for some $m \geq 1$ and $a_{-n} = 0 \ \forall n > m$
29.	Let C denote the unit circle centred at the origin in Argand's plane. The value of the integral $\frac{1}{2\pi i} \int_C \|1 + z + z^2\|^2 dz$, when integral in taken anti-clockwise along C, equals (1) 1 (2) 2 (3) 0 (4) 3

Question No.	Questions
30.	Let $f : \mathbb{C} \to \mathbb{C}$ be a non-constant entire function and let Image (f) = $\{w \in \mathbb{C} : z \in \mathbb{C} \text{ s.t. } f(z) = w\}$. Then (1) The interior of image (f) is empty (2) Image (f) intersects every line passing through the origin (3) There exists a disc in complex plane which is disjoint from image (f) (4) Image (f) contains all its limit points
31.	Let $C(t) = 3e^{it}$, $0 \le t \le 2\pi$ be the positively oriented circle of radius 3 centered at the origin. The value of λ for which $\oint_C \frac{\lambda}{z-2}\,dz = \oint_C \frac{1}{z^2-5z+4}\,dz$ is (1) $\lambda = 0$ (2) $\lambda = 1$ (3) $\lambda = \frac{1}{3}$ (4) $\lambda = \frac{-1}{3}$
32.	Let f be real valued harmonic function of complex variable and $g = f_x - if_y$, $h = f_x + if_y$. Then (1) g and h both are holomorphic functions (2) both g and h are identically equal to the zero function (3) g is holomorphic but h need not be holomorphic function (4) h is holomorphic but f need not be holomorphic function

Question No.	Questions
33.	How many elements does the set $\{z \in \mathbb{C} : z^{60} = -1,\ z^k \neq -1 \text{ for } 0 < k\ 60\}$ have ? (1) 45 (2) 32 (3) 30 (4) 24
34.	The number of roots of the equation $z^5 - 12z^2 + 14 = 0$ that lie in the region $\left\{z \in \mathbb{C} : 2 \le \lvert z \rvert < \frac{5}{2}\right\}$ is (1) 3 (2) 4 (3) 2 (4) 5
35.	The integral $\oint_{\lvert 1-z \rvert = 1} \frac{e^z}{z^2-1}\,dz$ is (1) 0 (2) $(i\pi)(e - e^{-1})$ (3) $e + e^{-1}$ (4) $ie\pi$
36.	Let p be a prime number. How many distinct sub-rings (with unity) of cardinality p does the field F_{p^2} have ? (1) p (2) p^2 (3) 1 (4) 0
37.	Let S_n denote the permutation group on n symbols and A_n be the subgroup of even permutations. Which of the following is true ? (1) there exists a finite group which is not a subgroup of S_n for any $n \ge 1$ (2) every finite group is a quotient of A_n for some $n \ge 1$ (3) every finite group is a subgroup of A_n for some $n \ge 1$ (4) no finite abelian group is a quotient of S_n for $n \ge 3$

Question No.	Questions
38.	Let S be the set of all integers from 100 to 999 which are neither divisible by 3 nor divisible by 5. Then the number of elements in 5 is (1) 240 (2) 360 (3) 420 (4) 480
39.	Consider the ideal $I = <(x^2+1), y>$ in the polynomial ring $\mathbb{C}[x, y]$. Which of the following statements is true ? (1) I is a maximal ideal (2) I is a maximal ideal but not a prime ideal (3) I is prime ideal but not a maximal ideal (4) I is neither a prime ideal nor a maximal ideal
40.	Let $f : \mathbb{Z} \to (\mathbb{Z} \to /4\mathbb{Z})$ X $(\mathbb{Z}/6\mathbb{Z})$ be the function defined by f (n) = (n mod 4, n mod 6). Then (1) Kernel of $f = 24\mathbb{Z}$ (2) Image of f has exactly 6 elements (3) (a mod 4, b mod 6) is in the Image of f, for all even integers a and b (4) (0 mod 4, 3 mod 6) is in the Image of f
41.	Let S_7 denote the group of permutations of the set {1, 2, 3, 4, 5, 6,7}. Which of the following is true ? (1) There are no elements of order 10 in S_7 (2) There are no elements of order 8 in S_7 (3) There are no elements of order 7 in S_7 (4) There are no elements of order 6 in S_7

Question No.	Questions
42.	The number of group homomorphism from $\mathbb{Z}_{10}$ to $\mathbb{Z}_{20}$ is (1) 0 (2) 1 (3) 5 (4) 10
43.	Let Aut (G) denote the group of automorphisms of G for a group G. Which of the following is true ? (1) If G is cyclic, then Aut (G) is cyclic (2) If G is finite, then Aut (G) is finite (3) If G is infinite, then Aut (G) is infinite (4) If Aut (G) is isomorphic to Aut (H), where G and H are two groups, then G is isomorphic to H.
44.	Let $f(x) = x^7 - 105x + 12$. Then which of the following is correct ? (1) f (x) is reducible over Q (2) ∃ an integer m s.t. f (m) = 2 (3) ∃ an integer m s.t. f (m) = 105 (4) f (m) is not a prime number for any integer m.
45.	The degree of splitting field of $x^4 - 1$ ove $\mathbb{Q}$ is (1) 2 (2) 4 (3) 1 (4) 0
46.	Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous map. Which of the following is a correct statement ? (1) f is bounded (2) Image of f is an open subset of $\mathbb{R}$ (3) f (A) is bounded for all bounded subsets A of $\mathbb{R}$ (4) f^{-1} (A) is compact for all compact subsets A of $\mathbb{R}$

Question No.	Questions
47.	Let A and B be topological spaces where B is Hausdorff. Let A × B be the given product topology. Then for a function $f : A \to B$ which of the following statements is necessarily true ? (1) if B is finite, then f is continuous (2) if f is continuous, then graph $(f) = \{(x, f(x)) / x \in A\}$ is closed in A × B (3) if group (f) is closed in A × B, then f is continuous (4) if group (f) is closed in A × B, then f is continuous and bounded
48.	Let A be a subset of $\mathbb{R}$ and $A = \bigcap_{n\geq 1} V_n$, where for each $n \geq 1, V_n$ is an open dense subset of $\mathbb{R}$. Which of the following are incorrect ? (1) A is countable (2) A is uncountable (3) A is dense in $\mathbb{R}$ (4) A is a non-empty set
49.	Let X be a topological space and let U be a proper dense open subset of X. Choose the correct statement : (1) If X is connected, then U is connected (2) If X is compact, then U is compact (3) If X is compact, then X\U is compact (4) If X\U is compact, then X is compact
50.	Let G be an open set in $\mathbb{R}^n$. Two points $x, y \in G$ are defined to be equivalent if they can be joined by a continuous path completely lying inside G. Then number of equivalent classes is (1) only one (2) at most countable (3) at most finite (4) can be finite or uncountable

Question No.	Questions
51.	A load slides without friction on a frictionless wire in the shape of cycloid with equation $x = a(\theta - \sin\theta),\ y = a(1+\cos\theta),\ 0 \le \theta \le 2\pi$. The the Lagrangian is (1) $ma^2(1+\cos\theta)\dot{\theta}^2 - mga(1+\cos\theta)$ (2) $ma^2(1+\cos\theta)\dot{\theta}^2 - mga(1-\cos\theta)$ (3) $ma^2(1-\cos\theta)\dot{\theta}^2 - mga(1+\cos\theta)$ (4) $ma^2(1-\cos\theta)\dot{\theta}^2 + mga(1+\cos\theta)$
52.	Consider the two dimensional motion of a mass m attached to one end of a spring whose other end is fixed. Let K be the spring constant. The kinetic energy T and the potential energy V of the system are given by $T = \frac{1}{2}m(\dot{r}^2 + (r\dot{\theta})^2),\ V = \frac{1}{2}Kr^2;\ \dot{r} = \frac{dr}{dt},\ \dot{\theta} = \frac{d\theta}{dt}$. Then which of the following statement is correct ? (1) r is ignorable co-ordinate (2) θ is not an ignorable co-ordinate (3) $r^2\dot{\theta}$ remains constant throughout the motion (4) $r\dot{\theta}$ remains constant throughout the motion

Question No.	Questions
53.	A body, under the action of no forces, moves so that the resolved part of its angular velocity about one of the principal axes at the centre of the gravity is constant. Let A, B, C be the principal moments and w_1, w_2, w_3 be the angular velocities about the principal axes at C.G. Consider the following statements : I : The angular velocity of the body is constant II : $A \dot{w}_1 w_1 + B \dot{w}_2 w_2 + C \dot{w}_3 w_3 = 0$ Then (1) Statement I is true but II is false (2) Statement I is false but II is true (3) Both the statements I and II are true (4) Both the statements I and II are false
54.	Given that the Lagrangian for the motion of a simple pendulum is $L = \frac{1}{2} m\ell^2 \dot{\theta}^2 + mg\ell \cos\theta$, where m is the mass of the pendulum bob suspended by a string of length ℓ, g is accelaration due to gravity, θ is the amplitude of the pendulum from the mean position. Then corresponding Hamiltonian is (1) $H = \frac{p^2}{2m\ell^2} + mg\ell\cos\theta$ (2) $\frac{p^2}{2m\ell^2} - mg\ell\cos\theta = H$ (3) $H = \frac{p^2}{m\ell^2} - mg\ell\cos\theta$ (4) $H = \frac{3p^2}{2m\ell^2} + mg\ell\cos\theta$

Question No.	Questions
55.	The admissible extremal for $I[y] = \int_0^{\log 3} \{e^{-x}y'^2 + 2e^x(y + y^1)\} dx$, where $y(\log 3) = 1$ and $y(0)$ is free, is (1) $4 - e^x$ (2) $10 - e^{2x}$ (3) $e^{2x} - 8$ (4) $e^x - 2$
56.	Consider the functional $I[y] = \int_0^1 \{(y'(x))^2 + (y'(x))^3\} dx$ subject to $y(0) = 1$ and $y(1) = 2$. Then (1) there exists an extremal $y \in c^1([0, 1]) \setminus c^2([0, 1])$ (2) there exists an extremal $y \in c([0, 1]) \setminus c^1([0, 1])$ (3) no extremal y belongs to $c^1([0, 1])$ (4) every extremal y belongs to $c^2([0, 1])$
57.	Given a problem of calculus of variation $J[y] = \int_0^1 [2y + (y^1)^2] dx$ subject to $y(0) = 0$, $y(1) = 1$. The value of Inf $J[y]$ is (1) $\frac{21}{24}$ (2) $\frac{18}{24}$ (3) $\frac{23}{12}$ (4) Does not exist
58.	If y is a solution of the integral equation $y(x) = 1 + \int_0^x (x - t)\, y(t)\, dt$. Then (1) $\int_{\mathbb{R}} y(x)\, dx < \infty$ (2) $\int_{\mathbb{R}} \frac{dx}{y(x)} < \infty$ (3) $y(x)$ is periodic in $\mathbb{R}$ (4) y is bounded but not periodic in $\mathbb{R}$

Question No.	Questions
59.	The value of the solution of the integral equation $y(x) - 1 + 2x + 4x^2 = \int_0^x \left\{6(x-t) + x - 4(x-t)^2\right\} y(t)\, dt$ at $x = \log 5$ is equal to (1) 5 (2) $\frac{1}{5}$ (3) 4 (4) $\frac{1}{4}$
60.	If g is the solution of $\int_0^x (1 - x^2 + t^2)\, g(t)\, dt = \frac{x^2}{2}$, Then $g\left(\sqrt{2}\right)$ equals (1) $\sqrt{2}\, e^{2\sqrt{2}}$ (2) $\sqrt{2}\, e^{\sqrt{2}}$ (3) $\sqrt{2}\, e^{2}$ (4) $\sqrt{2}\, e^{4}$
61.	If f (x) and g (x) are two solutions of $\cos x \frac{d^2y}{dx_2} + \sin x \frac{dy}{dx} - \left(1 + e^{-x^2}\right) y = 0,\ x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ such that $f(0) = \sqrt{2},\ f'(0) = 1;\ g(0) = -\sqrt{2},\ g'(0) = 2.$ Then Wronskian of f and g is (1) $3 \cos x$ (2) $3\sqrt{2} \cos x - x$ (3) $3\sqrt{2} \cos x$ (4) 0
62.	The critical point (0, 0) of the system $\frac{dx}{dt} = 2x - 7y,\ \frac{dy}{dt} = 3x - 8y$ is (1) unstable node (2) asymptotically stable node (3) unstable spiral point (4) asymptotically stable spiral point

Question No.	Questions
63.	Given a boundary value problem $y''(x)+\lambda\, y(x)=0;\ y(0)=0,\ y(\pi)=0$ Set of its eigen values is (1) $\mathbb{R}$ (2) $(-\infty, 0)$ (3) $\{n^2 : n \in \mathbb{Z}^+\}$ (4) $\{\sqrt{n} : n \in \mathbb{Z}^+\}$
64.	The limiting value of y (x), as $x \to \infty$, where y (x) is the solution of $y'(x)=ay-by^2$; a, b > 0, $y(0)=y_0$, will be (1) 0 (2) a/b (3) b/a (4) y_0
65.	Given a differential equation $x''(t)+p(t)\,x'(t)+q(t)\,x(t)=0;\ p(t), q(t) \in C^1[a, b]$. Let f (t) and g (t) be its two solutions on [a, b]. Then which of the following is incorrect ? (1) f and g are linearly dependent and $W(f, g)(t)=0\ \forall t \in [a, b]$ (2) f and g are linearly in dependent and $\exists$ to $\in (a, b)$ s.t. $W(t_0)=0$ (3) f and g are linearly in dependent and $W(f, g)(t) \neq 0$ for any $t \in [a, b]$ (4) f and g are linearly independent then every other solution can be written as their linear combination
66.	Let D denote the disc $\{(x, y) \mid x^2+y^2 \le 1\}$ and let D^c be its complement in the plane. The P.D.E $(x^2-1)\dfrac{\partial^2 u}{dx^2}+2y\dfrac{\partial^2 u}{dxdy}-\dfrac{\partial^2 u}{dy^2}=0$ is (1) Parabolic $\forall (x, y) \in D^c$ (2) Hyperbolic $\forall (x, y) \in D$ (3) Hyperbolic $\forall (x, y) \in D^c$ (4) Parabolic $\forall (x, y) \in D$

Question No.	Questions
67.	Given a partial differential equation $x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = (x+y)\,z$. Then which of the following is not the general solution of the differential equation ? (1) $F\left(\frac{xy}{z}, \frac{x-y}{z}\right) = 0$ for an arbitrary twice differentiable function F (2) $F\left(\frac{x-y}{z}, \frac{1}{x} - \frac{1}{y}\right) = 0$ for an arbitrary twice differentiable function F (3) $z = f\left(\frac{1}{x} - \frac{1}{y}\right)$ for an arbitrary twice differentiable function f (4) $z = xyf\left(\frac{1}{x} - \frac{1}{y}\right)$ for an arbitrary twice differentiable function f
68.	The Cauchy problem $2\,u_x + 3\,u_y = 5$, $u = 1$ on the line $3x - 2y = 0$ has (1) exactly one solution (2) exactly two solutions (3) infinitely many solutions (4) no solution
69.	Let u be the unique solution of $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, where $(x, t) \in (0, 1) \text{ X } (0, \infty)$ $u\,(x, 0) = \sin \pi x,\ x \in (0,1)$ $u(0, t) = u\,(1, t) = \theta,\ t \in (0, \infty)$ Then which of the following is true ? (1) $\exists\,(x, t) \in (0,1) \text{ X } (0, \infty)$ s.t. $u\,(x, t) = 0$ (2) $\exists\,(x, t) \in (0,1) \text{ X } (0, \infty)$ s.t. $\frac{\partial u}{\partial t}\,(x, t) = 0$ (3) the function $e^t u\,(x, t)$ is bounded for $(x, t) \in (0,1) \text{ X } (0, \infty)$ (4) $\exists\,(x, t) \in (0,1) \text{ X } (0, \infty)$ s.t. $u\,(x, t) > 1$

Question No.	Questions
70.	Let u (x, t) be the solution of the initial value problem $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$; $u(x, 0) = x^3$, $\frac{\partial u}{\partial t}(x, 0) = \sin x$ Then $u(\pi, \pi)$ is (1) $4\pi^3$ (2) $2\pi^3$ (3) 0 (4) 4
71.	Let f (x) be a polynomial of unknown degree taking the values x: 0, 1, 2, 3; f(x): 2, 7, 13, 16 All the fourth divided difference are $\frac{-1}{6}$. Then coefficient of x^3 is (1) $\frac{-2}{3}$ (2) $\frac{1}{3}$ (3) –1 (4) 16
72.	The iterative method $x_{n+1} = g(x_n)$ for the solution of the equation $x^2 - x - 2 = 0$ converges quadratically in a neighbourhood of the root x = 2 if g (x) equals (1) $x^2 - 2$ (2) $1+\frac{2}{x}$ (3) $(x-2)^2 - 6$ (4) $\frac{x^2+2}{2x-1}$
73.	The forward difference operator is defined as $\Delta u_n = u_{n+1} - u_n$. Then which of the following difference equation has bounded general solution ? (1) $\Delta^2 u_n - 3\Delta u_n + 2u_n = 0$ (2) $\Delta^2 u_n + \Delta u_n + \frac{1}{4} u_n = 0$ (3) $\Delta^2 u_n - 2\Delta u_n + 2u_n = 0$ (4) $\Delta^2 u_{n+1} - \frac{1}{3}\Delta^2 u_n = 0$

Question No.	Questions
74.	Given the following statements · I : $\nabla = 1 - E^{-1}$ II : $E = 1 + \frac{\delta^2}{2} + \sqrt{1 + \frac{\delta^2}{4}}$ Then (1) I is true but II is false (2) I is false but II is true (3) Both I and II are false (4) Both I and II are true
75.	The expression of local error of Runge-Kutta method of order 2 is of the form (1) $\frac{h^3}{10}\left(f_{xx} + 2f_{yy}\, f^2 - 2f_x f_y\right) + 0\left(h^4\right)$ (2) $\frac{h^3}{12}\left(f_{xx} + 2f_{yy}\, f^2 - 2f_x f_y\right) + 0\left(h^4\right)$ (3) $\frac{h^3}{12}\left(f_{xx} + 2f\,f_{xy} + f^2 f_{yy} - 2f_x f_y - 2f\,f_y^2\right) + \left(h^4\right)$ (4) None of these
76.	Let X_1, X_2, X_3, X_4, X_5 be i.i.d random variables having a continuous distribution function. Then $P\left(X_1 > X_2 > X_3 > X_4 > X_5 / X_1 = \max\ (X_1, X_2, X_3, X_4, X_5)\right)$ equals (1) $\frac{1}{5}$ (2) $\frac{1}{4!}$ (3) $\frac{1}{4}$ (4) $\frac{1}{5!}$

Question No.	Questions

77. Consider a Markov chain with state space {0, 1, 2, 3, 4} and transition matrix

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

Then $\lim_{n \to \infty} p_{23}^{(n)}$ equals

(1) $\frac{1}{3}$

(2) $\frac{1}{2}$

(3) 0

(4) 1

78. Consider the function f (x) defined as

$f(x) = ce^{-x^4}, \; x \in \mathbb{R}$

For what value of c is f a probability density function ?

