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(PHD/URS-EE-DEC.-2022)

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13/12/2022

Poonam
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Sr. No. **10057**

Code

A

MATHEMATICS

Time : 1¼ Hours

Total Questions : 100

Max. Marks : 100

Roll No. _____ (in figure) _____ (in words)

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6. There will be no negative marking. Each correct answer will be awarded one full mark. Cutting, erasing, overwriting and more than one answer in OMR Answer-Sheet will be treated as incorrect answer.
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Question No.	Questions
4.	<p>Every bounded sequence has at least one limit point. This represents</p> <p>(1) Archimedean Property (2) Heine-Borel theorem</p> <p>(3) Bolzano-Weierstress theorem (4) Denseness Property</p>
5.	<p>Let $f : [0, \infty) \rightarrow \mathbb{R}$ be a function, where \mathbb{R} denotes the set of all real numbers. Then which one of the following statements is true ?</p> <p>(1) If f is continuous and $\lim_{x \rightarrow \infty} f(x)$ is finite, then f is uniformly continuous.</p> <p>(2) If f is bounded and continuous, then f is uniformly continuous.</p> <p>(3) If f is uniformly continuous, then $\lim_{x \rightarrow \infty} f(x)$ exists.</p> <p>(4) None of these</p>
6.	<p>Which of the following is false ?</p> <p>(1) The sequence $\{f_n\}$, where $f_n(x) = \frac{1}{x+n}$, is uniformly convergent in any interval $[0, b]$, $b > 0$.</p> <p>(2) The sequence $\{f_n\}$, where $f_n(x) = \frac{n^2 x}{1+n^3 x^2}$, is uniformly convergent on the interval $[0, 1]$.</p> <p>(3) The sequence $\{f_n\}$, where $f_n(x) = \tan^{-1} nx$, $x \geq 0$, is uniformly convergent in any interval $[a, b]$, $a > 0$.</p> <p>(4) None of these</p>

Question No.	Questions
7.	<p>For which of the following function, Rolle's theorem is not applicable?</p> <p>(1) $f(x) = \cos 2x$ in $[-\pi/4, \pi/4]$ (2) $f(x) = \frac{\sin x}{e^x}$ in $[0, \pi]$</p> <p>(3) $f(x) = x^3 - 6x^2 + 11x - 6$ in $[1, 3]$ (4) $f(x) = x$ in $[-1, 1]$</p>
8.	<p>If $f(x) = x$, $x \in [0, 1]$ and let $P = \left\{0, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, 1\right\}$ be the partition of $[0, 1]$, then $U(f, P)$ is</p> <p>(1) $23/36$ (2) $31/36$</p> <p>(3) $49/36$ (4) None of these</p>
9.	<p>If a function f defined on $[0, 1]$ as $f(x) = \begin{cases} \sin\left(\frac{1}{x}\right), & \text{if } x \text{ is irrational} \\ 0, & \text{otherwise} \end{cases}$, then</p> <p>(1) f is not bounded</p> <p>(2) f is R-integrable</p> <p>(3) f is not R-integrable since f is not bounded</p> <p>(4) f is not R-integrable since lower and upper integrals of f are unequal</p>

