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MPHDURS-EE-2013

SUBJECT : Mathematics

18/9/13

10160

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Sr. No. ....

Time : 1¼ **Hours** Max. Marks : **100** Total Questions : **100**

Candidate's Name ____________________ Date of Birth ____________

Father's Name ____________________ Mother's Name ____________________

Roll No. (in figures) ____________ (in words) ____________________

Date of Examination ____________________

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1. ***All questions are compulsory and carry equal marks.***
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1. The result "Let $< f_n >$ be a sequence of non-negative measurable functions which converge almost everywhere on a set E to a function f. Then $\int_E f \le \underline{\lim} \int_E f_n$ " is known as :
   (1) F. Riesz Theorem
   (2) Bounded Convergence Theorem
   (3) Fatou's Lemma
   (4) Lebesgue Monotone Convergence Theorem

2. The members of the smallest σ-algebra which contains all of the open sets are called :
   (1) Lebesgue sets (2) Borel sets
   (3) σ-open sets (4) Lebesgue measurable sets

3. For $o \le p \le 1$, the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ is :
   (1) convergent but not absolutely (2) convergent
   (3) absolutely convergent (4) oscillatory

4. The series $\sum_{n=1}^{\infty} \frac{\cos n\theta}{n^p}$ converges uniformly for all values of θ if :
   (1) $p \ge 1$ (2) $p < 1$ (3) $p \le 1$ (4) $p > 1$

5. Outermeasure is a set function whose domain is :
   (1) P(R) (2) R
   (3) Collection of all measurable sets (4) Collection of all continuous functions

6. Which of the following is ***not true*** ?
   (1) Every absolutely continuous function is of bounded variation
   (2) Every bounded function is of bounded variation
   (3) Every monotone function on $[a, b]$ is of bounded variation
   (4) Every function of bounded variation is bounded

7. The word "Topologi" was introduced in Germany in 1847 by :
   (1) George Contor (2) Johann Benedict
   (3) Kazimierz Kuratowski (4) Felix Hausdroff

8. A function which is analytic for all finite values of Z and bounded is :
   (1) a constant (2) zero (3) a function of Z (4) continuous

9. The result "The order of a canonical product is equal to the exponent of convergence of its zeros" is known as :

(1) Borel's theorem (2) Jensen's formula

(3) Bloch's theorem (4) Morera's theorem

10. The constant

$$r = \lim_{n\to\infty}\left(1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}-\log n\right)$$

is called :

(1) Euler's constant (2) Euler's number

(3) Lebesgue constant (4) Lebesgue number

11. The function $D:R \to R$ such that

$$D(x)=\begin{cases}1 \text{ if } & x\in Q\\ 0 \text{ if } & x\notin Q\end{cases}$$

is known as :

(1) Step Function (2) Simple Function

(3) Characteristic Function (4) Dirichlet's Function

12. Every convergent sequence of measurable functions is nearly uniformly convergent. This result is known as :

(1) 1st principle of measurability (2) Littlewood's 2nd principle of measurability

(3) Littlewood's third principle (4) Egorov's theorem

13. If a_n and b_n are sequences of extended real numbers and $a_n \le b_n$ for all n sufficiently large. Which of the following is ***not true*** ?