(1) $\frac{4}{\Gamma\left(\frac{1}{4}\right)}$

(2) $\frac{3}{\Gamma\left(\frac{1}{3}\right)}$

(3) $\frac{2}{\Gamma\left(\frac{1}{4}\right)}$

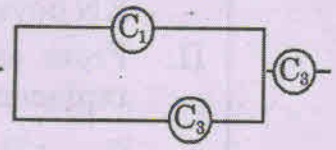
(4) $\frac{1}{4\Gamma\left(\frac{1}{4}\right)}$

Question No.	Questions
79.	Given the observations 0.8, 0.71, 0.9, 1.2, 1.68, 1.4, 0.88, 1.62 from the uniform distribution on $(\theta - 0.2, \theta + 0.8)$ with $-\infty < \theta < \infty$. Which of the following is a maximum likelihood estimate for θ ? (1) 0.7 (2) 1.1 (3) 1.3 (4) 0.9
80.	Suppose that $\lvert 3x \rvert + \lvert 2y \rvert \le 1;\ x \ge 0,\ y \ge 0$. Then the maximum value of $9x+4y$ is (1) 1 (2) 2 (3) 3 (4) 4
81.	Let $\psi(t) = e^{-\lvert t \rvert - \frac{t^2}{2}}$ and $\phi(t) = \dfrac{e^{-\lvert t \rvert} + e^{\frac{-t^2}{2}}}{2}$. Which of the following is true ? (1) ϕ is a characteristic function but ψ is not (2) ψ is a characteristic function but ϕ is not (3) neither ϕ nor ψ is a characteristic function (4) Both ϕ and ψ are characteristic functions
82.	If $b_{xy} = 0.20$ and $r_{xy} = 0.50$, Then b_{yx} is equal to (1) 0.20 (2) 0.25 (3) 0.50 (4) 1.25
83.	Let $X_1, X_2, \ldots\ldots, X_n$ be independent random variables ; X_i having exponential distribution with parameter θ_i, $i = 1, 2, \ldots, n$. Then $Z = \text{Min}(X_1, X_2, \ldots., X_n)$ has (1) normal distribution (2) geometric distribution (3) exponential distribution with parameter $\sum_{i=1}^{n} \theta_i$ (4) None of these

Question No.	Questions
84.	In queueing description M | M | 1, the arrival and departure distribution are (1) Binomial (2) General (3) Both Markovian (4) None of these
85.	Successful life of product, time, weight and height are classified as (1) continuous random variable (2) discrete random variable (3) continuous time variable (4) None of these
86.	If a random variable X has a Chi-Square distribution with 4 degree of freedom, then its mean is equal to (1) 2 (2) 3 (3) 4 (4) None of these
87.	In a Latin Square Design, if factors A, B, C and D have levels 8, then the total number of cells in the design is (1) 4096 (2) 64 (3) 512 (4) None of these
88.	If a system has two components in parallel with each of reliability 0.75, then the reliability of the system is equal to (1) 0.9375 (2) 0.9753 (3) 0.7935 (4) None of these

Question No.	Questions
89.	Let (v, b, r, k, w) be the standard parameters of a balanced incomplete block design (BIBD). Which of the following (v, b, r, k, w) can be parameters of BIBD ? (1) (v, b, r, k,w) = (44, 33, 9, 12, 3) (2) (v, b, r, k,w) = (17, 45, 8, 3, 1) (3) (v, b, r, k,w) = (35, 35, 17, 17, 9) (4) (v, b, r, k,w) = (16, 24, 9, 6, 3)
90.	100 tickets are marked 1, 2, ... , 100 and arranged at random. Four tickets are picked from these tickets and given to four persons A, B, C and D. What is the probability that A gets the ticket with the largest value (among A, B, C, D) and D gets the ticket with smallest value (among A, B, C, D) ? (1) $\frac{1}{4}$ (2) $\frac{1}{12}$ (3) $\frac{1}{2}$ (4) $\frac{1}{6}$
91.	Let $\underline{X} \sim N_3(\underline{\mu}, \Sigma)$, where $\underline{\mu} = (1, 1, 1)$ and $\Sigma = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 3 & C \\ 1 & C & 2 \end{pmatrix}$. The value of C such that X_2 and $-X_1 + X_2 - X_3$ are independent is (1) 0 (2) -2 (3) 2 (4) 1

Question No.	Questions
92.	Assume that X ~ Binomial (n, p) for some $n \geq 1$ and $0 < p < 1$ and Y ~ Poisson (λ) for some $\lambda > 0$. Suppose $\Sigma[X] = \Sigma[Y]$. Then (1) Var (X) = Var (Y) (2) Var (X) < Var (Y) (3) Var (Y) < Var (X) (4) Var (X) may be larger or smaller than Var (Y)
93.	Let X, Y be independent random variables and let $Z = \frac{X-Y}{2} + 3$ If X has characteristic function ϕ and Y has characteristic function ψ, then Z has characteristic function θ where (1) $\theta(t) = e^{-i3t}\phi(2t)\psi(-2t)$ (2) $\theta(t) = e^{i3t}\phi\left(\frac{t}{2}\right)\psi\left(-\frac{t}{2}\right)$ (3) $\theta(t) = e^{-i3t}\phi\left(\frac{t}{2}\right)\psi\left(\frac{t}{2}\right)$ (4) $\theta(t) = e^{-i3t}\phi\left(\frac{t}{2}\right)\psi\left(\frac{-t}{2}\right)$
94.	Consider LPP : Minimize : $z = -2x - 5y$ subject to : $3x + 4y \geq 5$, $x \geq 0$, $y \geq 0$. Which of the following is correct ? (1) Set of feasible solutions is empty (2) Set of feasible solutions is non-empty but there is no optimal solution (3) Optimal value is obtained at $\left(0, \frac{5}{4}\right)$ (4) Optimal value is attained at $\left(\frac{5}{3}, 0\right)$

Question No.	Questions
95.	(X, Y) follows the bivariate normal distribution N_2 (0, 0, 1, 1, ρ), $-1 < \rho < 1$. Then (1) X + Y and X – Y are uncorrelated only if $\rho = 0$ (2) X + Y and X – Y are uncorrelated only if $\rho < 0$ (3) X + Y and X – Y are uncorrelated only if $\rho > 0$ (4) X + Y and X – Y are uncorrelated for all values ρ
96.	Let (X_1, X_2, X_3, X_4) be an optimal solution to the problem of minimizing $X_1 + X_2 + X_3 + X_4$ subject to the constraints $X_1 + X_2 \geq 300$, $X_2 + X_3 \geq 500$, $X_3 + X_4 \geq 400$, $X_4 + X_1 \geq 200$, $X_1 \geq 0$, $X_2 \geq 0$, $X_3 \geq 0$, $X_4 \geq 0$. Which of the following is a possible value for any X_i ? (1) 300 (2) 400 (3) 500 (4) 600
97.	A system consists of 3 components arranged as –  Each of the components C_1, C_2, C_3 has independent and identically distributed (i.i.d) lifetimes whose distribution is exponential with mean 1. Then the survival function, S (t), of the system is given by (1) $S(t) = e^{-3t}, t > 0$ (2) $S(t) = (1 - e^{-t})^2 e^{-t}, t > 0$ (3) $S(t) = (1 - e^{-2t}) e^{-t}, t > 0$ (4) $S(t) = (1 - (1 - e^{-t})^2) e^{-t}, t > 0$

Question No.	Questions
98.	Let $X_1, X_2, \ldots, X_7$ be a random sample from $N(\mu, \sigma^2)$ where μ and σ^2 are unknown. Consider the problem of testing $H_0 : \mu = 2$ against $H_1 : \mu > 2$. Suppose the observed values of $x_1, x_2, \ldots, x_7$ are 1.2, 1.3, 1.7, 1.8, 2.1, 2.3, 2.7. If we use the uniformly most powerful test, which of the following is true ? (1) H_0 is accepted both at 5% and 1% levels of significance (2) H_0 is rejected both at 5% and 1% levels of significance (3) H_0 is rejected at 5% level of significance, but accepted at 1% level of significance (4) H_0 is rejected at 1% level of significance, but accepted at 5% level of significance
99.	Let N_t denote the number of accidents up to time t. Assume that $\{N_t\}$ is a Poisson process with intensity 2. Given that there are exactly 5 accidents during the time period $[20, 30]$. What is the conditional probability that there is exactly one accident dusing the time period $[15, 25]$? (1) $\frac{15}{32} e^{-20}$ (2) $\frac{15}{32} e^{-10}$ (3) $20\, e^{-20}$ (4) $\frac{1}{5}$
100.	Suppose there are K groups each consisting of N boys. We want to estimate the mean age μ of these μN boys. Fix $1 < n < N$ and consider the following two sampling schemes : I. Draw a simple random sample without replacement of size kn out of KN boys II. From each of the K groups, draw a simple random sample with replacement of size n. Let $\overline{Y}$ and $\overline{Y}_G$ be the respective sample mean ages for the two schemes. Which of the following is not true ? (1) $E(\overline{Y}) = \mu$ (2) $E(\overline{Y}_G) = \mu$ (3) Var $(\overline{Y})$ may be less than Var $(\overline{Y}_G)$ in some cases (4) Var $(\overline{Y})$ = Var $(\overline{Y}_G)$ if all the group means are same.

(DO NOT OPEN THIS QUESTION BOOKLET BEFORE TIME OR UNTIL YOU ARE ASKED TO DO SO)

(MPH/PHD/URS–EE–2019)

Subject : MATHEMATICS

Code **B**

Sr. No. 10278

SET–"X"

Time : 1¼ Hours **Total Questions : 100** **Max. Marks : 100**

Roll No. ________________ (in figure)______________________________ (in words)

Name : ______________________________ Father's Name : ____________________

Mother's Name : ________________________ Date of Examination : ______________

(Signature of the candidate) (Signature of the Invigilator)

CANDIDATES MUST READ THE FOLLOWING INFORMATION/ INSTRUCTIONS BEFORE STARTING THE QUESTION PAPER.

1. **Candidates are required to attempt any 75 questions out of the given 100 multiple choice questions of 4/3 marks each. No credit will be given for more than 75 correct responses.**
2. The candidates must return the Question book-let as well as OMR answer-sheet to the Invigilator concerned before leaving the Examination Hall, failing which a case of use of unfair-means / mis-behaviour will be registered against him / her, in addition to lodging of an FIR with the police. Further the answer-sheet of such a candidate will not be evaluated.
3. Keeping in view the transparency of the examination system, carbonless OMR Sheet is provided to the candidate so that a copy of OMR Sheet may be kept by the candidate.
4. Question Booklet along-with answer key of all the A,B,C and D code shall be got uploaded on the university website after the conduct of Entrance Examination. In case there is any discrepancy in the Question Booklet/Answer Key, the same may be brought to the notice of the Controller of Examination in writing/through E. Mail within 24 hours of uploading the same on the University Website. Thereafter, no complaint in any case, will be considered.
5. The candidate MUST NOT do any rough work or writing in the OMR Answer-Sheet. Rough work, if any, may be done in the question book-let itself. Answers MUST NOT be ticked in the Question book-let.
6. **There will be no negative marking. Each correct answer will be awarded 4/3 mark. Cutting, erasing, overwriting and more than one answer in OMR Answer-Sheet will be treated as incorrect answer.**
7. Use only Black or Blue **BALL POINT PEN** of good quality in the OMR Answer-Sheet.
8. BEFORE ANSWERING THE QUESTIONS, THE CANDIDATES SHOULD ENSURE THAT THEY HAVE BEEN SUPPLIED CORRECT AND COMPLETE BOOK-LET. COMPLAINTS, IF ANY, REGARDING MISPRINTING ETC. WILL NOT BE ENTERTAINED 30 MINUTES AFTER STARTING OF THE EXAMINATION.

Question No.	Questions
1.	Let f (x) be a polynomial of unknown degree taking the values x: 0, 1, 2, 3 ; f(x): 2, 7, 13, 16 All the fourth divided difference are $\frac{-1}{6}$. Then coefficient of x^3 is (1) $\frac{-2}{3}$ (2) $\frac{1}{3}$ (3) −1 (4) 16
2.	The iterative method $x_{n+1} = g(x_n)$ for the solution of the equation $x^2 - x - 2 = 0$ converges quadratically in a neighbourhood of the root $x = 2$ if g (x) equals (1) $x^2 - 2$ (2) $1 + \frac{2}{x}$ (3) $(x-2)^2 - 6$ (4) $\frac{x^2+2}{2x-1}$
3.	The forward difference operator is defined as $\Delta u_n = u_{n+1} - u_n$. Then which of the following difference equation has bounded general solution ? (1) $\Delta^2 u_n - 3\Delta u_n + 2u_n = 0$ (2) $\Delta^2 u_n + \Delta u_n + \frac{1}{4} u_n = 0$ (3) $\Delta^2 u_n - 2\Delta u_n + 2u_n = 0$ (4) $\Delta^2 u_{n+1} - \frac{1}{3}\Delta^2 u_n = 0$

Question No.	Questions
4.	Given the following statements : I : $\nabla = 1 - E^{-1}$ II : $E = 1 + \frac{\delta^2}{2} + \sqrt{1 + \frac{\delta^2}{4}}$ Then (1) I is true but II is false (2) I is false but II is true (3) Both I and II are false (4) Both I and II are true
5.	The expression of local error of Runge-Kutta method of order 2 is of the form (1) $\frac{h^3}{10}\left(f_{xx} + 2f_{yy}\, f^2 - 2f_x f_y\right) + 0\left(h^4\right)$ (2) $\frac{h^3}{12}\left(f_{xx} + 2f_{yy}\, f^2 - 2f_x f_y\right) + 0\left(h^4\right)$ (3) $\frac{h^3}{12}\left(f_{xx} + 2f\, f_{xy} + f^2 f_{yy} - 2f_x f_y - 2f\, f_y^2\right) + \left(h^4\right)$ (4) None of these
6.	Let X_1, X_2, X_3, X_4, X_5 be i.i.d random variables having a continuous distribution function. Then $P\left(X_1 > X_2 > X_3 > X_4 > X_5 / X_1 = \max\ (X_1, X_2, X_3, X_4, X_5)\right)$ equals (1) $\frac{1}{5}$ (2) $\frac{1}{4!}$ (3) $\frac{1}{4}$ (4) $\frac{1}{5!}$

Question No.	Questions
7.	Consider a Markov chain with state space {0, 1, 2, 3, 4} and transition matrix $$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$ Then $\lim_{n \to \infty} p_{23}^{(n)}$ equals (1) $\frac{1}{3}$ (2) $\frac{1}{2}$ (3) 0 (4) 1
8.	Consider the function f (x) defined as $f(x) = ce^{-x^4}, x \in \mathbb{R}$ For what value of c is f a probability density function ? (1) $\frac{4}{\Gamma\left(\frac{1}{4}\right)}$ (2) $\frac{3}{\Gamma\left(\frac{1}{3}\right)}$ (3) $\frac{2}{\Gamma\left(\frac{1}{4}\right)}$ (4) $\frac{1}{4\Gamma\left(\frac{1}{4}\right)}$

Question No.	Questions
9.	Given the observations 0.8, 0.71, 0.9, 1.2, 1.68, 1.4, 0.88, 1.62 from the uniform distribution on $(\theta - 0.2, \theta + 0.8)$ with $-\infty < \theta < \infty$. Which of the following is a maximum likelihood estimate for θ ? (1) 0.7 (2) 1.1 (3) 1.3 (4) 0.9
10.	Suppose that $\lvert 3x \rvert + \lvert 2y \rvert \leq 1$; $x \geq 0$, $y \geq 0$. Then the maximum value of $9x + 4y$ is (1) 1 (2) 2 (3) 3 (4) 4
11.	A load slides without friction on a frictionless wire in the shape of cycloid with equation $x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$, $0 \leq \theta \leq 2\pi$. The the Lagrangian is (1) $ma^2(1 + \cos\theta)\dot{\theta}^2 - mga(1 + \cos\theta)$ (2) $ma^2(1 + \cos\theta)\dot{\theta}^2 - mga(1 - \cos\theta)$ (3) $ma^2(1 - \cos\theta)\dot{\theta}^2 - mga(1 + \cos\theta)$ (4) $ma^2(1 - \cos\theta)\dot{\theta}^2 + mga(1 + \cos\theta)$
12.	Consider the two dimensional motion of a mass m attached to one end of a spring whose other end is fixed. Let K be the spring constant. The kinetic energy T and the potential energy V of the system are given by $T = \frac{1}{2} m (\dot{r}^2 + (r\dot{\theta})^2)$, $V = \frac{1}{2} Kr^2$; $\dot{r} = \frac{dr}{dt}$, $\dot{\theta} = \frac{d\theta}{dt}$. Then which of the following statement is correct ? (1) r is ignorable co-ordinate (2) θ is not an ignorable co-ordinate (3) $r^2\dot{\theta}$ remains constant throughout the motion (4) $r\dot{\theta}$ remains constant throughout the motion

Question No.	Questions
13.	A body, under the action of no forces, moves so that the resolved part of its angular velocity about one of the principal axes at the centre of the gravity is constant. Let A, B, C be the principal moments and w_1, w_2, w_3 be the angular velocities about the principal axes at C.G. Consider the following statements : I : The angular velocity of the body is constant II : $A\,\dot{w}_1\,w_1 + B\,\dot{w}_2\,w_2 + C\,\dot{w}_3\,w_3 = 0$ Then (1) Statement I is true but II is false (2) Statement I is false but II is true (3) Both the statements I and II are true (4) Both the statements I and II are false
14.	Given that the Lagrangian for the motion of a simple pendulum is $L = \frac{1}{2} m\ell^2 \dot{\theta}^2 + mg\ell\cos\theta$, where m is the mass of the pendulum bob suspended by a string of length ℓ, g is accelaration due to gravity, θ is the amplitude of the pendulum from the mean position. Then corresponding Hamiltonian is (1) $H = \frac{p^2}{2m\ell^2} + mg\ell\cos\theta$ (2) $\frac{p^2}{2m\ell^2} - mg\ell\cos\theta = H$ (3) $H = \frac{p^2}{m\ell^2} - mg\ell\cos\theta$ (4) $H = \frac{3p^2}{2m\ell^2} + mg\ell\cos\theta$

Question No.	Questions
15.	The admissible extremal for $I[y]=\int_0^{\log 3}\left\{e^{-x}y'^2+2e^x\left(y+y^1\right)\right\}dx$, where $y(\log 3)=1$ and $y(0)$ is free, is (1) $4-e^x$ (2) $10-e^{2x}$ (3) $e^{2x}-8$ (4) e^x-2
16.	Consider the functional $I[y]=\int_0^1\left\{(y'(x))^2+(y'(x))^3\right\}dx$ subject to $y(0)=1$ and $y(1)=2$. Then (1) there exists an extremal $y\in c^1([0,1])\setminus c^2([0,1])$ (2) there exists an extremal $y\in c^{s}([0,1])\setminus c^1([0,1])$ (3) no extremal y belongs to $c^1([0,1])$ (4) every extremal y belongs to $c^2([0,1])$
17.	Given a problem of calculus of variation $J[y]=\int_0^1\left[2y+(y^1)^2\right]dx$ subject to $y(0)=0$, $y(1)=1$. The value of Inf $J[y]$ is (1) $\frac{21}{24}$ (2) $\frac{18}{24}$ (3) $\frac{23}{12}$ (4) Does not exist
18.	If y is a solution of the integral equation $y(x)=1+\int_0^x(x-t)\,y(t)\,dt$. Then (1) $\int_{\mathbb{R}} y(x)\,dx<\infty$ (2) $\int_{\mathbb{R}}\frac{dx}{y(x)}<\infty$ (3) y (x) is periodic in $\mathbb{R}$ (4) y is bounded but not periodic in $\mathbb{R}$