Question No.	Questions
10.	<p>Consider the following improper integrals</p> $I_1 = \int_0^{\infty} \frac{\sin^2 x}{x^2} dx \text{ and } I_2 = \int_1^{\infty} \frac{x^3}{(1+x)^5} dx, \text{ then}$ <p>(1) Both are divergent (2) I_1 converges but not I_2 (3) I_2 converges but not I_1 (4) Both are convergent</p>
11.	<p>Which of the following functions is not a function of bounded variation ?</p> <p>(1) $f(x) = \begin{cases} x \sin(\pi/x), & \text{if } 0 < x \leq 1 \\ 0, & \text{if } x = 0 \end{cases}$ (2) $f(x) = \begin{cases} x^2 \sin(1/x), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$ (3) $f(x) = 3x^2 - 2x^3, -2 \leq x \leq 2$ (4) None of these</p>
12.	<p>Choose the incorrect statement.</p> <p>(1) The set of all irrational numbers in $[0, 1]$ is measurable. (2) Every non-empty one set has positive measure. (3) Every subset of a set of measure zero is not of measure zero. (4) None of these</p>
13.	<p>The directional derivative of the function $\phi(x, y) = \frac{xy}{x^2 + y^2}$ at the point $(0, 1)$ along a line making an angle of 30° with positive direction of x-axis is</p> <p>(1) $\frac{2}{\sqrt{3}}$ (2) $\frac{\sqrt{3}}{2}$ (3) $\sqrt{3}$ (4) None of these</p>

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Question No.	Questions
14.	The metric space (\mathbb{R}, d) , where d is a usual metric, is (1) compact (2) disconnected (3) connected but not compact (4) compact and connected
15.	In a metric space $(0, 1]$ with usual metric $d(x, y) = x - y $, the sequence $\langle \frac{1}{n} \rangle$ is a (1) Not a Cauchy sequence (2) Cauchy sequence but does not converge in $(0, 1]$ (3) Cauchy sequence that is convergent in $(0, 1]$ (4) None of these
16.	Let V be a vector space over \mathbb{R}^3 . Which one of the following is not a subspace of V ? (1) $\{(x, y, z) : 3x + y - z = 0, x, y, z \in \mathbb{R}\}$ (2) $\{(x, y, z) : x + y - z = 0, 2x + 3y - z = 0, x, y, z \in \mathbb{R}\}$ (3) $\{(x, y, z) : x - 3y + 4z = 0, x, y, z \in \mathbb{R}\}$ (4) $\{(x, y, z) : x + y \geq 0, x, y, z \in \mathbb{R}\}$
17.	The value of k for which the vector $u = (1, k, 5)$ in $V_3(\mathbb{R})$ can be expressed as a linear combination of vectors $v = (1, -3, 2)$ and $w = (2, -1, 1)$ is (1) 3 (2) -8 (3) -2 (4) None of these

Question No.	Questions
18.	<p>The dimension of the subspace W of \mathbb{R}^4 generated by $\{(3, 8, -3, -5), (1, -2, 5, -3), (2, 3, 1, -4)\}$ is</p> <p>(1) 1 (2) 3 (3) 2 (4) None of these</p>
19.	<p>The rank of the matrix $\begin{bmatrix} 1 & 2 & 1 \\ -3 & -6 & -3 \\ 5 & 10 & 5 \end{bmatrix}$ is</p> <p>(1) 1 (2) 2 (3) 3 (4) None of these</p>
20.	<p>The system of equations $2x - 3y + z = 9$; $x + y + z = 6$; $x - y + z = 2$ has</p> <p>(1) a unique solution (2) infinite solutions (3) no solution (4) none of these</p>
21.	<p>Consider the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$, then</p> <p>(1) A has no real eigen values. (2) A has both positive and negative real eigen values. (3) All real eigen values of A are positive. (4) All real eigen values of A are negative.</p>