(1) $\lim\inf a_n \ge \lim\inf b_n$ (2) $\lim\inf a_n \le \lim\inf b_n$

(3) $\lim\sup a_n \le \lim\sup b_n$ (4) None of these

14. The composition of two Lebesgue measurable functions is :

(1) not necessarily Lebesgue measurable

(2) Borel measurable

(3) always measurable

(4) always Lebesgue measurable

15. Every uniformly continuous function is :

(1) Absolutely continuous (2) Not absolutely continuous

(3) Not Continuous (4) None of these

16. Which of the following statements is ***not correct*** ?

(1) Ch. function of irrational numbers in [0, 1] is Riemann integrable

(2) Ch. functions are simple functions

(3) Ch. function of the set E of rational numbers in [0, 1] is measurable

(4) None of the above

17. Let A be the set of algebraic numbers. Then the outer measure of A is equal to :

(1) ∞

(2) a finite measure

(3) zero

(4) outer measure of the set of real numbers

18. The axiom of choice was formulated in 1904 by :

(1) Riemann (2) Ernst Zermelo (3) G. H. Moore (4) George Cantor

19. The result "Let (–1, 1) be interval of convergence for the power series $\sum a_n x^n$. If $\sum_{n=0}^{\infty} a_n = S$, then $\lim_{x \to 1-0} \sum_{n=0}^{\infty} a_n x^n = S$" is known as :

(1) Uniqueness theorem

(2) Weierstrass's theorem

(3) Tauber's theorem

(4) Abel's theorem

20. If a function f is convex and $f(0) \leq 0$, then :

(1) f is superadditive on the positive half axis

(2) f is additive

(3) f is subadditive on the positive half axis

(4) f is superconvex

21. A condition is said to be steady-state if the dependent variables are :

(1) Not present in Heat equation

(2) Independent of time t

(3) Dependent on time t

(4) None of these

22. The one-dimensional wave equation for an elastic string of length L under boundary conditions $y(0, t) = 0$, $y(L, t) = 0$ indicates that :

(1) the string is not fixed at $x = 0$

(2) the string is only fixed at $x = 0$

(3) the string is fastened at both ends

(4) none of these

23. If H represents Hamiltonian function, then $\frac{dH}{dt}$ is equal to :

(1) $\frac{\partial H}{\partial t}$ (2) $\frac{\partial^2 H}{\partial t^2}$ (3) $\frac{d^2 H}{dt^2}$ (4) None of these

24. The two dimensional Laplace equation in polar co-ordinates is given by :

(1) $\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}=0$

(2) $\frac{\partial^2 u}{\partial r^2}+\frac{\partial u}{\partial r}+\frac{1}{r}\frac{\partial^2 u}{\partial \theta^2}=0$

(3) $\frac{\partial u}{\partial r}+\frac{1}{r}\frac{\partial^2 u}{\partial \theta^2}+\frac{1}{r^2}\frac{\partial^2 u}{\partial r^2}=0$

(4) $\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}=0$

25. For the heat conduction equation $\frac{\partial u}{\partial t}=c\frac{\partial^2 u}{\partial x^2}$ in a bar subject to the boundary conditions that the end $x = 0$ is held at zero temperature and the end $x = 1$ is at temperature zero, the boundary conditions can be expressed at :

(1) $u(0, t) \neq 0; u(1, t) = 0$

(2) $u(1, t) \neq 0; u(0, t) = 0$

(3) $u(0, t) = 0; u(1, t) = 0$

(4) $u(0, t) \neq 0; u(1, t) \neq 0$

26. The boundary value problem which models the displacement function for a semi-infinite string which is initially undisturbed and is given an initial velocity is expressed as :

(1) $\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}$; $u(x, 0) \neq 0$

(2) $\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}$; $u(0, t)=0$; $u(x, 0)=0$

(3) $\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}$; $u(x, 0)=0$; $\frac{\partial u}{\partial t}(x, 0)=0$

(4) $\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}$; $u(x, 0)=0$; $\frac{\partial u}{\partial t}(x, 0)=f(x)$

27. For the Lagrangian function $L(t, q_i, q_i)$ the generalized momenta p_i is defined as :

(1) $p_i=\frac{\partial L}{\partial q_i}$ (2) $p_i=\frac{\partial L}{\partial \dot{q}_i}$ (3) $p_i=\frac{\partial^2 L}{\partial q_i^2}$ (4) None of these