Question No.	Questions
19.	The value of the solution of the integral equation $y(x)-1+2x+4x^2=\int_0^x\left\{6(x-t)+x-4(x-t)^2\right\}y(t)\,dt$ at $x=\log 5$ is equal to (1) 5 (2) $\frac{1}{5}$ (3) 4 (4) $\frac{1}{4}$
20.	If g is the solution of $\int_0^x(1-x^2+t^2)\,g(t)\,dt=\frac{x^2}{2}$, Then $g\left(\sqrt{2}\right)$ equals (1) $\sqrt{2}\,e^{2\sqrt{2}}$ (2) $\sqrt{2}\,e^{\sqrt{2}}$ (3) $\sqrt{2}\,e^{2}$ (4) $\sqrt{2}\,e^{4}$
21.	Let $C(t)=3e^{it},\ 0\le t\le 2\pi$ be the positively oriented circle of radius 3 centered at the origin. The value of λ for which $\oint_C \frac{\lambda}{z-2}\,dz=\oint_C \frac{1}{z^2-5z+4}\,dz$ is (1) $\lambda=0$ (2) $\lambda=1$ (3) $\lambda=\frac{1}{3}$ (4) $\lambda=\frac{-1}{3}$
22.	Let f be real valued harmonic function of complex variable and $g=f_x-if_y$, $h=f_x+if_y$. Then (1) g and h both are holomorphic functions (2) both g and h are identically equal to the zero function (3) g is holomorphic but h need not be holomorphic function (4) h is holomorphic but f need not be holomorphic function

Question No.	Questions
23.	How many elements does the set $\{z \in \mathbb{C} : z^{60} = -1,\ z^k \neq -1 \text{ for } 0 < k\ 60\}$ have ? (1) 45 (2) 32 (3) 30 (4) 24
24.	The number of roots of the equation $z^5 - 12z^2 + 14 = 0$ that lie in the region $\left\{z \in \mathbb{C} : 2 \leq \lvert z \rvert < \frac{5}{2}\right\}$ is (1) 3 (2) 4 (3) 2 (4) 5
25.	The integral $\oint_{\lvert 1-z \rvert = 1} \frac{e^z}{z^2 - 1}\, dz$ is (1) 0 (2) $(i\pi)(e - e^{-1})$ (3) $e + e^{-1}$ (4) $ie\pi$
26.	Let p be a prime number. How many distinct sub-rings (with unity) of cardinality p does the field F_{p^2} have ? (1) p (2) p^2 (3) 1 (4) 0
27.	Let S_n denote the permutation group on n symbols and A_n be the subgroup of even permutations. Which of the following is true ? (1) there exists a finite group which is not a subgroup of S_n for any $n \geq 1$ (2) every finite group is a quotient of A_n for some $n \geq 1$ (3) every finite group is a subgroup of A_n for some $n \geq 1$ (4) no finite abelian group is a quotient of S_n for $n \geq 3$

Question No.	Questions
28.	Let S be the set of all integers from 100 to 999 which are neither divisible by 3 nor divisible by 5. Then the number of elements in 5 is (1) 240 (2) 360 (3) 420 (4) 480
29.	Consider the ideal $I = < (x^2 + 1), y >$ in the polynomial ring $\mathbb{C}[x, y]$. Which of the following statements is true ? (1) I is a maximal ideal (2) I is a maximal ideal but not a prime ideal (3) I is prime ideal but not a maximal ideal (4) I is neither a prime ideal nor a maximal ideal
30.	Let $f : \mathbb{Z} \to (\mathbb{Z} \to /4\mathbb{Z}) \text{ X } (\mathbb{Z}/6\mathbb{Z})$ be the function defined by $f(n) = (n \bmod 4, n \bmod 6)$. Then (1) Kernel of $f = 24\mathbb{Z}$ (2) Image of f has exactly 6 elements (3) (a mod 4, b mod 6) is in the Image of f, for all even integers a and b (4) (0 mod 4, 3 mod 6) is in the Image of f
31.	Let $f(x, y) = \frac{x^2}{y^2}$, $(x, y) \in \left[\frac{1}{2}, \frac{3}{2}\right] \times \left[\frac{1}{2}, \frac{3}{2}\right]$. Then the derivative of f at (1, 1) along the direction (1, 1) is (1) 1 (2) 2 (3) 0 (4) −2

Question No.	Questions
32.	Which of the following statement is not correct (1) if F is closed and K is compact, then $F \cap K$ is compact (2) if $\{K_n\}$ is a sequence of nonempty compact sets s.t. $K_{n+1} \subset K_n$ $(n = 1, 2, ...)$, then $\bigcap_{n=1}^{\infty} K_n$ is empty (3) Every closed subset of a compact set is compact (4) The set $\{x : x \in \mathbb{R} \text{ and } x(x^2 - 6x + 8) = 0\}$ is compact
33.	Let $X = \{x : x = (x_1, x_2, x_3), x_i \in \mathbb{R}, x_1^2 + x_2^2 + x_3^2 < 16\}$. Then (1) X has no limit point in $\mathbb{R}^3$ (2) X has a limit point in $\mathbb{R}^3$ (3) X is compact (4) All of the above statement are true
34.	Let A, B be $n \times n$ matrices. Then trace of $A^2 B^2$ is equal to (1) trace $((AB)^2)$ (2) $(\text{trace}(AB))^2$ (3) trace (AB^2A) (4) trace (BABA)
35.	Let f, g, h be the functions from $\mathbb{R}^3$ to $\mathbb{R}^2$ such that $f\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}4\\x+y\end{pmatrix}, g\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}xy\\x+y\end{pmatrix}, h\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}z-x\\x+y\end{pmatrix}; \begin{pmatrix}x\\y\\z\end{pmatrix} \in \mathbb{R}^3$ Then (1) only f is a linear transformation (2) only g is a linear transformation (3) only h is a linear transformation (4) f and g are linear transformations but h is not a linear transformation

Question No.	Questions
36.	Let S denote the set of all the prime numbers p such that the matrix $\begin{bmatrix} 91 & 31 & 0 \\ 29 & 31 & 0 \\ 79 & 23 & 59 \end{bmatrix}$ has an inverse in the field Z/pZ. Then (1) $S = \{31\}$ (2) $\{31, 59\}$ (3) $S = \{7, 13, 59\}$ (4) S is infinite
37.	Let V be the vector space of all real polynomials of degree ≤ 10. Let $TP(x) = P'(x)$, for $P \in V$, be a linear transformation from V to V. Consider the basis $\{1, x, x^2, \ldots\ldots, x^{10}\}$ of V. Let A be the matrix of T with respect to this basis. Then (1) trace (A) = 1 (2) There is no $n \in \mathbb{N}$ s.t. $A^n = 0$ (3) A has non-zero eigen value (4) det (A) = 0
38.	Given a matrix $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, $\theta = \frac{2\pi}{31}$. Then A^{2015} is equal to (1) I (2) A (3) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ (4) $\begin{pmatrix} \cos 13\theta & \sin 13\theta \\ -\sin 13\theta & \cos 13\theta \end{pmatrix}$

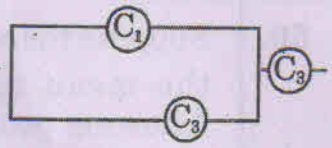
Question No.	Questions
39.	Given a matrix $\begin{pmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{pmatrix}$. Then this form is (1) negative definite (2) positive definite (3) non-negative definite but not positive definite (4) neither negative definite nor positive definite
40.	Let W_1 and W_2 be subspaces of $\mathbb{R}^3$ given by $W_1 = \{(x, y, z) \in \mathbb{R}^3 / x + y + z = 0\}$ $W_2 = \{(x, y, z) \in \mathbb{R}^3 / x - y + z = 0\}$ If W is a subspace of $\mathbb{R}^3$ such that (i) $W \cap W_2$ = space {0, 1, 1} (ii) $W \cap W_1$ = orthogonal to $W \cap W_2$ with respect to the usual inner product of $\mathbb{R}^3$. Then (1) W = Span {(0,1,–1), (0,1,1)} (2) W = Span {(1,0,–1), (0,1,–1)} (3) W = Span {(1,0,–1), (0,1,1)} (4) W = Span {(1,0,–1), (1,0,1)}

Question No.	Questions
41.	Let $\underline{X} \sim N_3(\underline{\mu}, \Sigma)$, where $\underline{\mu} = (1, 1, 1)$ and $\Sigma = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 3 & C \\ 1 & C & 2 \end{pmatrix}$. The value of C such that X_2 and $-X_1 + X_2 - X_3$ are independent is (1) 0 (2) -2 (3) 2 (4) 1
42.	Assume that $X \sim$ Binomial (n, p) for some $n \geq 1$ and $0 < p < 1$ and $Y \sim$ Poisson (λ) for some $\lambda > 0$. Suppose $\Sigma[X] = \Sigma[Y]$. Then (1) Var (X) = Var (Y) (2) Var (X) < Var (Y) (3) Var (Y) < Var (X) (4) Var (X) may be larger or smaller than Var (Y)
43.	Let X, Y be independent random variables and let $Z = \frac{X-Y}{2} + 3$ If X has characteristic function ϕ and Y has characteristic function ψ, then Z has characteristic function θ where (1) $\theta(t) = e^{-i3t} \phi(2t) \psi(-2t)$ (2) $\theta(t) = e^{i3t} \phi\left(\frac{t}{2}\right) \psi\left(-\frac{t}{2}\right)$ (3) $\theta(t) = e^{-i3t} \phi\left(\frac{t}{2}\right) \psi\left(\frac{t}{2}\right)$ (4) $\theta(t) = e^{-i3t} \phi\left(\frac{t}{2}\right) \psi\left(\frac{-t}{2}\right)$

Question No.	Questions
44.	Consider LPP : Minimize : $z = -2x - 5y$ subject to : $3x + 4y \geq 5,\ x \geq 0,\ y \geq 0$. Which of the following is correct ? (1) Set of feasible solutions is empty (2) Set of feasible solutions is non-empty but there is no optimal solution (3) Optimal value is obtained at $\left(0, \frac{5}{4}\right)$ (4) Optimal value is attained at $\left(\frac{5}{3}, 0\right)$
45.	(X, Y) follows the bivariate normal distribution N_2 (0, 0, 1, 1, ρ), $-1 < \rho < 1$. Then (1) $X + Y$ and $X - Y$ are uncorrelated only if $\rho = 0$ (2) $X + Y$ and $X - Y$ are uncorrelated only if $\rho < 0$ (3) $X + Y$ and $X - Y$ are uncorrelated only if $\rho > 0$ (4) $X + Y$ and $X - Y$ are uncorrelated for all values ρ
46.	Let (X_1, X_2, X_3, X_4) be an optimal solution to the problem of minimizing $X_1 + X_2 + X_3 + X_4$ subject to the constraints $X_1 + X_2 \geq 300$, $X_2 + X_3 \geq 500$, $X_3 + X_4 \geq 400$, $X_4 + X_1 \geq 200$, $X_1 \geq 0$, $X_2 \geq 0$, $X_3 \geq 0$, $X_4 \geq 0$. Which of the following is a possible value for any X_i ? (1) 300 (2) 400 (3) 500 (4) 600

Question No.	Questions

47. A system consists of 3 components arranged as –



Each of the components C_1, C_2, C_3 has independent and identically distributed (i.i.d) lifetimes whose distribution is exponential with mean 1. Then the survival function, S (t), of the system is given by

(1) $S(t) = e^{-3t}, t > 0$ (2) $S(t) = (1 - e^{-t})^2 e^{-t}, t > 0$

(3) $S(t) = (1 - e^{-2t}) e^{-t}, t > 0$ (4) $S(t) = (1 - (1 - e^{-t})^2) e^{-t}, t > 0$

48. Let $X_1, X_2, \ldots, X_7$ be a random sample from $N(\mu, \sigma^2)$ where μ and σ^2 are unknown. Consider the problem of testing $H_0 : \mu = 2$ against $H_1 : \mu > 2$. Suppose the observed values of $x_1, x_2, \ldots, x_7$ are 1.2, 1.3, 1.7, 1.8, 2.1, 2.3, 2.7. If we use the uniformly most powerful test, which of the following is true ?

(1) H_0 is accepted both at 5% and 1% levels of significance

(2) H_0 is rejected both at 5% and 1% levels of significance

(3) H_0 is rejected at 5% level of significance, but accepted at 1% level of significance

(4) H_0 is rejected at 1% level of significance, but accepted at 5% level of significance

49. Let N_t denote the number of accidents up to time t. Assume that $\{N_t\}$ is a Poisson process with intensity 2. Given that there are exactly 5 accidents during the time period $[20, 30]$. What is the conditional probability that there is exactly one accident dusing the time period $[15, 25]$?

(1) $\frac{15}{32} e^{-20}$ (2) $\frac{15}{32} e^{-10}$ (3) $20 e^{-20}$ (4) $\frac{1}{5}$

Question No.	Questions
50.	Suppose there are K groups each consisting of N boys. We want to estimate the mean age μ of these μN boys. Fix $1 < n < N$ and consider the following two sampling schemes : I. Draw a simple random sample without replacement of size kn out of KN boys II. From each of the K groups, draw a simple random sample with replacement of size n. Let $\overline{Y}$ and $\overline{Y}_G$ be the respective sample mean ages for the two schemes. Which of the following is not true ? (1) $E(\overline{Y}) = \mu$ (2) $E(\overline{Y}_G) = \mu$ (3) Var $(\overline{Y})$ may be less than Var $(\overline{Y}_G)$ in some cases (4) Var $(\overline{Y})$ = Var $(\overline{Y}_G)$ if all the group means are same.
51.	If f (x) and g (x) are two solutions of $\cos x \frac{d^2y}{dx_2} + \sin x \frac{dy}{dx} - \left(1 + e^{-x^2}\right) y = 0, \; x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ such that $f(0) = \sqrt{2}, \; f'(0) = 1; \; g(0) = -\sqrt{2}, \; g'(0) = 2.$ Then Wronskian of f and g is (1) $3 \cos x$ (2) $3\sqrt{2} \cos x - x$ (3) $3\sqrt{2} \cos x$ (4) 0
52.	The critical point (0, 0) of the system $\frac{dx}{dt} = 2x - 7y, \; \frac{dy}{dt} = 3x - 8y$ is (1) unstable node (2) asymptotically stable node (3) unstable spiral point (4) asymptotically stable spiral point

Question No.	Questions
53.	Given a boundary value problem $y''(x)+\lambda\, y(x)=0;\ y(0)=0,\ y(\pi)=0$ Set of its eigen values is (1) $\mathbb{R}$ (2) $(-\infty, 0)$ (3) $\{n^2 : n \in Z^+\}$ (4) $\{\sqrt{n} : n \in Z^+\}$
54.	The limiting value of y (x), as $x \to \infty$, where y (x) is the solution of $y'(x) = ay - by^2$; a, $b > 0$, $y(0) = y_0$, will be (1) 0 (2) a/b (3) b/a (4) y_0
55.	Given a differential equation $x''(t)+p(t)\,x'(t)+q(t)\,x(t)=0;\ p(t), q(t) \in C^1[a, b]$. Let f (t) and g (t) be its two solutions on [a, b]. Then which of the following is incorrect ? (1) f and g are linearly dependent and $W(f, g)(t) = 0\ \forall t \in [a, b]$ (2) f and g are linearly in dependent and $\exists\ t_0 \in (a, b)$ s. t. $W(t_0) = 0$ (3) f and g are linearly in dependent and $W(f, g)(t) \neq 0$ for any $t \in [a, b]$ (4) f and g are linearly independent then every other solution can be written as their linear combination
56.	Let D denote the disc $\{(x, y) \mid x^2 + y^2 \leq 1\}$ and let D^c be its complement in the plane. The P.D.E $(x^2-1)\dfrac{\partial^2 u}{dx^2}+2y\dfrac{\partial^2 u}{dxdy}-\dfrac{\partial^2 u}{dy^2}=0$ is (1) Parabolic $\forall (x, y) \in D^c$ (2) Hyperbolic $\forall (x, y) \in D$ (3) Hyperbolic $\forall (x, y) \in D^c$ (4) Parabolic $\forall (x, y) \in D$

Question No.	Questions
57.	Given a partial differential equation $x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = (x+y)\,z$. Then which of the following is not the general solution of the differential equation ? (1) $F\left(\frac{xy}{z}, \frac{x-y}{z}\right) = 0$ for an arbitrary twice differentiable function F (2) $F\left(\frac{x-y}{z}, \frac{1}{x}-\frac{1}{y}\right) = 0$ for an arbitrary twice differentiable function F (3) $z = f\left(\frac{1}{x}-\frac{1}{y}\right)$ for an arbitrary twice differentiable function f (4) $z = xyf\left(\frac{1}{x}-\frac{1}{y}\right)$ for an arbitrary twice differentiable function f
58.	The Cauchy problem $2\,u_x + 3\,u_y = 5$, $u = 1$ on the line $3x - 2y = 0$ has (1) exactly one solution (2) exactly two solutions (3) infinitely many solutions (4) no solution
59.	Let u be the unique solution of $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, where $(x, t) \in (0,1) \text{ X } (0, \infty)$ $u(x,0) = \sin \pi x,\ x \in (0,1)$ $u(0, t) = u(1, t) = \theta,\ t \in (0, \infty)$ Then which of the following is true ? (1) $\exists\, (x, t) \in (0,1) \text{ X } (0, \infty)$ s.t. $u(x, t) = 0$ (2) $\exists\, (x, t) \in (0,1) \text{ X } (0, \infty)$ s.t. $\frac{\partial u}{\partial t}(x, t) = 0$ (3) the function $e^t u(x, t)$ is bounded for $(x, t) \in (0,1) \text{ X } (0, \infty)$ (4) $\exists\, (x, t) \in (0,1) \text{ X } (0, \infty)$ s.t. $u(x, t) > 1$

Question No.	Questions
60.	Let u (x, t) be the solution of the initial value problem $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$; $u(x, 0) = x^3$, $\frac{\partial u}{\partial t}(x, 0) = \sin x$ Then $u(\pi, \pi)$ is (1) $4\pi^3$ (2) $2\pi^3$ (3) 0 (4) 4
61.	Let $\psi(t) = e^{-\lvert t \rvert - \frac{t^2}{2}}$ and $\phi(t) = \frac{e^{-\lvert t \rvert} + e^{\frac{-t^2}{2}}}{2}$. Which of the following is true ? (1) ϕ is a characteristic function but ψ is not (2) ψ is a characteristic function but ϕ is not (3) neither ϕ nor ψ is a characteristic function (4) Both ϕ and ψ are characteristic functions
62.	If $b_{xy} = 0.20$ and $r_{xy} = 0.50$, Then b_{yx} is equal to (1) 0.20 (2) 0.25 (3) 0.50 (4) 1.25
63.	Let $X_1, X_2, ..., X_n$ be independent random variables ; X_i having exponential distribution with parameter θ_i, $i = 1, 2, ..., n$. Then $Z = \text{Min}(X_1, X_2, ..., X_n)$ has (1) normal distribution (2) geometric distribution (3) exponential distribution with parameter $\sum_{i=1}^{n} \theta_i$ (4) None of these

Question No.	Questions
64.	In queueing description M|M| 1, the arrival and departure distribution are (1) Binomial (2) General (3) Both Markovian (4) None of these
65.	Successful life of product, time, weight and height are classified as (1) continuous random variable (2) discrete random variable (3) continuous time variable (4) None of these
66.	If a random variable X has a Chi-Square distribution with 4 degree of freedom, then its mean is equal to (1) 2 (2) 3 (3) 4 (4) None of these
67.	In a Latin Square Design, if factors A, B, C and D have levels 8, then the total number of cells in the design is (1) 4096 (2) 64 (3) 512 (4) None of these
68.	If a system has two components in parallel with each of reliability 0.75, then the reliability of the system is equal to (1) 0.9375 (2) 0.9753 (3) 0.7935 (4) None of these