Question No.	Questions
22.	<p>Which of the following is a linear transformation ?</p> <p>(1) $T(x, y) = (1 + x, y)$ for all $(x, y) \in \mathbb{R}^2$</p> <p>(2) $T(x, y) = (x, x + y)$ for all $(x, y) \in \mathbb{R}^2$</p> <p>(3) $T(x, y) = (x^2, y)$ for all $(x, y) \in \mathbb{R}^2$</p> <p>(4) None of these</p>
23.	<p>The matrix representing the linear transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined by $T(x_1, x_2) = (2x_2, 3x_1 - x_2)$ relative to the basis $\{(1, 3), (2, 5)\}$ is</p> <p>(1) $\begin{bmatrix} 30 & 48 \\ 18 & 29 \end{bmatrix}$ (2) $\begin{bmatrix} -30 & 48 \\ 18 & -29 \end{bmatrix}$</p> <p>(3) $\begin{bmatrix} 30 & 48 \\ -18 & -29 \end{bmatrix}$ (4) $\begin{bmatrix} -30 & -48 \\ 18 & 29 \end{bmatrix}$</p>
24.	<p>Let C^3 be a complex inner product space.</p> <p>If the vectors $u_1 = (1, 2i, i)$, $u_2 = (0, 1 + i, 1)$, $u_3 = (2, 1 - i, i) \in C^3$, then the vector orthogonal to both u_1 and u_3 is</p> <p>(1) $(-3 + i, -i, 1 - 5i)$ (2) $(-3 + i, -i, 1 + 5i)$</p> <p>(3) $(3 + i, -i, 1 + 5i)$ (4) None of these</p>
25.	<p>The quadratic form corresponding to symmetric matrix $\begin{bmatrix} 9 & 3 & -3 \\ 3 & 2 & -4 \\ -3 & -4 & 2 \end{bmatrix}$ is</p> <p>(1) $9x^2 - 2y^2 + 2z^2 - 6xy - 6xz - 8yz$</p> <p>(2) $9x^2 + 2y^2 + 2z^2 + 6xy - 6xz - 8yz$</p> <p>(3) $9x^2 + 2y^2 - 2z^2 + 6xy + 6xz + 8yz$</p> <p>(4) None of these</p>

Question No.	Questions
26.	<p>The radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!} z^n$ is</p> <p>(1) 4 (2) 3 (3) 2 (4) None of these</p>
27.	<p>Which one of the following functions $f(z)$, of the complex variable z, is analytic over the entire complex plane?</p> <p>(1) $f(z) = \ln(z)$ (2) $f(z) = e^{1/z}$ (3) $f(z) = \frac{1}{1-z}$ (4) $f(z) = \cos z$</p>
28.	<p>If $u = (x-1)^3 - 3xy^2 + 3y^2$ is the real part of an analytic function $f(z) = u + iv$, then the imaginary part v of $f(z)$ is</p> <p>(1) $3x^2y - 6xy - 3y + y^3 + c$ (2) $3x^2y + 6xy + 3y + y^3 + c$ (3) $3x^2y - 6xy + 3y - y^3 + c$ (4) None of these</p>
29.	<p>The value of the integral $\int_C \bar{z} dz$, from $z = 0$ to $z = 4 + 2i$ along the curve C given by $z = t^2 + it$, is equal to</p> <p>(1) $5 - \frac{8}{3}i$ (2) $10 - \frac{8}{3}i$ (3) $10 - \frac{4}{3}i$ (4) None of these</p>

Question No.	Questions
42.	<p>Let $Z[x]$ be the ring of polynomials over the ring of integers. Then</p> <p>(1) the ideal $\langle x \rangle$ is a prime ideal but not a maximal ideal.</p> <p>(2) the ideal $\langle x \rangle$ is not a prime ideal but a maximal ideal.</p> <p>(3) the ideal $\langle x \rangle$ is a prime ideal as well as a maximal ideal.</p> <p>(4) the ideal $\langle x \rangle$ is neither a prime ideal nor a maximal ideal.</p>
43.	<p>Which of the following is not a unique factorization domain ?</p> <p>(1) A Euclidean ring. (2) The ring $\langle Z, +, \cdot \rangle$ of integers.</p> <p>(3) $Z[\sqrt{-5}]$ (4) None of these</p>
44.	<p>Which one of the following polynomial is irreducible over the field Q of rational numbers ?</p> <p>(1) $2x^5 + 15x^3 + 3x + 6$ (2) $x^3 + 3x + 1$</p> <p>(3) $8x^3 - 6x - 1$ (4) All of the above</p>
45.	<p>The degree of splitting field of $x^4 - x^2 - 2$ over the field Q of rational numbers, is</p> <p>(1) 3 (2) 4</p> <p>(3) 2 (4) None of these</p>