28. If a lead is sliding on a uniformly rotating wire in a force free space, then the equations of motion are :

(1) $\ddot{r}=rw^2$ (2) $\dot{r}=rw^2$ (3) $\ddot{r}=rw$ (4) $r=\ddot{r}w^2$

29. Principle of least action states that the variation of the Lagrange action W^* is zero for :

(1) the parabolic path

(2) the circular path

(3) any path

(4) the straight line path

30. Which one of the following form a set of Routh's equations ?

(1) $\frac{dq_\alpha}{dt}=\frac{\partial R}{\partial p_\alpha}, \frac{dp_\alpha}{dt}=-\frac{\partial R}{\partial q_\alpha}$
(2) $\frac{dq_\alpha}{dt}=-\frac{\partial R}{\partial p_\alpha}, \frac{dp_\alpha}{dt}=-\frac{\partial R}{\partial q_\alpha}$

(3) $\frac{dq_\alpha}{dt}=-\frac{\partial R}{\partial p_\alpha}, \frac{dp_\alpha}{dt}=\frac{\partial R}{\partial q_\alpha}$
(4) $\frac{dq_\alpha}{dt}=\frac{\partial R}{\partial p_\alpha}=-\frac{\partial R}{\partial q_\alpha}$

31. Solution of the I. V. P.

$$\frac{dy}{dx}=-y,\ y(0)=1 \text{ is :}$$

(1) e^t (2) e^{-t} (3) $e^{-t/2}$ (4) $e^{t/2}$

32. Solution of the integral equation $\int_0^x e^{x-t}u(t)dt = x$ is :

(1) $x-1$ (2) x^2-1 (3) $1-x$ (4) x

33. The eigen values of the integral equation

$$u(x)=\lambda \int_{-1}^{1}(x+t)u(t)dt \text{ are :}$$

(1) $\pm\frac{\sqrt{3}}{2}$ (2) $\pm i\frac{\sqrt{3}}{2}$ (3) $\pm i\sqrt{3}$ (4) $1\pm i\sqrt{3}$

34. If the homogeneous Fredholm integral equation :

$$u(x)=\lambda \int_a^b k(x,t)\,u(t)\,dt$$

has only a trivial solution, then the corresponding non-homogeneous equation has always :

(1) no solution
(2) Infinite number of solutions
(3) a unique solution
(4) only trivial solution

35. Which of the following theorem expresses the symmetric Kernel of a Fredholm integral equation as an infinite series of product of its orthogonal eigen functions ?

(1) Poincare Bendixon Theorem
(2) Bendixon Theorem
(3) Hilbert-Schmidt Theorem
(4) Mercer's Theorem

36. The problem of Brachistochrone (shortest time) was first formulated in the year 1696 by :

(1) Newton (2) Jeans Bernouli (3) Leibnitz (4) Jacques Bernouli

37. The curve which minimizes the functional $J(y) = \int_a^b (x-y)^2 \, dx$ is :

(1) $x - y = 0$ (2) $x + y = 0$ (3) $x - 2y = 0$ (4) $y - 2x = 0$

38. The geodesics of the circular cylinder $\vec{r} = (a\cos\phi, a\sin\phi, z)$ is :

(1) Circle (2) Catenary (3) Straight line (4) Helix

39. In the Lipschitz condition $|f(t,y_1) - f(t,y_2)| \le k|y_1 - y_2|$ condition on k is :

(1) $k > 0$ (2) $k \ge 0$ (3) $0 < k \le 1$ (4) $k < 1$

40. If a rigid body rotates about a fixed point with an angular velocity $\vec{\omega}$ and has an angular momentum $\vec{H}$, then the kinetic energy T is given by :

(1) $\vec{\omega} \times \vec{H}$ (2) $\dfrac{\Delta.\vec{\omega}}{\vec{H}}$ (3) $\dfrac{1}{2}\vec{\omega}.\vec{H}$ (4) none of these

41. "A function $f(z)$ whose only singularities in the entire complex plane are poles" is called a :