Question No.	Questions
69.	Let (v, b, r, k, w) be the standard parameters of a balanced incomplete block design (BIBD). Which of the following (v, b, r, k, w) can be parameters of BIBD ? (1) (v, b, r, k,w) = (44, 33, 9, 12, 3) (2) (v, b, r, k,w) = (17, 45, 8, 3, 1) (3) (v, b, r, k,w) = (35, 35, 17, 17, 9) (4) (v, b, r, k,w) = (16, 24, 9, 6, 3)
70.	100 tickets are marked 1, 2, ... , 100 and arranged at random. Four tickets are picked from these tickets and given to four persons A, B, C and D. What is the probability that A gets the ticket with the largest value (among A, B, C, D) and D gets the ticket with smallest value (among A, B, C, D) ? (1) $\frac{1}{4}$ (2) $\frac{1}{12}$ (3) $\frac{1}{2}$ (4) $\frac{1}{6}$
71.	Let S_7 denote the group of permutations of the set {1, 2, 3, 4, 5, 6,7}. Which of the following is true ? (1) There are no elements of order 10 in S_7 (2) There are no elements of order 8 in S_7 (3) There are no elements of order 7 in S_7 (4) There are no elements of order 6 in S_7

Question No.	Questions
72.	The number of group homomorphism from $\mathbb{Z}_{10}$ to $\mathbb{Z}_{20}$ is (1) 0 (2) 1 (3) 5 (4) 10
73.	Let Aut (G) denote the group of automorphisms of G for a group G. Which of the following is true ? (1) If G is cyclic, then Aut (G) is cyclic (2) If G is finite, then Aut (G) is finite (3) If G is infinite, then Aut (G) is infinite (4) If Aut (G) is isomorphic to Aut (H), where G and H are two groups, then G is isomorphic to H.
74.	Let $f(x) = x^7 - 105x + 12$. Then which of the following is correct ? (1) f (x) is reducible over Q (2) $\exists$ an integer m s.t. f (m) = 2 (3) $\exists$ an integer m s.t. f (m) = 105 (4) f (m) is not a prime number for any integer m.
75.	The degree of splitting field of $x^4 - 1$ ove $\mathbb{Q}$ is (1) 2 (2) 4 (3) 1 (4) 0
76.	Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous map. Which of the following is a correct statement ? (1) f is bounded (2) Image of f is an open subset of $\mathbb{R}$ (3) f (A) is bounded for all bounded subsets A of $\mathbb{R}$ (4) f^{-1} (A) is compact for all compact subsets A of $\mathbb{R}$

Question No.	Questions
77.	Let A and B be topological spaces where B is Hausdorff. Let $A \times B$ be the given product topology. Then for a function $f : A \to B$ which of the following statements is necessarily true ? (1) if B is finite, then f is continuous (2) if f is continuous, then graph $(f) = \{(x, f(x)) / x \in A\}$ is closed in $A \times B$ (3) if group (f) is closed in $A \times B$, then f is continuous (4) if group (f) is closed in $A \times B$, then f is continuous and bounded
78.	Let A be a subset of $\mathbb{R}$ and $A = \bigcap_{n \geq 1} V_n$, where for each $n \geq 1, V_n$ is an open dense subset of $\mathbb{R}$. Which of the following are incorrect ? (1) A is countable (2) A is uncountable (3) A is dense in $\mathbb{R}$ (4) A is a non-empty set
79.	Let X be a topological space and let U be a proper dense open subset of X. Choose the correct statement : (1) If X is connected, then U is connected (2) If X is compact, then U is compact (3) If X is compact, then X\U is compact (4) If X\U is compact, then X is compact
80.	Let G be an open set in $\mathbb{R}^n$. Two points $x, y \in G$ are defined to be equivalent if they can be joined by a continuous path completely lying inside G. Then number of equivalent classes is (1) only one (2) at most countable (3) at most finite (4) can be finite or uncountable

Question No.	Questions
81.	Which of the following matrices is not diagonalizable over $\mathbb{R}$? (1) $\begin{pmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ (2) $\begin{pmatrix} 2 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ (3) $\begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix}$ (4) $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$
82.	The rank of the matrix $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 & 3 \\ 1 & 2 & 3 & 4 & 4 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}$ is (1) 2 (2) 3 (3) 4 (4) 5
83.	Let $A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 2 & 3 & \alpha \end{pmatrix}$, $b = \begin{pmatrix} 1 \\ 3 \\ \beta \end{pmatrix}$ Then the system $AX = b$ over the field of real numbers has (1) no solution wherever $\beta \neq 7$ (2) an infinite number of solutions if $\alpha = 2$ and $\beta \neq 7$ (3) unique solution if $\alpha \neq 2$ (4) an infinite number of solutions whenever $\alpha \neq 2$

Question No.	Questions
84.	Which of the following subset of $\mathbb{R}^4$ is basis of $\mathbb{R}^4$? $B_1 = \{(1,0,0,0), (1,1,0,0), (1,1,1,0), (1,1,1,1)\}$ $B_2 = \{(1,0,0,0), (1,2,0,0), (1,2,3,0), (1,2,3,4)\}$ $B_3 \{(1,2,0,0), (0,0,1,1), (2,1,0,0), (-5,5,0,0)\}$ (1) B_1 and B_3 but not B_2 (2) B_1, B_2 and B_3 (3) only B_1 (4) B_1 and B_2 but not B_3
85.	Let A be an invertible real n×n matrix. Let $F: \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ defined by $F(x, y) = (Ax, y)$, where (x, y) denote the inner product of x and y. Let DF (x, y) denote the derivative of F at (x, y) which is a linear transformation from $\mathbb{R}^n \times \mathbb{R}^n \to R$. The which of the following is incorrect ? (1) if $x \neq 0$, then $DF(x, 0) \neq 0$ (2) if $x \neq 0$, then $DF(0, y) \neq 0$ (3) if $x = 0$ or $y = 0$, then $DF(x, y) = 0$ (4) if $(x, y) \neq (0, 0)$, then $DF(x, y) \neq 0$
86.	Let P (n) be a polynomial of degree $d \geq 2$. The radius of convergence of the power series $\sum_{n=0}^{\infty} P(n) z^n$ is (1) 0 (2) 1 (3) ∞ (4) depends upon d
87.	The residue of the function $f(z) = e^{-e^{\frac{1}{z}}}$ at $z = 0$ is (1) $1 + e^{-1}$ (2) e^{-1} (3) $-e^{-1}m$ (4) $1 - e^{-1}$

Question No.	Questions
88.	Let f be a holomorphic function on $0<\|z\|<\varepsilon$, $\varepsilon>0$, given by a convergent Laurent series $\sum_{n=-\infty}^{\infty} a_n z^n$. Also given that $\lim_{z\to 0} \|f(z)\| = \infty$. We can conclude that (1) $a_{-1}\neq 0$ and $a_{-n}=0$ $\forall n\geq 2$ (2) $a_{-n}=0$ $\forall n\geq 1$ (3) $a_{-n}\neq 0$ $\forall n\geq 1$ (4) $a_{-m}\neq 0$ for some $m\geq 1$ and $a_{-n}=0$ $\forall n>m$
89.	Let C denote the unit circle centred at the origin in Argand's plane. The value of the integral $\frac{1}{2\pi i}\int_C \|1+z+z^2\|^2 dz$, when integral in taken anti-clockwise along C, equals (1) 1 (2) 2 (3) 0 (4) 3
90.	Let $f:\mathbb{C}\to\mathbb{C}$ be a non-constant entire function and let Image (f) = $\{w\in\mathbb{C} : z\in\mathbb{C} \text{ s.t. } f(z)=w\}$. Then (1) The interior of image (f) is empty (2) Image (f) intersects every line passing through the origin (3) There exists a disc in complex plane which is disjoint from image (f) (4) Image (f) contains all its limit points

Question No.	Questions
91.	Completeness of a metric space is preserved under (1) Isometry (2) Homeomorphism (3) Continuous function (4) Bijective function
92.	Given an interval (– 1, 1) and a sequence $\{a_n\}$ of elements in it. Then (1) Every limit point of $\{a_n\}$ is an (–1, 1) (2) The limit points of $\{a_n\}$ can only be in {–1, 0, 1} (3) Every limit point of $\{a_n\}$ is in [–1, 1] (4) The limit point of $\{a_n\}$ cannot be in {–1, 0, 1}
93.	If f is a function $f:\mathbb{R}\to\mathbb{R}$ s.t. $f(0)=0$ and $\lvert f'(x)\rvert \le \forall x$, then f (1) is in (1) (5, 6) (2) [– 4, 4] (3) $(-\infty, -5) \cup (5, \infty,)$ (4) [– 5, 5]
94.	Let $A=\{n \in \mathbb{N} : n=1 \text{ or the only prime factors of n are 2 or 3}\}$. Let $S=\sum_{n\in A}\frac{1}{n}$, then (1) S is divergent series (2) A is finite (3) S = 3 (4) S = 6
95.	Let $x_n(t)=te^{-nt^2}$, $t\in\mathbb{R}$, $n\ge 1$. Then the sequence $\{x_n\}$ is (1) uniformly convergent on $\mathbb{R}$ (2) a sequence of unbounded functions (3) bounded and not uniformly convergent on $\mathbb{R}$ (4) uniformly convergent only on compact subsets of $\mathbb{R}$

Question No.	Questions
96.	Which of the following is necessarily true for a function $f : A \to B$ (1) if f as injective, then $\exists\ g : B \to A$ s.t. $f(g(y)) = y\ \forall y \in B$ (2) if f is injective and B is countable, then A is finite (3) if f is subjective and A is countable, then B is countably infinite (4) if f is subjective, then $\exists\ g : B \to A$ s.t. $f(g(y)) = y\ \forall y \in B$
97.	The difference $\log(2) - \sum_{n=1}^{100} \frac{1}{2^n \cdot n}$ is (1) less than 0 (2) less than $\frac{1}{2^{100}.101}$ (3) greater than 1 (4) greater than $\frac{1}{2^{100}.101}$
98.	Let f be function defined on the set $S = \left\{x \in \mathbb{R},\ x \geq 0,\ x \neq n\pi + \frac{\pi}{2},\ n \in \mathbb{N} \cup \{0\}\right\}$ and f (x) = tan x. Then (1) f has a unique fixed point on S (2) there is no fixed point of f on S (3) f has infinitely many fixed points on S (4) f has finite number of fixed points on S
99.	A function $f : \mathbb{R} \to \mathbb{R}$ need not be lebesgue measurable if (1) $\{x \in \mathbb{R} : f(x) \geq \alpha\}$ is measurable for each $\alpha \in \mathbb{Q}$ (2) $\{x \in \mathbb{R} : f(x) = \alpha\}$ is measurable for each $\alpha \in \mathbb{R}$ (3) for each open set G in $\mathbb{R}$, $f^{-1}(G)$ is measurable (4) f is monotone
100.	Let $f(x) = \left(1 + \frac{1}{x}\right)^x$ and $g(x) = \left(1 + \frac{1}{x}\right)^{x+1}$, $x > 0$. then (1) f (x) and g (x) both are increasing functions (2) f (x) is increasing and g (x) is decreasing (3) f (x) is decreasing and g (x) is increasing (4) f (x) and g (x) both are decreasing functions

(DO NOT OPEN THIS QUESTION BOOKLET BEFORE TIME OR UNTIL YOU ARE ASKED TO DO SO)

(MPH/PHD/URS–EE–2019)

Subject : MATHEMATICS

Code **C**

Sr. No. 10271

SET–"X"

Time : 1¼ Hours | Total Questions : 100 | Max. Marks : 100

Roll No. ______________ (in figure) ______________________ (in words)

Name : ______________________ Father's Name : ______________________

Mother's Name : ______________________ Date of Examination : ______________

(Signature of the candidate) (Signature of the Invigilator)

CANDIDATES MUST READ THE FOLLOWING INFORMATION/ INSTRUCTIONS BEFORE STARTING THE QUESTION PAPER.

1. **Candidates are required to attempt any 75 questions out of the given 100 multiple choice questions of 4/3 marks each. No credit will be given for more than 75 correct responses.**
2. The candidates must return the Question book-let as well as OMR answer-sheet to the Invigilator concerned before leaving the Examination Hall, failing which a case of use of unfair-means / mis-behaviour will be registered against him / her, in addition to lodging of an FIR with the police. Further the answer-sheet of such a candidate will not be evaluated.
3. Keeping in view the transparency of the examination system, carbonless OMR Sheet is provided to the candidate so that a copy of OMR Sheet may be kept by the candidate.
4. Question Booklet along-with answer key of all the A,B,C and D code shall be got uploaded on the university website after the conduct of Entrance Examination. In case there is any discrepancy in the Question Booklet/Answer Key, the same may be brought to the notice of the Controller of Examination in writing/through E. Mail within 24 hours of uploading the same on the University Website. Thereafter, no complaint in any case, will be considered.
5. The candidate MUST NOT do any rough work or writing in the OMR Answer-Sheet. Rough work, if any, may be done in the question book-let itself. Answers **MUST NOT** be ticked in the Question book-let.
6. **There will be no negative marking. Each correct answer will be awarded 4/3 mark. Cutting, erasing, overwriting and more than one answer in OMR Answer-Sheet will be treated as incorrect answer.**
7. Use only Black or Blue **BALL POINT PEN** of good quality in the OMR Answer-Sheet.
8. BEFORE ANSWERING THE QUESTIONS, THE CANDIDATES SHOULD ENSURE THAT THEY HAVE BEEN SUPPLIED CORRECT AND COMPLETE BOOK-LET. COMPLAINTS, IF ANY, REGARDING MISPRINTING ETC. WILL NOT BE ENTERTAINED 30 MINUTES AFTER STARTING OF THE EXAMINATION.

Question No.	Questions
1.	Let S_7 denote the group of permutations of the set {1, 2, 3, 4, 5, 6,7}. Which of the following is true ? (1) There are no elements of order 10 in S_7 (2) There are no elements of order 8 in S_7 (3) There are no elements of order 7 in S_7 (4) There are no elements of order 6 in S_7
2.	The number of group homomorphism from $\mathbb{Z}_{10}$ to $\mathbb{Z}_{20}$ is (1) 0 (2) 1 (3) 5 (4) 10
3.	Let Aut (G) denote the group of automorphisms of G for a group G. Which of the following is true ? (1) If G is cyclic, then Aut (G) is cyclic (2) If G is finite, then Aut (G) is finite (3) If G is infinite, then Aut (G) is infinite (4) If Aut (G) is isomorphic to Aut (H), where G and H are two groups, then G is isomorphic to H.
4.	Let $f(x) = x^7 - 105x + 12$. Then which of the following is correct ? (1) f (x) is reducible over Q (2) $\exists$ an integer m s.t. f (m) = 2 (3) $\exists$ an integer m s.t. f (m) = 105 (4) f (m) is not a prime number for any integer m.

Question No.	Questions
5.	The degree of splitting field of $x^4 - 1$ ove $\mathbb{Q}$ is (1) 2 (2) 4 (3) 1 (4) 0
6.	Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous map. Which of the following is a correct statement ? (1) f is bounded (2) Image of f is an open subset of $\mathbb{R}$ (3) f (A) is bounded for all bounded subsets A of $\mathbb{R}$ (4) f^{-1} (A) is compact for all compact subsets A of $\mathbb{R}$
7.	Let A and B be topological spaces where B is Hausdorff. Let $A \times B$ be the given product topology. Then for a function $f : A \to B$ which of the following statements is necessarily true ? (1) if B is finite, then f is continuous (2) if f is continuous, then graph $(f) = \{(x, f(x)) / x \in A\}$ is closed in $A \times B$ (3) if group (f) is closed in $A \times B$, then f is continuous (4) if group (f) is closed in $A \times B$, then f is continuous and bounded
8.	Let A be a subset of $\mathbb{R}$ and $A = \bigcap_{n\geq1} V_n$, where for each $n \geq 1, V_n$ is an open dense subset of $\mathbb{R}$. Which of the following are incorrect ? (1) A is countable (2) A is uncountable (3) A is dense in $\mathbb{R}$ (4) A is a non-empty set

Question No.	Questions
9.	Let X be a topological space and let U be a proper dense open subset of X. Choose the correct statement : (1) If X is connected, then U is connected (2) If X is compact, then U is compact (3) If X is compact, then X\U is compact (4) If X\U is compact, then X is compact
10.	Let G be an open set in $\mathbb{R}^n$. Two points $x, y \in G$ are defined to be equivalent if they can be joined by a continuous path completely lying inside G. Then number of equivalent classes is (1) only one (2) at most countable (3) at most finite (4) can be finite or uncountable
11.	Which of the following matrices is not diagonalizable over $\mathbb{R}$? (1) $\begin{pmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ (2) $\begin{pmatrix} 2 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ (3) $\begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix}$ (4) $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$
12.	The rank of the matrix $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 & 3 \\ 1 & 2 & 3 & 4 & 4 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}$ is (1) 2 (2) 3 (3) 4 (4) 5

Question No.	Questions
13.	Let $A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 2 & 3 & \alpha \end{pmatrix}$, $b = \begin{pmatrix} 1 \\ 3 \\ \beta \end{pmatrix}$ Then the system $AX = b$ over the field of real numbers has (1) no solution wherever $\beta \neq 7$ (2) an infinite number of solutions if $\alpha = 2$ and $\beta \neq 7$ (3) unique solution if $\alpha \neq 2$ (4) an infinite number of solutions whenever $\alpha \neq 2$
14.	Which of the following subset of $\mathbb{R}^4$ is basis of $\mathbb{R}^4$? $B_1 = \{(1,0,0,0), (1,1,0,0), (1,1,1,0), (1,1,1,1)\}$ $B_2 = \{(1,0,0,0), (1,2,0,0), (1,2,3,0), (1,2,3,4)\}$ $B_3\ \{(1,2,0,0), (0,0,1,1), (2,1,0,0), (-5,5,0,0)\}$ (1) B_1 and B_3 but not B_2 (2) B_1, B_2 and B_3 (3) only B_1 (4) B_1 and B_2 but not B_3
15.	Let A be an invertible real n×n matrix. Let $F : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ defined by $F(x, y) = (Ax, y)$, where (x, y) denote the inner product of x and y. Let $DF(x, y)$ denote the derivative of F at (x, y) which is a linear transformation from $\mathbb{R}^n \times \mathbb{R}^n \to R$. The which of the following is incorrect ? (1) if $x \neq 0$, then $DF(x, 0) \neq 0$ (2) if $x \neq 0$, then $DF(0, y) \neq 0$ (3) if $x = 0$ or $y = 0$, then $DF(x, y) = 0$ (4) if $(x, y) \neq (0, 0)$, then $DF(x, y) \neq 0$

Question No.	Questions
16.	Let P (n) be a polynomial of degree $d \geq 2$. The radius of convergence of the power series $\sum_{n=0}^{\infty} P(n) z^n$ is (1) 0 (2) 1 (3) ∞ (4) depends upon d
17.	The residue of the function $f(z) = e^{-e^{\frac{1}{z}}}$ at $z = 0$ is (1) $1+e^{-1}$ (2) e^{-1} (3) $-e^{-1}$ (4) $1-e^{-1}$
18.	Let f be a holomorphic function on $0 < \|z\| < \varepsilon$, $\varepsilon > 0$, given by a convergent Laurent series $\sum_{n=-\infty}^{\infty} a_n z^n$. Also given that $\lim_{z \to 0} \|f(z)\| = \infty$. We can conclude that (1) $a_{-1} \neq 0$ and $a_{-n} = 0 \;\forall n \geq 2$ (2) $a_{-n} = 0 \;\forall n \geq 1$ (3) $a_{-n} \neq 0 \;\forall n \geq 1$ (4) $a_{-m} \neq 0$ for some $m \geq 1$ and $a_{-n} = 0 \;\forall n > m$
19.	Let C denote the unit circle centred at the origin in Argand's plane. The value of the integral $\frac{1}{2\pi i} \int_C \|1+z+z^2\|^2 dz$, when integral in taken anti-clockwise along C, equals (1) 1 (2) 2 (3) 0 (4) 3