Question No.	Questions
46.	<p>Let F be a finite field and let K/F be a field extension of degree 6. Then the Galois group of K/F is isomorphic to</p> <ol style="list-style-type: none"> (1) the cyclic group of order 6 (2) the permutation group on $\{1, 2, 3\}$ (3) the permutation group on $\{1, 2, 3, 4, 5, 6\}$ (4) the permutation group on $\{1\}$
47.	<p>Which of the following spaces is not separable ?</p> <ol style="list-style-type: none"> (1) \mathbb{R} with the trivial topology (2) The Cantor set as a subspace of \mathbb{R} (3) \mathbb{R} with the discrete topology (4) None of these
48.	<p>Which one of the following topological spaces is not compact ?</p> <ol style="list-style-type: none"> (1) Indiscrete topological space (2) Infinite discrete topological space (3) A topological space with cofinite topology (4) None of these
49.	<p>Let X be a topological space and U be a proper dense open subset of X. Then which of the following statement is true ?</p> <ol style="list-style-type: none"> (1) If X is connected, then U is connected. (2) If X is compact, then U is compact. (3) If $X \setminus U$ is compact, then X is compact. (4) If X is compact, then $X \setminus U$ is compact.

Question No.	Questions
50.	<p>Let X and Y be two topological spaces and let $f : X \rightarrow Y$ be a continuous function. Then</p> <p>(1) $f^{-1}(K)$ is connected if $K \subset Y$ is connected</p> <p>(2) $f^{-1}(K)$ is compact if $K \subset Y$ is compact</p> <p>(3) $f(K)$ is connected if $K \subset X$ is connected</p> <p>(4) None of these</p>
51.	<p>For the initial value problem (IVP) $y' = f(x, y)$, $y(0) = 0$, which of the following statement is true ?</p> <p>(1) $f(x, y) = \sqrt{xy}$ satisfies Lipschitz's condition and so IVP has unique solution.</p> <p>(2) $f(x, y) = \sqrt{xy}$ does not satisfy Lipschitz's condition and so IVP has no solution.</p> <p>(3) $f(x, y) = y$ satisfies Lipschitz's condition and so IVP has unique solution.</p> <p>(4) $f(x, y) = y$ does not satisfy Lipschitz's condition still the IVP has unique solution.</p>
52.	<p>The solution of the differential equation $\frac{dy}{dx} = 2xy$, $y(0) = 1$ by Picard's method upto third approximation is</p> <p>(1) $1 + x^2 + \frac{x^4}{2} + \frac{x^6}{6}$</p> <p>(2) $1 + x^2 + \frac{3x^4}{2} + \frac{x^6}{6}$</p> <p>(3) $1 + x^2 + \frac{x^4}{4} + \frac{x^6}{6}$</p> <p>(4) None of these</p>

Question No.	Questions
53.	<p>Using method of variation of parameters, the solution of the differential equation $y'' - 2y' = e^x \sin x$, is</p> <p>(1) $y = c_1 + c_2 e^{2x} - \frac{1}{2} e^x \sin x$ (2) $y = c_1 e^x + c_2 e^{-x} - \frac{1}{2} e^x \sin x$</p> <p>(3) $y = c_1 + c_2 e^x + \frac{1}{2} e^x \sin x$ (4) None of these</p>
54.	<p>Let n be non-negative integer. The eigen values of the Sturm-Liouville problem $y'' + \lambda y = 0$ with boundary conditions $y(0) = y(2\pi)$, $y'(0) = y'(2\pi)$ are</p> <p>(1) n (2) $n^2 \pi^2$</p> <p>(3) $n \pi$ (4) n^2</p>
55.	<p>Green's function of the boundary value problem</p> $\frac{d^2 u}{dx^2} + u = 0, \quad u(0) = 0, \quad u\left(\frac{\pi}{2}\right) = 0$ <p>is given by</p> <p>(1) $G(x, \xi) = \cos \xi \sin x, 0 \leq x < \xi$</p> <p>(2) $G(x, \xi) = \cosh \xi \sinh x, 0 \leq x < \xi$</p> <p>(3) $G(x, \xi) = x(1 - \xi), 0 \leq x < \xi$</p> <p>(4) None of these</p>
56.	<p>The solution of Partial differential equation $xz p + yz q = xy$ is</p> <p>(1) $\phi(xy, yz - y^2) = 0$ (2) $\phi(x/y, z + y^2) = 0$</p> <p>(3) $\phi(x/y, xy - z^2) = 0$ (4) None of these</p>