(1) Analytic Function (2) Harmonic Function

(3) Entire Function (4) Meromorphic Function

42. Which of the following statement is ***not correct*** ?

(1) Subspace of Hausdorff space is Hausdorff

(2) Product of two Hausdorff spaces is Hausdorff

(3) The space X is Hausdorff if and only if the diagonal $\Delta = \{x \times x ; x \in X\}$ is open in XXX

(4) The space X is Hausdorff if and only if the diagonal $\Delta = \{x \times x ; x \in X\}$ is closed in XXX

43. The result "Let $X = A \cup B$ where A and B are closed in X. Let $f : A \to Y$ and $g : B \to Y$ be continuous. If $f(x) = g(x)$ for every $x \in A \cap B$, then f and g combine to give a continuous function $h : X \to Y$ defined by setting $h(x) = f(x)$ if $x \in A$ and $h(x) = g(x)$ if $x \in B$" is called :

(1) Pasting Lemma (2) Zorn's Lemma

(3) Embedding Lemma (4) Sequence Lemma

44. Every metric space is :

(1) Normed space (2) Paracompact

(3) Compact (4) Not first axiom sapce

45. If J is the Jacobian of functions u and v w.r.t. x and y and J_0 is the Jacobian of x and y. w.r.t. u and v, then :

(1) $JJ_0 = 1$ (2) $JJ_0 = 0$ (3) $JJ_0 = -1$ (4) $JJ_0 = 2$

46. Any infinite cyclic group has exactly k generators where :

(1) $k = 1$ (2) $k = 3$ (3) $k = 2$ (4) $k = 7$

47. The index of a saddle point is :

(1) 0 (2) 1 (3) -1 (4) does not exist

48. Let $F = \{f\}$ be an equicontinuous family of functions defined on a real interval I, then each function f is :

(1) continuous on I (2) uniformly continuous on I

(3) not continuous on I (4) constant on I

49. The critical point (0, 0) of the system $\frac{dx}{dt} = 4y, \frac{dy}{dt} = x$ is :

(1) stable (2) asymptotically stable

(3) not stable (4) stable but not asymptotically stable

50. Consider the linear autonomous system

$$\frac{dx}{dt} = ax + by, \frac{dy}{dt} = cx + dy$$

where a, b, c, d are real constants. If $a = d$ and b and c are of same sign such that $\sqrt{bc} < |a|$, then the critical point (0, 0) of the system is :

(1) saddle point (2) spiral point (3) node (4) centre

51. The concept of reflexivity was introduced by :

(1) H. Hahn (2) F. Riesz (3) R. C. James (4) D. Hilbert

52. Which of the following is ***not*** a Hilbert space ?

(1) R^n (2) l_2 (3) $L_2[0, 1]$ (4) $L_1[0, 1]$

53. In a normed linear space, weak convergence implies strong convergence if :

(1) $\dim X < \infty$ (2) $\dim X > \infty$ (3) $\dim X = \infty$ (4) none of these