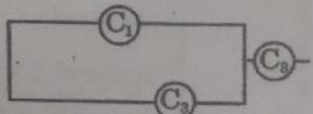
Question No.	Questions
16.	Let P (n) be a polynomial of degree $d \geq 2$. The radius of convergence of the power series $\sum_{n=0}^{\infty} P(n) z^n$ is (1) 0 (2) 1 (3) ∞ (4) depends upon d
17.	The residue of the function $f(z) = e^{-e^{\frac{1}{z}}}$ at $z = 0$ is (1) $1+e^{-1}$ (2) e^{-1} (3) $-e^{-1}$ (4) $1-e^{-1}$
18.	Let f be a holomorphic function on $0 < \|z\| < \varepsilon$, $\varepsilon > 0$, given by a convergent Laurent series $\sum_{n=-\infty}^{\infty} a_n z^n$. Also given that $\lim_{z \to 0} \|f(z)\| = \infty$. We can conclude that (1) $a_{-1} \neq 0$ and $a_{-n} = 0 \ \forall n \geq 2$ (2) $a_{-n} = 0 \ \forall n \geq 1$ (3) $a_{-n} \neq 0 \ \forall n \geq 1$ (4) $a_{-m} \neq 0$ for some $m \geq 1$ and $a_{-n} = 0 \ \forall n > m$
19.	Let C denote the unit circle centred at the origin in Argand's plane. The value of the integral $\frac{1}{2\pi i} \int_C \|1+z+z^2\|^2 dz$, when integral in taken anti-clockwise along C, equals (1) 1 (2) 2 (3) 0 (4) 3

Question No.	Questions
20.	Let $f : \mathbb{C} \to \mathbb{C}$ be a non-constant entire function and let Image (f) = $\{w \in \mathbb{C} : z \in \mathbb{C}$ s.t. $f(z) = w\}$. Then (1) The interior of image (f) is empty (2) Image (f) intersects every line passing through the origin (3) There exists a disc in complex plane which is disjoint from image (f) (4) Image (f) contains all its limit points
21.	Completeness of a metric space is preserved under (1) Isometry (2) Homeomorphism (3) Continuous function (4) Bijective function
22.	Given an interval (– 1, 1) and a sequence $\{a_n\}$ of elements in it. Then (1) Every limit point of $\{a_n\}$ is an (–1, 1) (2) The limit points of $\{a_n\}$ can only be in {–1, 0, 1} (3) Every limit point of $\{a_n\}$ is in [–1, 1] (4) The limit point of $\{a_n\}$ cannot be in {–1, 0, 1}
23.	If f is a function $f : \mathbb{R} \to \mathbb{R}$ s.t. $f(0) = 0$ and $\left|f'(x)\right| \leq \forall x$, then f (1) is in (1) (5, 6) (2) [– 4, 4] (3) $(-\infty, -5) \cup (5, \infty,)$ (4) [– 5, 5]

Question No.	Questions
24.	Let $A = \{n \in \mathbb{N} : n = 1$ or the only prime factors of n are 2 or 3$\}$. Let $S = \sum_{n \in A} \frac{1}{n}$, then (1) S is divergent series (2) A is finite (3) $S = 3$ (4) $S = 6$
25.	Let $x_n(t) = te^{-nt^2}$, $t \in \mathbb{R}$, $n \geq 1$. Then the sequence $\{x_n\}$ is (1) uniformly convergent on $\mathbb{R}$ (2) a sequence of unbounded functions (3) bounded and not uniformly convergent on $\mathbb{R}$ (4) uniformly convergent only on compact subsets of $\mathbb{R}$
26.	Which of the following is necessarily true for a function $f : A \to B$ (1) if f as injective, then $\exists\, g : B \to A$ s.t. $f(g(y)) = y\ \forall y \in B$ (2) if f is injective and B is countable, then A is finite (3) if f is subjective and A is countable, then B is countably infinite (4) if f is subjective, then $\exists\, g : B \to A$ s.t. $f(g(y)) = y\ \forall y \in B$
27.	The difference $\log(2) - \sum_{n=1}^{100} \frac{1}{2^n \cdot n}$ is (1) less than 0 (2) less than $\frac{1}{2^{100}.101}$ (3) greater than 1 (4) greater than $\frac{1}{2^{100}.101}$

Question No.	Questions
28.	Let f be function defined on the set $S=\left\{x \in \mathbb{R},\ x \geq 0,\ x \neq n\pi+\frac{\pi}{2},\ n \in \mathbb{N} \cup \{0\}\right\}$ and f (x) = tan x. Then (1) f has a unique fixed point on S (2) there is no fixed point of f on S (3) f has infinitely many fixed points on S (4) f has finite number of fixed points on S
29.	A function $f : \mathbb{R} \to \mathbb{R}$ need not be lebesgue measurable if (1) $\{x \in \mathbb{R} : f(x) \geq \alpha\}$ is measurable for each $\alpha \in \mathbb{Q}$ (2) $\{x \in \mathbb{R} : f(x) = \alpha\}$ is measurable for each $\alpha \in \mathbb{R}$ (3) for each open set G in $\mathbb{R}$, f^{-1} (G) is measurable (4) f is monotone
30.	Let $f(x)=\left(1+\frac{1}{x}\right)^x$ and $g(x)=\left(1+\frac{1}{x}\right)^{x+1}$, $x > 0$. then (1) f (x) and g (x) both are increasing functions (2) f (x) is increasing and g (x) is decreasing (3) f (x) is decreasing and g (x) is increasing (4) f (x) and g (x) both are decreasing functions
31.	Let $\underline{X} \sim N_3(\underline{\mu}, \Sigma)$, where $\underline{\mu} = (1, 1, 1)$ and $\Sigma = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 3 & C \\ 1 & C & 2 \end{pmatrix}$. The value of C such that X_2 and $-X_1 + X_2 - X_3$ are independent is (1) 0 (2) −2 (3) 2 (4) 1

Question No.	Questions
32.	Assume that X ~ Binomial (n, p) for some $n \geq 1$ and $0 < p < 1$ and Y ~ Poisson (λ) for some $\lambda > 0$. Suppose $\Sigma[X] = \Sigma[Y]$. Then (1) Var (X) = Var (Y) (2) Var (X) < Var (Y) (3) Var (Y) < Var (X) (4) Var (X) may be larger or smaller than Var (Y)
33.	Let X, Y be independent random variables and let $Z = \frac{X-Y}{2} + 3$ If X has characteristic function ϕ and Y has characteristic function ψ, then Z has characteristic function θ where (1) $\theta(t) = e^{-i3t}\phi(2t)\psi(-2t)$ (2) $\theta(t) = e^{i3t}\phi\left(\frac{t}{2}\right)\psi\left(-\frac{t}{2}\right)$ (3) $\theta(t) = e^{-i3t}\phi\left(\frac{t}{2}\right)\psi\left(\frac{t}{2}\right)$ (4) $\theta(t) = e^{-i3t}\phi\left(\frac{t}{2}\right)\psi\left(\frac{-t}{2}\right)$
34.	Consider LPP : Minimize : $z = -2x - 5y$ subject to : $3x + 4y \geq 5$, $x \geq 0$, $y \geq 0$. Which of the following is correct ? (1) Set of feasible solutions is empty (2) Set of feasible solutions is non-empty but there is no optimal solution (3) Optimal value is obtained at $\left(0, \frac{5}{4}\right)$ (4) Optimal value is attained at $\left(\frac{5}{3}, 0\right)$

Question No.	Questions
35.	(X, Y) follows the bivariate normal distribution N_2 (0, 0, 1, 1, ρ), $-1 < \rho < 1$. Then (1) X + Y and X – Y are uncorrelated only if $\rho = 0$ (2) X + Y and X – Y are uncorrelated only if $\rho < 0$ (3) X + Y and X – Y are uncorrelated only if $\rho > 0$ (4) X + Y and X – Y are uncorrelated for all values ρ
36.	Let (X_1, X_2, X_3, X_4) be an optimal solution to the problem of minimizing $X_1 + X_2 + X_3 + X_4$ subject to the constraints $X_1 + X_2 \geq 300, X_2 + X_3 \geq 500, X_3 + X_4 \geq 400, X_4 + X_1 \geq 200, X_1 \geq 0, X_2 \geq 0,$ $X_3 \geq 0, X_4 \geq 0.$ Which of the following is a possible value for any X_i ? (1) 300 (2) 400 (3) 500 (4) 600
37.	A system consists of 3 components arranged as –  Each of the components C_1, C_2, C_3 has independent and identically distributed (i.i.d) lifetimes whose distribution is exponential with mean 1. Then the survival function, S (t), of the system is given by (1) $S(t) = e^{-3t}, t > 0$ (2) $S(t) = (1 - e^{-t})^2 e^{-t}, t > 0$ (3) $S(t) = (1 - e^{-2t}) e^{-t}, t > 0$ (4) $S(t) = (1 - (1 - e^{-t})^2) e^{-t}, t > 0$

Question No.	Questions
38.	Let $X_1, X_2, \ldots, X_7$ be a random sample from $N(\mu, \sigma^2)$ where μ and σ^2 are unknown. Consider the problem of testing $H_0: \mu = 2$ against $H_1: \mu > 2$. Suppose the observed values of $x_1, x_2, \ldots, x_7$ are 1.2, 1.3, 1.7, 1.8, 2.1, 2.3, 2.7. If we use the uniformly most powerful test, which of the following is true ? (1) H_0 is accepted both at 5% and 1% levels of significance (2) H_0 is rejected both at 5% and 1% levels of significance (3) H_0 is rejected at 5% level of significance, but accepted at 1% level of significance (4) H_0 is rejected at 1% level of significance, but accepted at 5% level of significance
39.	Let N_t denote the number of accidents up to time t. Assume that $\{N_t\}$ is a Poisson process with intensity 2. Given that there are exactly 5 accidents during the time period $[20, 30]$. What is the conditional probability that there is exactly one accident dusing the time period $[15, 25]$? (1) $\frac{15}{32} e^{-20}$ (2) $\frac{15}{32} e^{-10}$ (3) $20 e^{-20}$ (4) $\frac{1}{5}$
40.	Suppose there are K groups each consisting of N boys. We want to estimate the mean age μ of these μN boys. Fix $1 < n < N$ and consider the following two sampling schemes : I. Draw a simple random sample without replacement of size kn out of KN boys II. From each of the K groups, draw a simple random sample with replacement of size n. Let $\overline{Y}$ and $\overline{Y}_G$ be the respective sample mean ages for the two schemes. Which of the following is not true ? (1) $E(\overline{Y}) = \mu$ (2) $E(\overline{Y}_G) = \mu$ (3) Var $(\overline{Y})$ may be less than Var $(\overline{Y}_G)$ in some cases (4) Var $(\overline{Y})$ = Var $(\overline{Y}_G)$ if all the group means are same.

Question No.	Questions
41.	If f (x) and g (x) are two solutions of $\cos x \frac{d^2y}{dx_2} + \sin x \frac{dy}{dx} - \left(1+e^{-x^2}\right) y = 0,\ x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ such that $f(0)=\sqrt{2},\ f'(0)=1;\ g(0)=-\sqrt{2},\ g'(0)=2.$ Then Wronskian of f and g is (1) $3 \cos x$ (2) $3\sqrt{2} \cos x - x$ (3) $3\sqrt{2} \cos x$ (4) 0
42.	The critical point (0, 0) of the system $\frac{dx}{dt} = 2x - 7y,\ \frac{dy}{dt} = 3x - 8y$ is (1) unstable node (2) asymptotically stable node (3) unstable spiral point (4) asymptotically stable spiral point
43.	Given a boundary value problem $y''(x) + \lambda y(x) = 0;\ y(0) = 0,\ y(\pi) = 0$ Set of its eigen values is (1) $\mathbb{R}$ (2) $(-\infty, 0)$ (3) $\{n^2 : n \in \mathbb{Z}^+\}$ (4) $\{\sqrt{n} : n \in \mathbb{Z}^+\}$
44.	The limiting value of y (x), as $x \to \infty$, where y (x) is the solution of $y'(x) = ay - by^2$; a, b > 0, $y(0) = y_0$, will be (1) 0 (2) a/b (3) b/a (4) y_0

Question No.	Questions
45.	Given a differential equation $x''(t)+p(t)x'(t)+q(t)x(t)=0;\ p(t), q(t) \in C^1[a,b]$. Let f (t) and g (t) be its two solutions on [a, b]. Then which of the following is incorrect ? (1) f and g are linearly dependent and $W(f,g)(t)=0\ \forall t \in [a,b]$ (2) f and g are linearly in dependent and $\exists$ to $\in (a,b)$ s.t. $W(t_0)=0$ (3) f and g are linearly in dependent and $W(f,g)(t) \neq 0$ for any $t \in [a,b]$ (4) f and g are linearly independent then every other solution can be written as their linear combination
46.	Let D denote the disc $\{(x,y) \mid x^2+y^2 \le 1\}$ and let D^c be its complement in the plane. The P.D.E $(x^2-1)\dfrac{\partial^2 u}{dx^2}+2y\dfrac{\partial^2 u}{dxdy}-\dfrac{\partial^2 u}{dy^2}=0$ is (1) Parabolic $\forall (x,y) \in D^c$ (2) Hyperbolic $\forall (x,y) \in D$ (3) Hyperbolic $\forall (x,y) \in D^c$ (4) Parabolic $\forall (x,y) \in D$
47.	Given a partial differential equation $x^2\dfrac{\partial z}{\partial x}+y^2\dfrac{\partial z}{\partial y}=(x+y)z$. Then which of the following is not the general solution of the differential equation ? (1) $F\left(\dfrac{xy}{z}, \dfrac{x-y}{z}\right)=0$ for an arbitrary twice differentiable function F (2) $F\left(\dfrac{x-y}{z}, \dfrac{1}{x}-\dfrac{1}{y}\right)=0$ for an arbitrary twice differentiable function F (3) $z=f\left(\dfrac{1}{x}-\dfrac{1}{y}\right)$ for an arbitrary twice differentiable function f (4) $z=xyf\left(\dfrac{1}{x}-\dfrac{1}{y}\right)$ for an arbitrary twice differentiable function f

Question No.	Questions
48.	The Cauchy problem $2u_x + 3u_y = 5$, $u = 1$ on the line $3x - 2y = 0$ has (1) exactly one solution (2) exactly two solutions (3) infinitely many solutions (4) no solution
49.	Let u be the unique solution of $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, where $(x, t) \in (0, 1) \times (0, \infty)$ $u(x, 0) = \sin \pi x$, $x \in (0, 1)$ $u(0, t) = u(1, t) = \theta$, $t \in (0, \infty)$ Then which of the following is true ? (1) $\exists (x, t) \in (0, 1) \times (0, \infty)$ s.t. $u(x, t) = 0$ (2) $\exists (x, t) \in (0, 1) \times (0, \infty)$ s.t. $\frac{\partial u}{\partial t}(x, t) = 0$ (3) the function $e^t u(x, t)$ is bounded for $(x, t) \in (0, 1) \times (0, \infty)$ (4) $\exists (x, t) \in (0, 1) \times (0, \infty)$ s.t. $u(x, t) > 1$
50.	Let u (x, t) be the solution of the initial value problem $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$; $u(x, 0) = x^3$, $\frac{\partial u}{\partial t}(x, 0) = \sin x$ Then $u(\pi, \pi)$ is (1) $4\pi^3$ (2) $2\pi^3$ (3) 0 (4) 4

Question No.	Questions
51.	Let $C(t) = 3e^{it}$, $0 \le t \le 2\pi$ be the positively oriented circle of radius 3 centered at the origin. The value of λ for which $\oint_C \frac{\lambda}{z-2}\,dz = \oint_C \frac{1}{z^2-5z+4}\,dz$ is (1) $\lambda = 0$ (2) $\lambda = 1$ (3) $\lambda = \frac{1}{3}$ (4) $\lambda = \frac{-1}{3}$
52.	Let f be real valued harmonic function of complex variable and $g = f_x - if_y$, $h = f_x + if_y$. Then (1) g and h both are holomorphic functions (2) both g and h are identically equal to the zero function (3) g is holomorphic but h need not be holomorphic function (4) h is holomorphic but f need not be holomorphic function
53.	How many elements does the set $\{z \in \mathbb{C} : z^{60} = -1,\ z^k \ne -1 \text{ for } 0 < k\ 60\}$ have ? (1) 45 (2) 32 (3) 30 (4) 24
54.	The number of roots of the equation $z^5 - 12z^2 + 14 = 0$ that lie in the region $\left\{z \in \mathbb{C} : 2 \le \lvert z \rvert < \frac{5}{2}\right\}$ is (1) 3 (2) 4 (3) 2 (4) 5

Question No.	Questions
55.	The integral $\oint_{\lvert 1-z\rvert=1} \frac{e^z}{z^2-1}\,dz$ is (1) 0 (2) $(i\pi)(e-e^{-1})$ (3) $e+e^{-1}$ (4) $ie\pi$
56.	Let p be a prime number. How many distinct sub-rings (with unity) of cardinality p does the field F_{p^2} have ? (1) p (2) p^2 (3) 1 (4) 0
57.	Let S_n denote the permutation group on n symbols and A_n be the subgroup of even permutations. Which of the following is true ? (1) there exists a finite group which is not a subgroup of S_n for any $n \geq 1$ (2) every finite group is a quotient of A_n for some $n \geq 1$ (3) every finite group is a subgroup of A_n for some $n \geq 1$ (4) no finite abelian group is a quotient of S_n for $n \geq 3$
58.	Let S be the set of all integers from 100 to 999 which are neither divisible by 3 nor divisible by 5. Then the number of elements in 5 is (1) 240 (2) 360 (3) 420 (4) 480
59.	Consider the ideal $I = <(x^2+1), y>$ in the polynomial ring $\mathbb{C}[x, y]$. Which of the following statements is true ? (1) I is a maximal ideal (2) I is a maximal ideal but not a prime ideal (3) I is prime ideal but not a maximal ideal (4) I is neither a prime ideal nor a maximal ideal

Question No.	Questions
60.	Let $f : \mathbb{Z} \rightarrow (\mathbb{Z} \rightarrow / 4\mathbb{Z}) \times (\mathbb{Z}/6\mathbb{Z})$ be the function defined by $f(n) = (n \bmod 4, n \bmod 6)$. Then (1) Kernel of $f = 24\mathbb{Z}$ (2) Image of f has exactly 6 elements (3) (a mod 4, b mod 6) is in the Image of f, for all even integers a and b (4) (0 mod 4, 3 mod 6) is in the Image of f
61.	Let f (x) be a polynomial of unknown degree taking the values x: 0, 1, 2, 3; f(x): 2, 7, 13, 16 All the fourth divided difference are $\frac{-1}{6}$. Then coefficient of x^3 is (1) $\frac{-2}{3}$ (2) $\frac{1}{3}$ (3) -1 (4) 16
62.	The iterative method $x_{n+1} = g(x_n)$ for the solution of the equation $x^2 - x - 2 = 0$ converges quadratically in a neighbourhood of the root $x = 2$ if g (x) equals (1) $x^2 - 2$ (2) $1 + \frac{2}{x}$ (3). $(x-2)^2 - 6$ (4) $\frac{x^2+2}{2x-1}$
63.	The forward difference operator is defined as $\Delta u_n = u_{n+1} - u_n$. Then which of the following difference equation has bounded general solution ? (1) $\Delta^2 u_n - 3\Delta u_n + 2u_n = 0$ (2) $\Delta^2 u_n + \Delta u_n + \frac{1}{4} u_n = 0$ (3) $\Delta^2 u_n - 2\Delta u_n + 2u_n = 0$ (4) $\Delta^2 u_{n+1} - \frac{1}{3}\Delta^2 u_n = 0$