Question No.	Questions
57.	<p>The integral surface for the Cauchy problem $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 1$ which passes through the circle $z = 0, x^2 + y^2 = 1$ is</p> <p>(1) $x^2 + y^2 + 2z^2 + 2zx - 2yz - 1 = 0$</p> <p>(2) $x^2 + y^2 + 2z^2 + 2zx + 2yz - 1 = 0$</p> <p>(3) $x^2 + y^2 + 2z^2 - 2zx - 2yz - 1 = 0$</p> <p>(4) None of these</p>
58.	<p>The partial differential equation $(x+1)\frac{\partial^2 u}{\partial x^2} - 2(x+2)\frac{\partial^2 u}{\partial x \partial y} + (x+3)\frac{\partial^2 u}{\partial y^2} = 0$ is</p> <p>(1) Elliptic (2) Hyperbolic</p> <p>(3) Parabolic (4) None of these</p>
59.	<p>Using method of separation of variables, the solution of Partial differential equation</p> $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ <p>subject to the boundary conditions :</p> <p>$u(0, y) = \sin y$ for all y</p> <p>and $u(\infty, y) = 0$ for all y</p> <p>is given by</p> <p>(1) $u(x, y) = e^{-x} \sin y$ (2) $u(x, y) = e^x \sin y$</p> <p>(3) $u(x, y) = e^{-2x} \sin y$ (4) None of these</p>

Question No.	Questions										
60.	<p>Let $u(x, t) = e^{iwx} v(t)$ with $v(0) = 1$ be a solution to $\frac{\partial u}{\partial t} = \frac{\partial^3 u}{\partial x^3}$. Then</p> <p>(1) $u(x, t) = e^{iwx(x-w^2t)}$ (2) $u(x, t) = e^{iwx-w^2t}$</p> <p>(3) $u(x, t) = e^{iwx(x+w^2t)}$ (4) $u(x, t) = e^{iwx^3(x-t)}$</p>										
61.	<p>Using Newton Raphson's method, the smallest positive root of the equation $3x^3 - 9x^2 + 8 = 0$, lying between 1 and 2, is</p> <p>(1) 1.0327 (2) 1.2261</p> <p>(3) 1.6514 (4) None of these</p>										
62.	<p>Using Gauss Elimination method, the solution of following equations</p> $4x + 3y + 2z = 8, \quad x + y + 2z = 7, \quad 3x + 2y + 4z = 13$ <p>is given by</p> <p>(1) $x = -1, y = 2, z = 3$ (2) $x = 1, y = 2, z = 3$</p> <p>(3) $x = -1, y = -2, z = 3$ (4) $x = 1, y = -2, z = 3$</p>										
63.	<p>Given that</p> <table border="1" data-bbox="311 1355 1093 1489"> <tbody> <tr> <td>x</td> <td>3</td> <td>7</td> <td>9</td> <td>10</td> </tr> <tr> <td>f(x)</td> <td>168</td> <td>120</td> <td>72</td> <td>63</td> </tr> </tbody> </table> <p>The value of third divided difference of the function f(x) is</p> <p>(1) 5 (2) 1</p> <p>(3) -1 (4) -2</p>	x	3	7	9	10	f(x)	168	120	72	63
x	3	7	9	10							
f(x)	168	120	72	63							