54. Which of the following is ***not true*** ?

(1) C as a real vector space is of dimension two

(2) $\mathbb{C}$ as a complex vector space is of dimension one

(3) l_1 is reflexive

(4) Space $C[a, b]$ is dense in $L_p[a, b]$

55. If x and y are orthonormal vectors in a Hilbert space H, then :

(1) $\|x-y\|=2$ (2) $\|x-y\|=\sqrt{2}$ (3) $\|x-y\|=0$ (4) $\|x-y\|=1$

56. L^p-spaces are complete. This result is known as :

(1) F. Riesz Theorem (2) Riesz Fisher Theorem

(3) Lebesgue Theorem (4) Jordan Decomposition Theorem

57. If P is a projection on a closed linear subspace M of H, then M is invariant under :

(1) $TP = PT$ (2) $P = TPT$ (3) $T = PTP$ (4) $TP = PTP$

58. If P is a projection on a Hilbert space H, then which of the following is ***not true*** ?

(1) P is a positive operator (2) $O \leq P \leq 1$

(3) $\|P\| > 1$ (4) $\|Px\| \leq \|x\| \quad \forall x \in H$

59. A one to one continuous linear transformation of a Banach space onto another Banach space is a :

(1) Homomorphism (2) Homeomorphism

(3) Closed Mapping (4) Open Mapping

60. A subspace Y of a Banach space Y is complete if and only if :

(1) The set Y is open in X (2) The set Y is complete in X

(3) The set Y is closed in X (4) None of the above

61. Which one of the following is ***not*** a topological property ?

(1) Boundedness (2) Compact (3) Closed (4) Open

62. Every metric space is paracompact. This theorem is named after :

(1) Stone (2) Michael (3) Lindelof (4) Hausdorff

63. Every convergent sequence in a topological space has a unique limit if X is :

(1) First countable Hausdorff space (2) T_1-space

(3) Hausdorff space (4) Second countable space

64. Regular spaces were first studied in 1921 by :

(1) Victoris (2) Hausdorff (3) Kolmogorov (4) Tietz

65. The result "A topological space is a Tychonoff space if and only if it is embeddable into a cube" is known as :

(1) Embedding Lemma
(2) Tychonoff Embedding Theorem
(3) Urysohn's Metrization Theorem
(4) None of these

66. The space $C[0, 1]$ is ***not*** a :

(1) Complete space
(2) Normed linear space
(3) Metric space
(4) Regular space

67. If (X, T) is an indiscrete topological space, then it has :

(1) no component
(2) compact component
(3) finite number of components
(4) only X as the component

68. For an empty set ϕ, which statement is ***true*** ?

(1) $d(\phi) = +\infty$ (2) $d(\phi) = -\infty$ (3) $\inf(\phi) = -\infty$ (4) none of these

69. Which of the following statement is ***not true*** ?

(1) R^n is connected
(2) R is connected
(3) Q is connected
(4) C^n is connected

70. The norm $||.||$ from a vector space X to R is a :

(1) Linear functional
(2) Sublinear functional
(3) Bi-linear functional
(4) Superlinear functional

71. The basis and the degree of the extension $Q(\sqrt{2}, \sqrt{3})$ over Q is :

(1) $\{\sqrt{2}, \sqrt{3}\}$, 4
(2) $\{1, \sqrt{2}, \sqrt{3}\}$, 4
(3) $\{1, \sqrt{2}, \sqrt{3}, \sqrt{6}\}$, 4
(4) $\{1, \sqrt{2}, \sqrt{3}\}$, 2

72. The set R of real numbers is :

(1) totally bounded
(2) locally compact
(3) countably compact
(4) sequentially compact

73. Every Lindelof metric space is :

(1) Compact
(2) First countable
(3) Second countable
(4) Reducible

74. Which of the following topology is coarser than the usual topology of R ?

(1) lower limit topology on R
(2) upper limit topology on R
(3) co-countable topology on R
(4) finite complement topology on R

75. Which of the following properties is Hereditary ?

(1) 2nd axiom of countablility
(2) Compactness
(3) Lindelofness
(4) Seperability

76. The concept of normality of a topological space was introduced by :

(1) Urysohn (2) Tichonov (3) Hausdorff (4) Tietze

77. Which of the following properties is ***not*** invariant under continuous map ?

(1) Lindeloffness
(2) Separability
(3) 1st axiom of countability
(4) Compactness

78. Which of the following statement is ***not correct*** ?

(1) Cantor set is perfect
(2) Contor set is totally disconnected
(3) Cantor set is closed
(4) Cantor set is countable

79. Let N be the set of non-negative integers. Then the collection

$$H = \{F \; ; \; N - F \text{ is finite}\}$$ is known as :

(1) Atomic Filter
(2) Cofinite Filter
(3) Frechet Filter
(4) Nbd Filter

80. Which of the following statement is ***not true*** ?

(1) Usual topological space (R, V) is Hausdorff
(2) Every indiscrete space containing at least two points is metrizable
(3) Every Discrete topological space is Hausdorff
(4) All metric spaces are Hausdorff