Question No.	Questions
64.	Given the following statements : I : $\nabla = 1 - E^{-1}$ II : $E = 1 + \frac{\delta^2}{2} + \sqrt{1 + \frac{\delta^2}{4}}$ Then (1) I is true but II is false (2) I is false but II is true (3) Both I and II are false (4) Both I and II are true
65.	The expression of local error of Runge-Kutta method of order 2 is of the form (1) $\frac{h^3}{10}\left(f_{xx} + 2f_{yy}\, f^2 - 2f_x f_y\right) + 0\left(h^4\right)$ (2) $\frac{h^3}{12}\left(f_{xx} + 2f_{yy}\, f^2 - 2f_x f_y\right) + 0\left(h^4\right)$ (3) $\frac{h^3}{12}\left(f_{xx} + 2f\,f_{xy} + f^2 f_{yy} - 2f_x f_y - 2f\,f_y^2\right) + \left(h^4\right)$ (4) None of these
66.	Let X_1, X_2, X_3, X_4, X_5 be i.i.d random variables having a continuous distribution function. Then $P\left(X_1 > X_2 > X_3 > X_4 > X_5 / X_1 = \max\left(X_1, X_2, X_3, X_4, X_5\right)\right)$ equals (1) $\frac{1}{5}$ (2) $\frac{1}{4!}$ (3) $\frac{1}{4}$ (4) $\frac{1}{5!}$

Question No.	Questions
67.	Consider a Markov chain with state space {0, 1, 2, 3, 4} and transition matrix $$P = \begin{matrix} & 0 & 1 & 2 & 3 & 4 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 2 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 3 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 4 & 0 & 0 & 0 & 0 & 1 \end{matrix}$$ Then $\lim_{n \to \infty} p_{23}^{(n)}$ equals (1) $\frac{1}{3}$ (2) $\frac{1}{2}$ (3) 0 (4) 1
68.	Consider the function f (x) defined as $f(x) = ce^{-x^4}, \; x \in \mathbb{R}$ For what value of c is f a probability density function ? (1) $\frac{4}{\Gamma\left(\frac{1}{4}\right)}$ (2) $\frac{3}{\Gamma\left(\frac{1}{3}\right)}$ (3) $\frac{2}{\Gamma\left(\frac{1}{4}\right)}$ (4) $\frac{1}{4\Gamma\left(\frac{1}{4}\right)}$

Question No.	Questions
69.	Given the observations 0.8, 0.71, 0.9, 1.2, 1.68, 1.4, 0.88, 1.62 from the uniform distribution on $(\theta - 0.2, \theta + 0.8)$ with $-\infty < \theta < \infty$. Which of the following is a maximum likelihood estimate for θ ? (1) 0.7 (2) 1.1 (3) 1.3 (4) 0.9
70.	Suppose that $\lvert 3x \rvert + \lvert 2y \rvert \le 1;\ x \ge 0,\ y \ge 0$. Then the maximum value of $9x+4y$ is (1) 1 (2) 2 (3) 3 (4) 4
71.	Let $\psi(t) = e^{-\lvert t \rvert - \frac{t^2}{2}}$ and $\phi(t) = \dfrac{e^{-\lvert t \rvert} + e^{\frac{-t^2}{2}}}{2}$. Which of the following is true ? (1) ϕ is a characteristic function but ψ is not (2) ψ is a characteristic function but ϕ is not (3) neither ϕ nor ψ is a characteristic function (4) Both ϕ and ψ are characteristic functions
72.	If $b_{xy} = 0.20$ and $r_{xy} = 0.50$, Then b_{yx} is equal to (1) 0.20 (2) 0.25 (3) 0.50 (4) 1.25
73.	Let $X_1, X_2, \ldots\ldots, X_n$ be independent random variables ; X_i having exponential distribution with parameter θ_i, $i = 1, 2, \ldots, n$. Then $Z = \text{Min}(X_1, X_2, \ldots, X_n)$ has (1) normal distribution (2) geometric distribution (3) exponential distribution with parameter $\sum_{i=1}^{n} \theta_i$ (4) None of these

Question No.	Questions
74.	In queueing description M | M | 1, the arrival and departure distribution are (1) Binomial (2) General (3) Both Markovian (4) None of these
75.	Successful life of product, time, weight and height are classified as (1) continuous random variable (2) discrete random variable (3) continuous time variable (4) None of these
76.	If a random variable X has a Chi-Square distribution with 4 degree of freedom, then its mean is equal to (1) 2 (2) 3 (3) 4 (4) None of these
77.	In a Latin Square Design, if factors A, B, C and D have levels 8, then the total number of cells in the design is (1) 4096 (2) 64 (3) 512 (4) None of these
78.	If a system has two components in parallel with each of reliability 0.75, then the reliability of the system is equal to (1) 0.9375 (2) 0.9753 (3) 0.7935 (4) None of these

Question No.	Questions
79.	Let (v, b, r, k, w) be the standard parameters of a balanced incomplete block design (BIBD). Which of the following (v, b, r, k, w) can be parameters of BIBD ? (1) (v, b, r, k,w) = (44, 33, 9, 12, 3) (2) (v, b, r, k,w) = (17, 45, 8, 3, 1) (3) (v, b, r, k,w) = (35, 35, 17, 17, 9) (4) (v, b, r, k,w) = (16, 24, 9, 6, 3)
80.	100 tickets are marked 1, 2, ... , 100 and arranged at random. Four tickets are picked from these tickets and given to four persons A, B, C and D. What is the probability that A gets the ticket with the largest value (among A, B, C, D) and D gets the ticket with smallest value (among A, B, C, D) ? (1) $\frac{1}{4}$ (2) $\frac{1}{12}$ (3) $\frac{1}{2}$ (4) $\frac{1}{6}$
81.	Let $f(x, y) = \frac{x^2}{y^2}$, $(x, y) \in \left[\frac{1}{2}, \frac{3}{2}\right] \times \left[\frac{1}{2}, \frac{3}{2}\right]$. Then the derivative of f at (1, 1) along the direction (1, 1) is (1) 1 (2) 2 (3) 0 (4) −2
82.	Which of the following statement is not correct (1) if F is closed and K is compact, then $F \cap K$ is compact (2) if $\{K_n\}$ is a sequence of nonempty compact sets s.t. $K_{n+1} \subset K_n$ (n = 1, 2, ...), then $\bigcap_{n=1}^{\infty} K_n$ is empty (3) Every closed subset of a compact set is compact (4) The set $\{x : x \in \mathbb{R} \text{ and } x(x^2 - 6x + 8) = 0\}$ is compact

Question No.	Questions
83.	Let $X = \{x : x = (x_1, x_2, x_3),\ x_i \in \mathbb{R},\ x_1^2 + x_2^2 + x_3^2 < 16\}$. Then (1) X has no limit point in $\mathbb{R}^3$ (2) X has a limit point in $\mathbb{R}^3$ (3) X is compact (4) All of the above statement are true
84.	Let A, B be n × n matrices. Then trace of $A^2 B^2$ is equal to (1) trace $((AB)^2)$ (2) $(\text{trace } (AB))^2$ (3) trace (AB^2A) (4) trace (BABA)
85.	Let f, g, h be the functions from $\mathbb{R}^3$ to $\mathbb{R}^2$ such that $f\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ x+y \end{pmatrix}, g\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} xy \\ x+y \end{pmatrix}, h\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} z-x \\ x+y \end{pmatrix}; \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3$ Then (1) only f is a linear transformation (2) only g is a linear transformation (3) only h is a linear transformation (4) f and g are linear transformations but h is not a linear transformation
86.	Let S denote the set of all the prime numbers p such that the matrix $\begin{bmatrix} 91 & 31 & 0 \\ 29 & 31 & 0 \\ 79 & 23 & 59 \end{bmatrix}$ has an inverse in the field Z/pZ. Then (1) S = {31} (2) {31, 59} (3) S = {7, 13, 59} (4) S is infinite

Question No.	Questions
87.	Let V be the vector space of all real polynomials of degree ≤ 10. Let $TP(x) = P'(x)$, for $P \in V$, be a linear transformation from V to V. Consider the basis $\{1, x, x^2, ..., x^{10}\}$ of V. Let A be the matrix of T with respect to this basis. Then (1) trace (A) = 1 (2) There is no $n \in \mathbb{N}$ s.t. $A^n = 0$ (3) A has non-zero eigen value (4) det (A) = 0
88.	Given a matrix $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}, \theta = \frac{2\pi}{31}$. Then A^{2015} is equal to (1) I (2) A (3) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ (4) $\begin{pmatrix} \cos 13\theta & \sin 13\theta \\ -\sin 13\theta & \cos 13\theta \end{pmatrix}$
89.	Given a matrix $\begin{pmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{pmatrix}$. Then this form is (1) negative definite (2) positive definite (3) non-negative definite but not positive definite (4) neither negative definite nor positive definite

Question No.	Questions
90.	Let W_1 and W_2 be subspaces of $\mathbb{R}^3$ given by $W_1 = \{(x, y, z) \in \mathbb{R}^3 / x + y + z = 0\}$ $W_2 = \{(x, y, z) \in \mathbb{R}^3 / x - y + z = 0\}$ If W is a subspace of $\mathbb{R}^3$ such that (i) $W \cap W_2 =$ space {0, 1, 1} (ii) $W \cap W_1 =$ orthogonal to $W \cap W_2$ with respect to the usual inner product of $\mathbb{R}^3$. Then (1) $W = \text{Span}\{(0,1,-1), (0,1,1)\}$ (2) $W = \text{Span}\{(1,0,-1), (0,1,-1)\}$ (3) $W = \text{Span}\{(1,0,-1), (0,1,1)\}$ (4) $W = \text{Span}\{(1,0,-1), (1,0,1)\}$
91.	A load slides without friction on a frictionless wire in the shape of cycloid with equation $x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$, $0 \le \theta \le 2\pi$. The the Lagrangian is (1) $ma^2(1 + \cos\theta)\dot{\theta}^2 - mga(1 + \cos\theta)$ (2) $ma^2(1 + \cos\theta)\dot{\theta}^2 - mga(1 - \cos\theta)$ (3) $ma^2(1 - \cos\theta)\dot{\theta}^2 - mga(1 + \cos\theta)$ (4) $ma^2(1 - \cos\theta)\dot{\theta}^2 + mga(1 + \cos\theta)$

Question No.	Questions
92.	Consider the two dimensional motion of a mass m attached to one end of a spring whose other end is fixed. Let K be the spring constant. The kinetic energy T and the potential energy V of the system are given by $T=\frac{1}{2}m\left(\dot{r}^2+(r\dot{\theta})^2\right),\ V=\frac{1}{2}Kr^2;\ \dot{r}=\frac{dr}{dt},\ \dot{\theta}=\frac{d\theta}{dt}.$ Then which of the following statement is correct ? (1) r is ignorable co-ordinate (2) θ is not an ignorable co-ordinate (3) $r^2\dot{\theta}$ remains constant throughout the motion (4) $r\dot{\theta}$ remains constant throughout the motion
93.	A body, under the action of no forces, moves so that the resolved part of its angular velocity about one of the principal axes at the centre of the gravity is constant. Let A, B, C be the principal moments and w_1, w_2, w_3 be the angular velocities about the principal axes at C.G. Consider the following statements : I : The angular velocity of the body is constant II : $A\dot{w}_1 w_1 + B\dot{w}_2 w_2 + C\dot{w}_3 w_3 = 0$ Then (1) Statement I is true but II is false (2) Statement I is false but II is true (3) Both the statements I and II are true (4) Both the statements I and II are false

Question No.	Questions
94.	Given that the Lagrangian for the motion of a simple pendulum is $L=\frac{1}{2}m\ell^2\dot{\theta}^2+mg\ell\cos\theta$, where m is the mass of the pendulum bob suspended by a string of length ℓ, g is accelaration due to gravity, θ is the amplitude of the pendulum from the mean position. Then corresponding Hamiltonian is (1) $H=\frac{p^2}{2m\ell^2}+mg\ell\cos\theta$ (2) $\frac{p^2}{2m\ell^2}-mg\ell\cos\theta=H$ (3) $H=\frac{p^2}{m\ell^2}-mg\ell\cos\theta$ (4) $H=\frac{3p^2}{2m\ell^2}+mg\ell\cos\theta$
95.	The admissible extremal for $I[y]=\int_0^{\log 3}\{e^{-x}y'^2+2e^x(y+y^1)\}dx$, where $y(\log 3)=1$ and y(0) is free, is (1) $4-e^x$ (2) $10-e^{2x}$ (3) $e^{2x}-8$ (4) e^x-2
96.	Consider the functional $I[y]=\int_0^1\{(y'(x))^2+(y'(x))^3\}dx$ subject to y (0) = 1 and y (1) = 2. Then (1) there exists an extremal $y\in c^1([0,1])\setminus c^2([0,1])$ (2) there exists an extremal $y\in c([0,1])\setminus c^1([0,1])$ (3) no extremal y belongs to $c^1([0,1])$ (4) every extremal y belongs to $c^2([0,1])$

Question No.	Questions
97.	Given a problem of calculus of variation $J[y]=\int_0^1\left[2y+(y^1)^2\right]dx$ subject to $y(0)=0,\ y(1)=1.$ The value of Inf $J[y]$ is (1) $\frac{21}{24}$ (2) $\frac{18}{24}$ (3) $\frac{23}{12}$ (4) Does not exist
98.	If y is a solution of the integral equation $y(x)=1+\int_0^x(x-t)\,y(t)\,dt$. Then (1) $\int_{\mathbb{R}} y(x)\,dx<\infty$ (2) $\int_{\mathbb{R}}\frac{dx}{y(x)}<\infty$ (3) $y(x)$ is periodic in $\mathbb{R}$ (4) y is bounded but not periodic in $\mathbb{R}$
99.	The value of the solution of the integral equation $y(x)-1+2x+4x^2=\int_0^x\left\{6(x-t)+x-4(x-t)^2\right\}y(t)\,dt$ at $x=\log 5$ is equal to (1) 5 (2) $\frac{1}{5}$ (3) 4 (4) $\frac{1}{4}$
100.	If g is the solution of $\int_0^x\left(1-x^2+t^2\right)g(t)\,dt=\frac{x^2}{2}$, Then $g\left(\sqrt{2}\right)$ equals (1) $\sqrt{2}\,e^{2\sqrt{2}}$ (2) $\sqrt{2}\,e^{\sqrt{2}}$ (3) $\sqrt{2}\,e^{2}$ (4) $\sqrt{2}\,e^{4}$

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(MPH/PHD/URS–EE–2019)

Subject : MATHEMATICS

Code **D**

Sr. No. 10344

SET–"X"

Time : 1¼ Hours **Total Questions : 100** **Max. Marks : 100**

Roll No. ________________ (in figure)____________________________ (in words)

Name : ________________________________ Father's Name : ________________

Mother's Name : ________________________ Date of Examination : ____________

(Signature of the candidate) (Signature of the Invigilator)

CANDIDATES MUST READ THE FOLLOWING INFORMATION INSTRUCTIONS BEFORE STARTING THE QUESTION PAPER.

1. **Candidates are required to attempt any 75 questions out of the give 100 multiple choice questions of 4/3 marks each. No credit will be give for more than 75 correct responses.**
2. The candidates must return the Question book-let as well as OMR answer-sh to the Invigilator concerned before leaving the Examination Hall, failing whic case of use of unfair-means / mis-behaviour will be registered against him / h in addition to lodging of an FIR with the police. Further the answer-sheet of such a candidate will not be evaluated.
3. Keeping in view the transparency of the examination system, carbonless OMR Sheet is provided to the candidate so that a copy of OMR Sheet may be kept by the candidate.
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6. **There will be no negative marking. Each correct answer will be awarded 4/3 mark. Cutting, erasing, overwriting and more than one answer in OMR Answer-Sheet will be treated as incorrect answer.**
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8. BEFORE ANSWERING THE QUESTIONS, THE CANDIDATES SHOULD ENSURE THAT THEY HAVE BEEN SUPPLIED CORRECT AND COMPLETE BOOK-LET. COMPLAINTS, IF ANY, REGARDING MISPRINTING ETC. WILL NOT BE ENTERTAINED 30 MINUTES AFTER STARTING OF THE EXAMINATION.

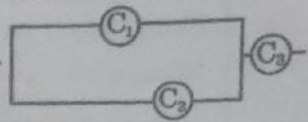
Question No.	Questions
1.	Let $f(x, y) = \frac{x^2}{y^2}$, $(x, y) \in \left[\frac{1}{2}, \frac{3}{2}\right] \times \left[\frac{1}{2}, \frac{3}{2}\right]$. Then the derivative of f at (1, 1) along the direction (1, 1) is (1) 1 (2) 2 (3) 0 (4) −2
2.	Which of the following statement is not correct (1) if F is closed and K is compact, then $F \cap K$ is compact (2) if $\{K_n\}$ is a sequence of nonempty compact sets s.t. $K_{n+1} \subset K_n$ (n = 1, 2, ...), then $\bigcap_{n=1}^{\infty} K_n$ is empty (3) Every closed subset of a compact set is compact (4) The set $\{x : x \in \mathbb{R} \text{ and } x(x^2 - 6x + 8) = 0\}$ is compact
3.	Let $X = \{x : x = (x_1, x_2, x_3), x_i \in \mathbb{R}, x_1^2 + x_2^2 + x_3^2 < 16\}$. Then (1) X has no limit point in $\mathbb{R}^3$ (2) X has a limit point in $\mathbb{R}^3$ (3) X is compact (4) All of the above statement are true
4.	Let A, B be n × n matrices. Then trace of $A^2 B^2$ is equal to (1) trace $((AB)^2)$ (2) $(\text{trace}(AB))^2$ (3) trace (AB^2A) (4) trace (BABA)

Question No.	Questions
5.	Let f, g, h be the functions from $\mathbb{R}^3$ to $\mathbb{R}^2$ such that $f\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}4\\x+y\end{pmatrix}, g\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}xy\\x+y\end{pmatrix}, h\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}z-x\\x+y\end{pmatrix}; \begin{pmatrix}x\\y\\z\end{pmatrix}\in\mathbb{R}^3$ Then (1) only f is a linear transformation (2) only g is a linear transformation (3) only h is a linear transformation (4) f and g are linear transformations but h is not a linear transformation
6.	Let S denote the set of all the prime numbers p such that the matrix $\begin{bmatrix}91 & 31 & 0\\29 & 31 & 0\\79 & 23 & 59\end{bmatrix}$ has an inverse in the field Z/pZ. Then (1) S = {31} (2) {31, 59} (3) S = {7, 13, 59} (4) S is infinite
7.	Let V be the vector space of all real polynomials of degree ≤ 10. Let $TP(x) = P'(x)$, for $P \in V$, be a linear transformation from V to V. Consider the basis $\{1, x, x^2, \ldots\ldots, x^{10}\}$ of V. Let A be the matrix of T with respect to this basis. Then (1) trace (A) = 1 (2) There is no $n \in \mathbb{N}$ s.t. $A^n = 0$ (3) A has non-zero eigen value (4) det (A) = 0