Question No.	Questions
67.	<p>The extremal of the functional</p> $I[y(x)] = \int_1^2 (y'^2 - 2xy) dx \quad \text{subject to } y(1) = 0, y(2) = -1 \text{ is}$ <p>(1) $y = \frac{1}{6}(x - x^3)$ (2) $y = x^2 - 1$</p> <p>(3) $y = \frac{1}{6}(7x - x^3)$ (4) None of these</p>
68.	<p>The extremal of $\int_1^2 \frac{\dot{x}^2}{t^3} dt$; $x(1) = 3, x(2) = 18$ (where $\dot{x} \equiv \frac{dx}{dt}$) using Lagrange's equation is given by which of the following?</p> <p>(1) $x = \frac{15}{7}t^3 + \frac{6}{7}$ (2) $x = 5t^2 - 2$</p> <p>(3) $x = 5t^3 + 3$ (4) $x = t^4 + 2$</p>
69.	<p>The solution of the linear integral equation</p> $\phi(x) = (1+x)^2 + \int_{-1}^1 (x\xi + x^2\xi^2)\phi(\xi) d\xi, \text{ is}$ <p>(1) $\phi(x) = 1 + 6x + \frac{25}{9}x^2$ (2) $\phi(x) = 1 - 6x + \frac{5}{9}x^2$</p> <p>(3) $\phi(x) = 1 + 3x - \frac{25}{9}x^2$ (4) None of these</p>

Question No.	Questions
73.	<p>The Lagrange's equation for a simple pendulum is, (where symbols have their usual meanings)</p> <p>(1) $\dot{\theta} + \frac{g}{\ell} \sin \theta = 0$ (2) $\ddot{\theta} + \frac{g}{\ell} \sin \theta = 0$</p> <p>(3) $\ddot{\theta} + \frac{g}{\ell} \tan \theta = 0$ (4) None of these</p>
74.	<p>Let q_i and \dot{q}_i respectively are the generalized coordinates and velocity of a mechanical system and p_i are its generalized momenta. If H is the Hamiltonian of the system, then Hamilton's equations of motion are</p> <p>(1) $\dot{q}_i = \frac{\partial H}{\partial p_i}, \dot{p}_i = \frac{\partial H}{\partial q_i}$ (2) $\dot{q}_i = -\frac{\partial H}{\partial p_i}, \dot{p}_i = \frac{\partial H}{\partial q_i}$</p> <p>(3) $\dot{q}_i = \frac{\partial H}{\partial p_i}, \dot{p}_i = -\frac{\partial H}{\partial q_i}$ (4) $\dot{q}_i = -\frac{\partial H}{\partial p_i}, \dot{p}_i = -\frac{\partial H}{\partial q_i}$</p>
75.	<p>If $\mathbf{L} = [L_1, L_2, L_3]$ is the vector moment of the external forces about a fixed point O, $\boldsymbol{\omega} = [\omega_1, \omega_2, \omega_3]$ the angular velocity, and A, B, C are the principal moments of inertia, then Euler's dynamical equations of motion are</p> <p>(1) $L_1 = A\omega_1 - (B - C)\omega_2\omega_3, L_2 = B\omega_2 - (C - A)\omega_3\omega_1, L_3 = C\omega_3 - (A - B)\omega_1\omega_2$</p> <p>(2) $L_1 = A\dot{\omega}_1 - (B - C)\omega_2\omega_3, L_2 = B\dot{\omega}_2 - (C - A)\omega_3\omega_1, L_3 = C\dot{\omega}_3 - (A - B)\omega_1\omega_2$</p> <p>(3) $L_1 = A\dot{\omega}_1 + (B - C)\omega_2\omega_3, L_2 = B\dot{\omega}_2 + (C - A)\omega_3\omega_1, L_3 = C\dot{\omega}_3 + (A - B)\omega_1\omega_2$</p> <p>(4) None of these</p>