81. The product of two odd permutations is :

(1) even and odd (2) odd (3) even (4) none of these

82. A group has almost one composition series. This result is known as :

(1) Cayley's theorem
(2) Sylow's theorem
(3) Lagrange's theorem
(4) Jordan-Holder theorem

83. If every non-constant polynomial over a field F has all its roots in F, then F is :

(1) Algebraically Closed Field (2) Prime Field

(3) Perfect Field (4) None of the above

84. Let $R = F[x]$ be a polynomial ring over a field F. Then R is :

(1) Artinian but not Noetherian (2) Artinian and Noetherian both

(3) Neither Artinian nor Noetherian (4) Noetherian but not Artinian

85. Which of the following is a prime field ?

(1) Q (2) R (3) $\mathbb{C}$ (4) Z_n

86. Let G be a commutative group having composition series. Then G must be :

(1) Infinite (2) Finite

(3) Finite with $G' = G$ (4) Infinite with $Z(G) = < e >$

87. Let M be a simple R-module and $T \in \text{Home}_R(M, M)$ such that $T \neq 0$, then :

(1) $I_m(T) = O$ (2) $\ker(T) = M$ (3) T is singular (4) T is non-singular

88. A composition series for a group is :

(1) Central series (2) Derived series

(3) Solvable series (4) None of these

89. The degree of the splitting field of the polynomial $f(x) = x^{10} - 1$ over Q is :

(1) 10 (2) 4 (3) 6 (4) 8

90. Any group of order 15 is :

(1) Abelian (2) Simple (3) Cyclic (4) p-group

91. If a function $f(z)$ is analytic except at finite number of singularities (including that at infinity), then the sum of residues of these singularities is :

(1) $2\pi i$ (2) πi (3) finite (4) zero

92. The transformation $f(z) = x - iy$ is :

(1) analytic (2) conformal (3) isogonal (4) none of these

93. The set of all bilinear transformation under the product of transformations form a :

(1) Monoid (2) Abelion group (3) Semi group (4) Non-Abelion group

94. The function $f(z) = e^{1/z}$ has essential singularity at :

(1) $Z=1$ (2) $Z=0$ (3) $Z=2$ (4) $Z=-1$

95. Which of the following statement is ***not true*** ?

(1) Exponential function is analytic

(2) Absolute value function when defined on the set of real or complex numbers is analytic

(3) Power functions are analytic

(4) Any polynomial is an analytic function

96. The simple poles of Gamma function are at :

(1) $Z=0, 1, 2, \ldots, n, \ldots$ (2) $Z=0, -1, -2, \ldots, -n, \ldots$

(3) $Z=1, 2, \ldots, n, \ldots$ (4) None of these

97. If $f(z)$ and $g(z)$ are analytic inside and on a closed contonic and $|g(z)|<|f(z)|$ on C, then $f(z)$ and $f(z)+g(z)$ have same :

(1) value (2) number of poles

(3) number of singularities (4) number of zeros

98. The residue of $f(z) = \dfrac{z^3}{z^2-1}$ at $z=\infty$ is :

(1) –1 (2) 1 (3) 0 (4) 3

99. The Taylor series of the function $\log(1+Z)$ about the point $Z=0$ is convergent for the region :

(1) $|Z|\leq 1$ (2) $|Z|<1$ (3) $|Z|\geq 1$ (4) $|Z|>1$

100. Which of the following statement is ***not correct*** ?

(1) v is a harmonic conjugate of u if and only if u is a harmonic conjugate of $-v$.

(2) An analytic function with constant modulus is constant.

(3) If v is a harmonic conjugate of u in the same domain, then u is a harmonic conjugate of v.

(4) Both the real and imaginary parts of an analytic function are harmonic.