Question No.	Questions
8.	Given a matrix $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}, \theta = \frac{2\pi}{31}$. Then A^{2016} is equal to (1) I (2) A (3) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ (4) $\begin{pmatrix} \cos 13\theta & \sin 13\theta \\ -\sin 13\theta & \cos 13\theta \end{pmatrix}$
9.	Given a matrix $\begin{pmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{pmatrix}$. Then this form is (1) negative definite (2) positive definite (3) non-negative definite but not positive definite (4) neither negative definite nor positive definite
10.	Let W_1 and W_2 be subspaces of $\mathbb{R}^3$ given by $W_1 = \{(x, y, z) \in \mathbb{R}^3 / x + y + z = 0\}$ $W_2 = \{(x, y, z) \in \mathbb{R}^3 / x - y + z = 0\}$ If W is a subspace of $\mathbb{R}^3$ such that (i) $W \cap W_2$ = space {0, 1, 1} (ii) $W \cap W_1$ = orthogonal to $W \cap W_2$ with respect to the usual inner product of $\mathbb{R}^3$. Then (1) W = Span $\{(0,1,-1), (0,1,1)\}$ (2) W = Span $\{(1,0,-1), (0,1,-1)\}$ (3) W = Span $\{(1,0,-1), (0,1,1)\}$ (4) W = Span $\{(1,0,-1), (1,0,1)\}$

Question No.	Questions
11.	Let $\underline{X} \sim N_3(\underline{\mu}, \Sigma)$, where $\underline{\mu} = (1, 1, 1)$ and $\Sigma = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 3 & C \\ 1 & C & 2 \end{pmatrix}$. The value of C such that X_2 and $-X_1 + X_2 - X_3$ are independent is (1) 0 (2) -2 (3) 2 (4) 1
12.	Assume that X ~ Binomial (n, p) for some $n \geq 1$ and $0 < p < 1$ and Y ~ Poisson (λ) for some $\lambda > 0$. Suppose $\Sigma[X] = \Sigma[Y]$. Then (1) Var (X) = Var (Y) (2) Var (X) < Var (Y) (3) Var (Y) < Var (X) (4) Var (X) may be larger or smaller than Var (Y)
13.	Let X, Y be independent random variables and let $Z = \frac{X - Y}{2} + 3$ If X has characteristic function ϕ and Y has characteristic function ψ, then Z has characteristic function θ where (1) $\theta(t) = e^{-i3t}\phi(2t)\psi(-2t)$ (2) $\theta(t) = e^{i3t}\phi\left(\frac{t}{2}\right)\psi\left(-\frac{t}{2}\right)$ (3) $\theta(t) = e^{-i3t}\phi\left(\frac{t}{2}\right)\psi\left(\frac{t}{2}\right)$ (4) $\theta(t) = e^{-i3t}\phi\left(\frac{t}{2}\right)\psi\left(\frac{-t}{2}\right)$

Question No.	Questions
14.	Consider LPP : Minimize : $z = -2x - 5y$ subject to : $3x + 4y \geq 5,\ x \geq 0,\ y \geq 0$. Which of the following is correct ? (1) Set of feasible solutions is empty (2) Set of feasible solutions is non-empty but there is no optimal solution (3) Optimal value is obtained at $\left(0, \frac{5}{4}\right)$ (4) Optimal value is attained at $\left(\frac{5}{3}, 0\right)$
15.	(X, Y) follows the bivariate normal distribution N_2 (0, 0, 1, 1, ρ), $-1 < \rho < 1$. Then (1) $X + Y$ and $X - Y$ are uncorrelated only if $\rho = 0$ (2) $X + Y$ and $X - Y$ are uncorrelated only if $\rho < 0$ (3) $X + Y$ and $X - Y$ are uncorrelated only if $\rho > 0$ (4) $X + Y$ and $X - Y$ are uncorrelated for all values ρ
16.	Let (X_1, X_2, X_3, X_4) be an optimal solution to the problem of minimizing $X_1 + X_2 + X_3 + X_4$ subject to the constraints $X_1 + X_2 \geq 300,\ X_2 + X_3 \geq 500,\ X_3 + X_4 \geq 400,\ X_4 + X_1 \geq 200,\ X_1 \geq 0,\ X_2 \geq 0,$ $X_3 \geq 0,\ X_4 \geq 0$. Which of the following is a possible value for any X_i ? (1) 300 (2) 400 (3) 500 (4) 600

Question No.	Questions
17.	A system consists of 3 components arranged as –  Each of the components C_1, C_2, C_3 has independent and identically distributed (i.i.d) lifetimes whose distribution is exponential with mean 1. Then the survival function, S (t), of the system is given by (1) $S(t) = e^{-3t}, t > 0$ (2) $S(t) = (1 - e^{-t})^2 e^{-t}, t > 0$ (3) $S(t) = (1 - e^{-2t}) e^{-t}, t > 0$ (4) $S(t) = (1 - (1 - e^{-t})^2) e^{-t}, t > 0$
18.	Let $X_1, X_2, \ldots, X_7$ be a random sample from $N(\mu, \sigma^2)$ where μ and σ^2 are unknown. Consider the problem of testing $H_0 : \mu = 2$ against $H_1 : \mu > 2$. Suppose the observed values of $x_1, x_2, \ldots, x_7$ are 1.2, 1.3, 1.7, 1.8, 2.1, 2.3, 2.7. If we use the uniformly most powerful test, which of the following is true ? (1) H_0 is accepted both at 5% and 1% levels of significance (2) H_0 is rejected both at 5% and 1% levels of significance (3) H_0 is rejected at 5% level of significance, but accepted at 1% level of significance (4) H_0 is rejected at 1% level of significance, but accepted at 5% level of significance
19.	Let N_t denote the number of accidents up to time t. Assume that $\{N_t\}$ is a Poisson process with intensity 2. Given that there are exactly 5 accidents during the time period $[20, 30]$. What is the conditional probability that there is exactly one accident dusing the time period $[15, 25]$? (1) $\frac{15}{32} e^{-20}$ (2) $\frac{15}{32} e^{-10}$ (3) $20 e^{-20}$ (4) $\frac{1}{5}$

Question No.	Questions
20.	Suppose there are K groups each consisting of N boys. We want to estimate the mean age μ of these μN boys. Fix $1 < n < N$ and consider the following two sampling schemes : I. Draw a simple random sample without replacement of size kn out of KN boys II. From each of the K groups, draw a simple random sample with replacement of size n. Let $\overline{Y}$ and $\overline{Y}_G$ be the respective sample mean ages for the two schemes. Which of the following is not true ? (1) $E\left(\overline{Y}\right) = \mu$ (2) $E\left(\overline{Y}_G\right) = \mu$ (3) Var $\left(\overline{Y}\right)$ may be less than Var $\left(\overline{Y}_G\right)$ in some cases (4) Var $\left(\overline{Y}\right)$ = Var $\left(\overline{Y}_G\right)$ if all the group means are same.
21.	Let f (x) be a polynomial of unknown degree taking the values <table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>f (x)</td><td>2</td><td>7</td><td>13</td><td>16</td></tr></table>All the fourth divided difference are $\frac{-1}{6}$. Then coefficient of x^3 is (1) $\frac{-2}{3}$ (2) $\frac{1}{3}$ (3) –1 (4) 16
22.	The iterative method $x_{n+1} = g(x_n)$ for the solution of the equation $x^2 - x - 2 = 0$ converges quadratically in a neighbourhood of the root $x = 2$ if g (x) equals (1) $x^2 - 2$ (2) $1 + \frac{2}{x}$ (3) $(x-2)^2 - 6$ (4) $\frac{x^2+2}{2x-1}$

Question No.	Questions
23.	The forward difference operator is defined as $\Delta u_n = u_{n+1} - u_n$. Then which of the following difference equation has bounded general solution ? (1) $\Delta^2 u_n - 3\Delta u_n + 2u_n = 0$ (2) $\Delta^2 u_n + \Delta u_n + \frac{1}{4} u_n = 0$ (3) $\Delta^2 u_n - 2\Delta u_n + 2u_n = 0$ (4) $\Delta^2 u_{n+1} - \frac{1}{3}\Delta^2 u_n = 0$
24.	Given the following statements : I : $\nabla = 1 - E^{-1}$ II : $E = 1 + \frac{\delta^2}{2} + \sqrt{1 + \frac{\delta^2}{4}}$ Then (1) I is true but II is false (2) I is false but II is true (3) Both I and II are false (4) Both I and II are true
25.	The expression of local error of Runge-Kutta method of order 2 is of the form (1) $\frac{h^3}{10}\left(f_{xx} + 2f_{yy} f^2 - 2f_x f_y\right) + 0\left(h^4\right)$ (2) $\frac{h^3}{12}\left(f_{xx} + 2f_{yy} f^2 - 2f_x f_y\right) + 0\left(h^4\right)$ (3) $\frac{h^3}{12}\left(f_{xx} + 2f\,f_{xy} + f^2 f_{yy} - 2f_x f_y - 2f\,f_y^2\right) + \left(h^4\right)$ (4) None of these

Question No.	Questions
26.	Let X_1, X_2, X_3, X_4, X_5 be i.i.d random variables having a continuous distribution function. Then $P(X_1 > X_2 > X_3 > X_4 > X_5 / X_1 = \max(X_1, X_2, X_3, X_4, X_5))$ equals (1) $\frac{1}{5}$ (2) $\frac{1}{4!}$ (3) $\frac{1}{4}$ (4) $\frac{1}{5!}$
27.	Consider a Markov chain with state space {0, 1, 2, 3, 4} and transition matrix $$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$ Then $\lim_{n \to \infty} p_{23}^{(n)}$ equals (1) $\frac{1}{3}$ (2) $\frac{1}{2}$ (3) 0 (4) 1

Question No.	Questions
28.	Consider the function f (x) defined as $f(x) = ce^{-x^4}$, $x \in \mathbb{R}$ For what value of c is f a probability density function ? (1) $\frac{4}{\Gamma\left(\frac{1}{4}\right)}$ (2) $\frac{3}{\Gamma\left(\frac{1}{3}\right)}$ (3) $\frac{2}{\Gamma\left(\frac{1}{4}\right)}$ (4) $\frac{1}{4\Gamma\left(\frac{1}{4}\right)}$
29.	Given the observations 0.8, 0.71, 0.9, 1.2, 1.68, 1.4, 0.88, 1.62 from the uniform distribution on $(\theta - 0.2, \theta + 0.8)$ with $-\infty < \theta < \infty$. Which of the following is a maximum likelihood estimate for θ ? (1) 0.7 (2) 1.1 (3) 1.3 (4) 0.9
30.	Suppose that $\lvert 3x\rvert + \lvert 2y\rvert \leq 1$; $x \geq 0$, $y \geq 0$. Then the maximum value of $9x+4y$ is (1) 1 (2) 2 (3) 3 (4) 4
31.	A load slides without friction on a frictionless wire in the shape of cycloid with equation $x = a(\theta - \sin\theta)$, $y = a(1+\cos\theta)$, $0 \leq \theta \leq 2\pi$. The the Lagrangian is (1) $ma^2(1+\cos\theta)\dot{\theta}^2 - mga(1+\cos\theta)$ (2) $ma^2(1+\cos\theta)\dot{\theta}^2 - mga(1-\cos\theta)$ (3) $ma^2(1-\cos\theta)\dot{\theta}^2 - mga(1+\cos\theta)$ (4) $ma^2(1-\cos\theta)\dot{\theta}^2 + mga(1+\cos\theta)$

Question No.	Questions
32.	Consider the two dimensional motion of a mass m attached to one end of a spring whose other end is fixed. Let K be the spring constant. The kinetic energy T and the potential energy V of the system are given by $T = \frac{1}{2} m(\dot{r}^2 + (r\dot{\theta})^2)$, $V = \frac{1}{2} Kr^2$; $\dot{r} = \frac{dr}{dt}$, $\dot{\theta} = \frac{d\theta}{dt}$. Then which of the following statement is correct ? (1) r is ignorable co-ordinate (2) θ is not an ignorable co-ordinate (3) $r^2 \dot{\theta}$ remains constant throughout the motion (4) $r\dot{\theta}$ remains constant throughout the motion
33.	A body, under the action of no forces, moves so that the resolved part of its angular velocity about one of the principal axes at the centre of the gravity is constant. Let A, B, C be the principal moments and w_1, w_2, w_3 be the angular velocities about the principal axes at C.G. Consider the following statements : I : The angular velocity of the body is constant II : $A\dot{w}_1 w_1 + B\dot{w}_2 w_2 + C\dot{w}_3 w_3 = 0$ Then (1) Statement I is true but II is false (2) Statement I is false but II is true (3) Both the statements I and II are true (4) Both the statements I and II are false

Question No.	Questions
34.	Given that the Lagrangian for the motion of a simple pendulum is $L = \frac{1}{2} m\ell^2 \dot{\theta}^2 + mg\ell\cos\theta$, where m is the mass of the pendulum bob suspended by a string of length ℓ, g is accelaration due to gravity, θ is the amplitude of the pendulum from the mean position. Then corresponding Hamiltonian is (1) $H = \frac{p^2}{2m\ell^2} + mg\ell\cos\theta$ (2) $\frac{p^2}{2m\ell^2} - mg\ell\cos\theta = H$ (3) $H = \frac{p^2}{m\ell^2} - mg\ell\cos\theta$ (4) $H = \frac{3p^2}{2m\ell^2} + mg\ell\cos\theta$
35.	The admissible extremal for $I[y] = \int_0^{\log 3} \{e^{-x}y'^2 + 2e^x(y + y^1)\}\, dx$, where $y(\log 3) = 1$ and $y(0)$ is free, is (1) $4 - e^x$ (2) $10 - e^{2x}$ (3) $e^{2x} - 8$ (4) $e^x - 2$
36.	Consider the functional $I[y] = \int_0^1 \{(y'(x))^2 + (y'(x))^3\}\, dx$ subject to y (0) = 1 and y (1) = 2. Then (1) there exists an extremal $y \in c^1([0, 1]) \setminus c^2([0, 1])$ (2) there exists an extremal $y \in c([0, 1]) \setminus c^1([0, 1])$ (3) no extremal y belongs to $c^1([0, 1])$ (4) every extremal y belongs to $c^2([0, 1])$

Question No.	Questions
37.	Given a problem of calculus of variation $J[y]=\int_0^1\left[2y+(y')^2\right]dx$ subject to $y(0)=0,\ y(1)=1$. The value of Inf $J[y]$ is (1) $\frac{21}{24}$ (2) $\frac{18}{24}$ (3) $\frac{23}{12}$ (4) Does not exist
38.	If y is a solution of the integral equation $y(x)=1+\int_0^x(x-t)\,y(t)\,dt$. Then (1) $\int_{\mathbb{R}}y(x)\,dx<\infty$ (2) $\int_{\mathbb{R}}\frac{dx}{y(x)}<\infty$ (3) $y(x)$ is periodic in $\mathbb{R}$ (4) y is bounded but not periodic in $\mathbb{R}$
39.	The value of the solution of the integral equation $y(x)-1+2x+4x^2=\int_0^x\left\{6(x-t)+x-4(x-t)^2\right\}y(t)\,dt$ at $x=\log 5$ is equal to (1) 5 (2) $\frac{1}{5}$ (3) 4 (4) $\frac{1}{4}$
40.	If g is the solution of $\int_0^x(1-x^2+t^2)\,g(t)\,dt=\frac{x^2}{2}$, Then $g(\sqrt{2})$ equals (1) $\sqrt{2}\,e^{2\sqrt{2}}$ (2) $\sqrt{2}\,e^{\sqrt{2}}$ (3) $\sqrt{2}\,e^2$ (4) $\sqrt{2}\,e^4$

Question No.	Questions
41.	Let $C(t)=3e^{it}$, $0\le t\le 2\pi$ be the positively oriented circle of radius 3 centered at the origin. The value of λ for which $\oint_C \frac{\lambda}{z-2}\,dz=\oint_C \frac{1}{z^2-5z+4}\,dz$ is (1) $\lambda=0$ (2) $\lambda=1$ (3) $\lambda=\frac{1}{3}$ (4) $\lambda=\frac{-1}{3}$
42.	Let f be real valued harmonic function of complex variable and $g=f_x-if_y$, $h=f_x+if_y$. Then (1) g and h both are holomorphic functions (2) both g and h are identically equal to the zero function (3) g is holomorphic but h need not be holomorphic function (4) h is holomorphic but f need not be holomorphic function
43.	How many elements does the set $\{z\in\mathbb{C}: z^{60}=-1,\ z^k\ne -1 \text{ for } 0<k\ 60\}$ have ? (1) 45 (2) 32 (3) 30 (4) 24
44.	The number of roots of the equation $z^5-12z^2+14=0$ that lie in the region $\left\{z\in\mathbb{C}: 2\le \lvert z\rvert<\frac{5}{2}\right\}$ is (1) 3 (2) 4 (3) 2 (4) 5

Question No.	Questions
45.	The integral $\oint_{\lvert 1-z\rvert=1} \frac{e^z}{z^2-1}\,dz$ is (1) 0 (2) $(i\pi)\left(e-e^{-1}\right)$ (3) $e+e^{-1}$ (4) $ie\pi$
46.	Let p be a prime number. How many distinct sub-rings (with unity) of cardinality p does the field F_{p^2} have ? (1) p (2) p^2 (3) 1 (4) 0
47.	Let S_n denote the permutation group on n symbols and A_n be the subgroup of even permutations. Which of the following is true ? (1) there exists a finite group which is not a subgroup of S_n for any $n \geq 1$ (2) every finite group is a quotient of A_n for some $n \geq 1$ (3) every finite group is a subgroup of A_n for some $n \geq 1$ (4) no finite abelian group is a quotient of S_n for $n \geq 3$
48.	Let S be the set of all integers from 100 to 999 which are neither divisible by 3 nor divisible by 5. Then the number of elements in 5 is (1) 240 (2) 360 (3) 420 (4) 480
49.	Consider the ideal $I = <(x^2+1), y>$ in the polynomial ring $\mathbb{C}[x, y]$. Which of the following statements is true ? (1) I is a maximal ideal (2) I is a maximal ideal but not a prime ideal (3) I is prime ideal but not a maximal ideal (4) I is neither a prime ideal nor a maximal ideal

Question No.	Questions
50.	Let $f : \mathbb{Z} \to (\mathbb{Z} \to /4\mathbb{Z}) \text{ X } (\mathbb{Z}/6\mathbb{Z})$ be the function defined by $f(n) = (n \bmod 4, n \bmod 6)$. Then (1) Kernel of $f = 24\mathbb{Z}$ (2) Image of f has exactly 6 elements (3) (a mod 4, b mod 6) is in the Image of f, for all even integers a and b (4) (0 mod 4, 3 mod 6) is in the Image of f
51.	Which of the following matrices is not diagonalizable over $\mathbb{R}$? (1) $\begin{pmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ (2) $\begin{pmatrix} 2 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ (3) $\begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix}$ (4) $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$
52.	The rank of the matrix $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 & 3 \\ 1 & 2 & 3 & 4 & 4 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}$ is (1) 2 (2) 3 (3) 4 (4) 5
53.	Let $A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 2 & 3 & \alpha \end{pmatrix}, b = \begin{pmatrix} 1 \\ 3 \\ \beta \end{pmatrix}$ Then the system $AX = b$ over the field of real numbers has (1) no solution wherever $\beta \neq 7$ (2) an infinite number of solutions if $\alpha = 2$ and $\beta \neq 7$ (3) unique solution if $\alpha \neq 2$ (4) an infinite number of solutions whenever $\alpha \neq 2$