Question No.	Questions
76.	<p>An integer is chosen at random from two hundred digits. Then the probability that the integer is divisible by 6 or 8 is</p> <p>(1) $\frac{3}{4}$ (2) $\frac{1}{2}$ (3) $\frac{3}{8}$ (4) $\frac{1}{4}$</p>
77.	<p>In a bolts factory, machines I, II and III manufacture respectively 25%, 35% and 40% of the total bolts. Of their total output 5%, 4% and 2% are respectively defective bolts. A bolt is drawn at random from the product. If the bolt drawn is found to be defective, what is the probability that it is manufactured by machine II ?</p> <p>(1) 0.41 (2) 0.27 (3) 0.13 (4) None of these</p>
78.	<p>Let X be a continuous random variable with probability density function (p.d.f.) defined as $f(x) = 6x(1-x)$, $0 \leq x \leq 1$.</p> <p>Then the value of number b such that $P(X < b) = P(X > b)$ is</p> <p>(1) $\frac{1}{4}$ (2) $\frac{3}{4}$ (3) $\frac{1}{2}$ (4) None of these</p>
79.	<p>If X and Y are two random variables having joint density function given by</p> $f(x, y) = \begin{cases} 6x^2y & ; 0 < x < 1, 0 < y < 1 \\ 0 & ; \text{otherwise} \end{cases}$ <p>Then $P(X + Y < 1)$ is</p> <p>(1) $\frac{1}{4}$ (2) $\frac{1}{10}$ (3) $\frac{3}{8}$ (4) None of these</p>

Question No.	Questions
88.	<p>Let X and Y be independent and identically distributed (i.i.d.) random variables uniformly distributed on (0, 4). Then $P(X > Y X < 2Y)$ is</p> <p>(1) $2/3$ (2) $5/6$ (3) $1/4$ (4) $1/3$</p>
89.	<p>If X and Y are independent normal variates with zero expectations and variances σ_1^2 and σ_2^2, then $Z = \frac{XY}{\sqrt{X^2 + Y^2}}$ is normal with variance</p> <p>(1) $\sigma_z^2 = \frac{1}{\left(\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}\right)}$ (2) $\sigma_z^2 = \sigma_1^2 + \sigma_2^2$ (3) $\sigma_z^2 = \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}$ (4) None of these</p>
90.	<p>In testing $H_0: \mu = 100$ against $A: \mu \neq 100$ at the 10% level of significance, H_0 is rejected if</p> <p>(1) 100 is contained in the 90% confidence interval (2) The value of the test statistic is in the acceptance region (3) The p-value is less than 0.10 (4) The p-value is greater than 0.10</p>

Question No.	Questions
91.	<p>Let p be the probability that a coin will fall head in a single toss in order to test $H_0 : p = \frac{1}{2}$ against $H_1 : p = \frac{3}{4}$. The coin is tossed 5 times and H_0 is rejected if more than 3 heads are obtained. Then the probability of type I error is</p> <p>(1) $1/16$ (2) $3/16$ (3) $5/16$ (4) None of these</p>
92.	<p>Given the observations 0.8, 0.71, 0.9, 1.2, 1.68, 1.4, 0.88, 1.62 from the uniform distribution on $(\theta - 0.2, \theta + 0.8)$ with $-\infty < \theta < \infty$, which of the following is a maximum likelihood estimate for θ?</p> <p>(1) 0.7 (2) 0.9 (3) 1.1 (4) 1.3</p>
93.	<p>If $X_1, X_2, X_3, \dots, X_n$ is a random sample from a normal population $N(\mu, 1)$. Then $t = \frac{1}{n} \sum_{i=1}^n X_i^2$ is a unbiased estimator of</p> <p>(1) μ^2 (2) $\mu^2 - 1$ (3) $1 - \mu^2$ (4) $\mu^2 + 1$</p>
94.	<p>Let X_1, X_2, \dots, X_n be a random sample of size n from a p-variate Normal distribution with mean μ and positive definite covariance matrix Σ. Choose the correct statement</p> <p>(1) $(X_1 - \mu)' \Sigma^{-1} (X_1 - \mu)$ has chi-square distribution with 1 d.f. (2) $\bar{X} \bar{X}'$ has Wishart distribution with p d.f. (3) $\sum_{i=1}^n (X_i - \mu)(X_i - \mu)'$ has Wishart distribution with n d.f. (4) $X_1 + X_2$ and $X_1 - X_2$ are independently distributed.</p>