Question No.	Questions
54.	Which of the following subset of $\mathbb{R}^4$ is basis of $\mathbb{R}^4$? $B_1 = \{(1,0,0,0), (1,1,0,0), (1,1,1,0), (1,1,1,1)\}$ $B_2 = \{(1,0,0,0), (1,2,0,0), (1,2,3,0), (1,2,3,4)\}$ $B_3 \{(1,2,0,0), (0,0,1,1), (2,1,0,0), (-5,5,0,0)\}$ (1) B_1 and B_3 but not B_2 (2) B_1, B_2 and B_3 (3) only B_1 (4) B_1 and B_2 but not B_3
55.	Let A be an invertible real n×n matrix. Let $F : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ defined by F (x, y) = (Ax, y), where (x, y) denote the inner product of x and y. Let DF (x, y) denote the derivative of F at (x, y) which is a linear transformation from $\mathbb{R}^n \times \mathbb{R}^n \to R$. The which of the following is incorrect ? (1) if $x \neq 0$, then DF (x, 0) $\neq 0$ (2) if $x \neq 0$, then DF (0, y) $\neq 0$ (3) if x = 0 or y = 0, then DF (x, y) = 0 (4) if (x, y) $\neq$ (0, 0), then DF (x, y) $\neq 0$
56.	Let P (n) be a polynomial of degree $d \geq 2$. The radius of convergence of the power series $\sum_{n=0}^{\infty} P(n) z^n$ is (1) 0 (2) 1 (3) ∞ (4) depends upon d
57.	The residue of the function $f(z) = e^{-e^{\frac{1}{z}}}$ at z = 0 is (1) $1 + e^{-1}$ (2) e^{-1} (3) $-e^{-1}$ (4) $1 - e^{-1}$

Question No.	Questions
58.	Let f be a holomorphic function on $0 < \lvert z \rvert < \varepsilon$, $\varepsilon > 0$, given by a convergent Laurent series $\sum_{n=-\infty}^{\infty} a_n z^n$. Also given that $\lim_{z \to 0} \lvert f(z) \rvert = \infty$. We can conclude that (1) $a_{-1} \neq 0$ and $a_{-n} = 0 \ \forall n \geq 2$ (2) $a_{-n} = 0 \ \forall n \geq 1$ (3) $a_{-n} \neq 0 \ \forall n \geq 1$ (4) $a_{-m} \neq 0$ for some $m \geq 1$ and $a_{-n} = 0 \ \forall n > m$
59.	Let C denote the unit circle centred at the origin in Argand's plane. The value of the integral $\frac{1}{2\pi i} \int_C \lvert 1 + z + z^2 \rvert^2 dz$, when integral in taken anti-clockwise along C, equals (1) 1 (2) 2 (3) 0 (4) 3
60.	Let $f : \mathbb{C} \to \mathbb{C}$ be a non-constant entire function and let Image (f) = $\{w \in \mathbb{C} : z \in \mathbb{C}$ s.t. $f(z) = w\}$. Then (1) The interior of image (f) is empty (2) Image (f) intersects every line passing through the origin (3) There exists a disc in complex plane which is disjoint from image (f) (4) Image (f) contains all its limit points

Question No.	Questions
61.	Let S_7 denote the group of permutations of the set {1, 2, 3, 4, 5, 6,7}. Which of the following is true ? (1) There are no elements of order 10 in S_7 (2) There are no elements of order 8 in S_7 (3) There are no elements of order 7 in S_7 (4) There are no elements of order 6 in S_7
62.	The number of group homomorphism from $\mathbb{Z}_{10}$ to $\mathbb{Z}_{20}$ is (1) 0 (2) 1 (3) 5 (4) 10
63.	Let Aut (G) denote the group of automorphisms of G for a group G. Which of the following is true ? (1) If G is cyclic, then Aut (G) is cyclic (2) If G is finite, then Aut (G) is finite (3) If G is infinite, then Aut (G) is infinite (4) If Aut (G) is isomorphic to Aut (H), where G and H are two groups, then G is isomorphic to H.
64.	Let $f(x) = x^7 - 105x + 12$. Then which of the following is correct ? (1) f (x) is reducible over Q (2) $\exists$ an integer m s.t. $f(m) = 2$ (3) $\exists$ an integer m s.t. $f(m) = 105$ (4) f (m) is not a prime number for any integer m.
65.	The degree of splitting field of $x^4 - 1$ ove $\mathbb{Q}$ is (1) 2 (2) 4 (3) 1 (4) 0

Question No.	Questions
66.	Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous map. Which of the following is a correct statement ? (1) f is bounded (2) Image of f is an open subset of $\mathbb{R}$ (3) f (A) is bounded for all bounded subsets A of $\mathbb{R}$ (4) f^{-1} (A) is compact for all compact subsets A of $\mathbb{R}$.
67.	Let A and B be topological spaces where B is Hausdorff. Let A × B be the given product topology. Then for a function $f : A \to B$ which of the following statements is necessarily true ? (1) if B is finite, then f is continuous (2) if f is continuous, then graph $(f) = \{(x, f(x)) / x \in A\}$ is closed in A × B (3) if group (f) is closed in A × B, then f is continuous (4) if group (f) is closed in A × B, then f is continuous and bounded
68.	Let A be a subset of $\mathbb{R}$ and $A = \bigcap_{n \geq 1} V_n$, where for each $n \geq 1, V_n$ is an open dense subset of $\mathbb{R}$. Which of the following are incorrect ? (1) A is countable (2) A is uncountable (3) A is dense in $\mathbb{R}$ (4) A is a non-empty set
69.	Let X be a topological space and let U be a proper dense open subset of X. Choose the correct statement : (1) If X is connected, then U is connected (2) If X is compact, then U is compact (3) If X is compact, then X\U is compact (4) If X\U is compact, then X is compact

Question No.	Questions
70.	Let G be an open set in $\mathbb{R}^n$. Two points $x, y \in G$ are defined to be equivalent if they can be joined by a continuous path completely lying inside G. Then number of equivalent classes is (1) only one (2) at most countable (3) at most finite (4) can be finite or uncountable
71.	If f (x) and g (x) are two solutions of $\cos x \frac{d^2y}{dx_2} + \sin x \frac{dy}{dx} - \left(1 + e^{-x^2}\right) y = 0,\ x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ such that $f(0) = \sqrt{2},\ f'(0) = 1;\ g(0) = -\sqrt{2},\ g'(0) = 2.$ Then Wronskian of f and g is (1) $3 \cos x$ (2) $3\sqrt{2} \cos x - x$ (3) $3\sqrt{2} \cos x$ (4) 0
72.	The critical point (0, 0) of the system $\frac{dx}{dt} = 2x - 7y,\ \frac{dy}{dt} = 3x - 8y$ is (1) unstable node (2) asymptotically stable node (3) unstable spiral point (4) asymptotically stable spiral point
73.	Given a boundary value problem $y''(x) + \lambda y(x) = 0;\ y(0) = 0,\ y(\pi) = 0$ Set of its eigen values is (1) $\mathbb{R}$ (2) $(-\infty, 0)$ (3) $\{n^2 : n \in \mathbb{Z}^+\}$ (4) $\{\sqrt{n} : n \in \mathbb{Z}^+\}$

Question No.	Questions
74.	The limiting value of y (x), as $x \to \infty$, where y (x) is the solution of $y'(x) = ay - by^2$; a, b > 0, $y(0) = y_0$, will be (1) 0 (2) a/b (3) b/a (4) y_0
75.	Given a differential equation $x''(t) + p(t)\,x'(t) + q(t)\,x(t) = 0;\ p(t), q(t) \in C^1[a, b]$. Let f (t) and g (t) be its two solutions on [a, b]. Then which of the following is incorrect ? (1) f and g are linearly dependent and $W(f, g)(t) = 0\ \forall t \in [a, b]$ (2) f and g are linearly in dependent and $\exists$ to $\in (a, b)$ s.t. $W(t_0) = 0$ (3) f and g are linearly in dependent and $W(f, g)(t) \neq 0$ for any $t \in [a, b]$ (4) f and g are linearly independent then every other solution can be written as their linear combination
76.	Let D denote the disc $\{(x, y) \mid x^2 + y^2 \le 1\}$ and let D^c be its complement in the plane. The P.D.E $(x^2 - 1)\dfrac{\partial^2 u}{dx^2} + 2y\dfrac{\partial^2 u}{dxdy} - \dfrac{\partial^2 u}{dy^2} = 0$ is (1) Parabolic $\forall\ (x, y) \in D^c$ (2) Hyperbolic $\forall\ (x, y) \in D$ (3) Hyperbolic $\forall\ (x, y) \in D^c$ (4) Parabolic $\forall\ (x, y) \in D$

Question No.	Questions
77.	Given a partial differential equation $x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = (x+y)\, z$. Then which of the following is not the general solution of the differential equation ? (1) $F\left(\frac{xy}{z}, \frac{x-y}{z}\right) = 0$ for an arbitrary twice differentiable function F (2) $F\left(\frac{x-y}{z}, \frac{1}{x} - \frac{1}{y}\right) = 0$ for an arbitrary twice differentiable function F (3) $z = f\left(\frac{1}{x} - \frac{1}{y}\right)$ for an arbitrary twice differentiable function f (4) $z = xyf\left(\frac{1}{x} - \frac{1}{y}\right)$ for an arbitrary twice differentiable function f
78.	The Cauchy problem $2u_x + 3u_y = 5$, $u = 1$ on the line $3x - 2y = 0$ has (1) exactly one solution (2) exactly two solutions (3) infinitely many solutions (4) no solution
79.	Let u be the unique solution of $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, where $(x, t) \in (0, 1) \times (0, \infty)$ $u(x, 0) = \sin \pi x,\ x \in (0,1)$ $u(0, t) = u(1, t) = 0,\ t \in (0, \infty)$ Then which of the following is true ? (1) $\exists\, (x, t) \in (0,1) \times (0, \infty)$ s.t. $u(x, t) = 0$ (2) $\exists\, (x, t) \in (0,1) \times (0, \infty)$ s.t. $\frac{\partial u}{\partial t}(x, t) = 0$ (3) the function $e^t u(x, t)$ is bounded for $(x, t) \in (0,1) \times (0, \infty)$ (4) $\exists\, (x, t) \in (0,1) \times (0, \infty)$ s.t. $u(x, t) > 1$

Question No.	Questions
80.	Let u (x, t) be the solution of the initial value problem $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$; $u(x, 0) = x^3$, $\frac{\partial u}{\partial t}(x, 0) = \sin x$ Then $u(\pi, \pi)$ is (1) $4\pi^3$ (2) $2\pi^3$ (3) 0 (4) 4
81.	Completeness of a metric space is preserved under (1) Isometry (2) Homeomorphism (3) Continuous function (4) Bijective function
82.	Given an interval (– 1, 1) and a sequence $\{a_n\}$ of elements in it. Then (1) Every limit point of $\{a_n\}$ is an (–1, 1) (2) The limit points of $\{a_n\}$ can only be in {–1, 0, 1} (3) Every limit point of $\{a_n\}$ is in [–1, 1] (4) The limit point of $\{a_n\}$ cannot be in {–1, 0, 1}
83.	If f is a function $f : \mathbb{R} \to \mathbb{R}$ s.t. $f(0) = 0$ and $\lvert f'(x) \rvert \leq \forall x$, then f (1) is in (1) (5, 6) (2) [– 4, 4] (3) (– ∞, – 5) ∪ (5, ∞,) (4) [– 5, 5]
84.	Let $A = \{n \in \mathbb{N} : n = 1$ or the only prime factors of n are 2 or $3\}$. Let $S = \sum_{n \in A} \frac{1}{n}$, then (1) S is divergent series (2) A is finite (3) S = 3 (4) S = 6

Question No.	Questions
85.	Let $x_n(t) = te^{-nt^2}$, $t \in \mathbb{R}$, $n \geq 1$. Then the sequence $\{x_n\}$ is (1) uniformly convergent on $\mathbb{R}$ (2) a sequence of unbounded functions (3) bounded and not uniformly convergent on $\mathbb{R}$ (4) uniformly convergent only on compact subsets of $\mathbb{R}$
86.	Which of the following is necessarily true for a function $f : A \to B$ (1) if f as injective, then $\exists\ g : B \to A$ s.t. $f(g(y)) = y\ \forall y \in B$ (2) if f is injective and B is countable, then A is finite (3) if f is subjective and A is countable, then B is countably infinite (4) if f is subjective, then $\exists\ g : B \to A$ s.t. $f(g(y)) = y\ \forall y \in B$
87.	The difference $\log(2) - \sum_{n=1}^{100} \frac{1}{2^n \cdot n}$ is (1) less than 0 (2) less than $\frac{1}{2^{100}.101}$ (3) greater than 1 (4) greater than $\frac{1}{2^{100}.101}$
88.	Let f be function defined on the set $S = \left\{x \in \mathbb{R}, x \geq 0, x \neq n\pi + \frac{\pi}{2}, n \in \mathbb{N} \cup \{0\}\right\}$ and $f(x) = \tan x$. Then (1) f has a unique fixed point on S (2) there is no fixed point of f on S (3) f has infinitely many fixed points on S (4) f has finite number of fixed points on S

Question No.	Questions
89.	A function $f: \mathbb{R} \to \mathbb{R}$ need not be lebesgue measurable if (1) $\{x \in \mathbb{R} : f(x) \geq \alpha\}$ is measurable for each $\alpha \in \mathbb{Q}$ (2) $\{x \in \mathbb{R} : f(x) = \alpha\}$ is measurable for each $\alpha \in \mathbb{R}$ (3) for each open set G in $\mathbb{R}$, $f^{-1}(G)$ is measurable (4) f is monotone
90.	Let $f(x) = \left(1+\frac{1}{x}\right)^x$ and $g(x) = \left(1+\frac{1}{x}\right)^{x+1}$, $x > 0$. then (1) f (x) and g (x) both are increasing functions (2) f (x) is increasing and g (x) is decreasing (3) f (x) is decreasing and g (x) is increasing (4) f (x) and g (x) both are decreasing functions
91.	Let $\psi(t) = e^{-\lvert t\rvert \frac{-t^2}{2}}$ and $\phi(t) = \frac{e^{-\lvert t\rvert} + e^{\frac{-t^2}{2}}}{2}$. Which of the following is true ? (1) ϕ is a characteristic function but ψ is not (2) ψ is a characteristic function but ϕ is not (3) neither ϕ nor ψ is a characteristic function (4) Both ϕ and ψ are characteristic functions
92.	If $b_{xy} = 0.20$ and $r_{xy} = 0.50$, Then b_{yx} is equal to (1) 0.20 (2) 0.25 (3) 0.50 (4) 1.25

Question No.	Questions
93.	Let $X_1, X_2, \ldots\ldots, X_n$ be independent random variables ; X_i having exponential distribution with parameter θ_i, $i = 1, 2, \ldots, n$. Then $Z = \text{Min}(X_1, X_2, \ldots, X_n)$ has (1) normal distribution (2) geometric distribution (3) exponential distribution with parameter $\sum_{i=1}^{n} \theta_i$ (4) None of these
94.	In queueing description M | M | 1, the arrival and departure distribution are (1) Binomial (2) General (3) Both Markovian (4) None of these
95.	Successful life of product, time, weight and height are classified as (1) continuous random variable (2) discrete random variable (3) continuous time variable (4) None of these
96.	If a random variable X has a Chi-Square distribution with 4 degree of freedom, then its mean is equal to (1) 2 (2) 3 (3) 4 (4) None of these
97.	In a Latin Square Design, if factors A, B, C and D have levels 8, then the total number of cells in the design is (1) 4096 (2) 64 (3) 512 (4) None of these

Question No.	Questions
98.	If a system has two components in parallel with each of reliability 0.75, then the reliability of the system is equal to (1) 0.9375 (2) 0.9753 (3) 0.7935 (4) None of these
99.	Let (v, b, r, k, w) be the standard parameters of a balanced incomplete block design (BIBD). Which of the following (v, b, r, k, w) can be parameters of BIBD ? (1) (v, b, r, k,w) = (44, 33, 9, 12, 3) (2) (v, b, r, k,w) = (17, 45, 8, 3, 1) (3) (v, b, r, k,w) = (35, 35, 17, 17, 9) (4) (v, b, r, k,w) = (16, 24, 9, 6, 3)
100.	100 tickets are marked 1, 2, ... , 100 and arranged at random. Four tickets are picked from these tickets and given to four persons A, B, C and D. What is the probability that A gets the ticket with the largest value (among A, B, C, D) and D gets the ticket with smallest value (among A, B, C, D) ? (1) $\frac{1}{4}$ (2) $\frac{1}{12}$ (3) $\frac{1}{2}$ (4) $\frac{1}{6}$

ANSWER KEY 'A'

1	2	3	4	5	6	7	8	9	10
1	3	4	3	1	4	2	3	4	2
11	12	13	14	15	16	17	18	19	20
3	2	2	3	3	4	4	1	2	1
21	22	23	24	25	26	27	28	29	30
1	4	3	4	3	2	3	4	2	2
31	32	33	34	35	36	37	38	39	40
4	3	2	1	4	3	3	4	4	3
41	42	43	44	45	46	47	48	49	50
2	4	2	4	1	3	2	1	3	2
51	52	53	54	55	56	57	58	59	60
3	3	3	2	1	4	3	2	1	3
61	62	63	64	65	66	67	68	69	70
3	2	3	2	2	3	3	4	3	1
71	72	73	74	75	76	77	78	79	80
2	4	2	1	3	2	3	3	4	3
81	82	83	84	85	86	87	88	89	90
4	4	3	3	1	3	2	1	4	2
91	92	93	94	95	96	97	98	99	100
3	2	2	2	4	1	4	1	2	4

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ANSWER KEY 'B'

1	2	3	4	5	6	7	8	9	10
2	4	2	1	3	2	3	3	4	3
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
3	3	3	2	1	4	3	2	1	3
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
4	3	2	1	4	3	3	4	4	3
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
3	2	2	3	3	4	4	1	2	1
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
3	2	2	2	4	1	4	1	2	4
51	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
3	2	3	2	2	3	3	4	3	1
61	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**
4	4	3	3	1	3	2	1	4	2
71	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
2	4	2	4	1	3	2	1	3	2
81	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**
1	4	3	4	3	2	3	4	2	2
91	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**	**100**
1	3	4	3	1	4	2	3	4	2

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ANSWER KEY 'C'

1	2	3	4	5	6	7	8	9	10
2	4	2	4	1	3	2	1	3	2
11	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
1	4	3	4	3	2	3	4	2	2
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
1	3	4	3	1	4	2	3	4	2
31	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
3	2	2	2	4	1	4	1	2	4
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**
3	2	3	2	2	3	3	4	3	1
51	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
4	3	2	1	4	3	3	4	4	3
61	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**
2	4	2	1	3	2	3	3	4	3
71	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
4	4	3	3	1	3	2	1	4	2
81	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**
3	2	2	3	3	4	4	1	2	1
91	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**	**100**
3	3	3	2	1	4	3	2	1	3

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ANSWER KEY 'D'

1	2	3	4	5	6	7	8	9	10
3	2	2	3	3	4	4	1	2	1
11	12	13	14	15	16	17	18	19	20
3	2	2	2	4	1	4	1	2	4
21	22	23	24	25	26	27	28	29	30
2	4	2	1	3	2	3	3	4	3
31	32	33	34	35	36	37	38	39	40
3	3	3	2	1	4	3	2	1	3
41	42	43	44	45	46	47	48	49	50
4	3	2	1	4	3	3	4	4	3
51	52	53	54	55	56	57	58	59	60
1	4	3	4	3	2	3	4	2	2
61	62	63	64	65	66	67	68	69	70
2	4	2	4	1	3	2	1	3	2
71	72	73	74	75	76	77	78	79	80
3	2	3	2	2	3	3	4	3	1
81	82	83	84	85	86	87	88	89	90
1	3	4	3	1	4	2	3	4	2
91	92	93	94	95	96	97	98	99	100
4	4	3	3	1	3	2	1	4	2

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