Question No.	Questions
95.	<p>In a trivariate distribution : $\sigma_1 = 2, \sigma_2 = \sigma_3 = 3, r_{12} = 0.7, r_{23} = r_{31} = 0.5$. Then $R_{1.23}$ is</p> <p>(1) 0.1423 (2) 0.7211 (3) 0.4892 (4) None of these</p>
96.	<p>The total number of all possible Latin squares of order 3 is</p> <p>(1) 12 (2) 9 (3) 6 (4) None of these</p>
97.	<p>The Mean time to failure (MTTF) for an exponential distribution with parameter λ is</p> <p>(1) λ (2) λ^2 (3) $1/\lambda$ (4) None of these</p>
98.	<p>The maximum value of $Z = 2x + 3y$ subject to the constraints : $x + y \leq 30 ; 3 \leq y \leq 12 ; x - y \geq 0 ; 0 \leq x \leq 20$, is</p> <p>(1) 72 (2) 60 (3) 49 (4) None of these</p>

Question No.	Questions
99.	<p>Men arrive in a queue according to a Poisson process with rate λ_1 and women arrive in the same queue according to another Poisson process with rate λ_2. The arrivals of men and women are independent. The probability that the first arrival in the queue is a man is</p> <p>(1) $\frac{\lambda_1}{\lambda_1 + \lambda_2}$ (2) $\frac{\lambda_2}{\lambda_1 + \lambda_2}$</p> <p>(3) $\frac{\lambda_1}{\lambda_2}$ (4) $\frac{\lambda_2}{\lambda_1}$</p>
100.	<p>Arrivals at a telephone booth are considered to be Poisson with an average time of 12 minutes between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 4 minutes. Then the probability that a person arriving at the booth will have to wait in the queue is</p> <p>(1) $\frac{2}{3}$ (2) $\frac{1}{3}$</p> <p>(3) $\frac{1}{6}$ (4) None of these</p>

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Question No.	Questions	Question No.
1.	<p>The resolvent kernel for the integral equation</p> $\phi(x) = 29 + 6x + \int_0^x [5 - 6(x - \xi)]\phi(\xi) d\xi, \text{ is}$ <p>(1) $9e^{3(x-\xi)} - 4e^{2(x-\xi)}$ (2) $9e^{2(x-\xi)} - 4e^{3(x-\xi)}$ (3) $9e^{3(x-\xi)} - e^{-2(x-\xi)}$ (4) None of these</p>	
2.	<p>Consider the two dimensional motion of a mass m attached to one end of a spring whose other end is fixed. Let k be the spring constant. The kinetic energy T and the potential energy V of the system are given by</p> $T = \frac{1}{2} m (\dot{r}^2 + (r\dot{\theta})^2) \text{ and } V = \frac{1}{2} kr^2,$ <p>where $\dot{r} = \frac{dr}{dt}$ and $\dot{\theta} = \frac{d\theta}{dt}$ with t as time.</p> <p>Then which of the following statement is correct ?</p> <p>(1) r is an ignorable coordinate (2) θ is not an ignorable coordinate (3) $r^2 \dot{\theta}$ remains constant throughout the motion (4) $r \dot{\theta}$ remains constant throughout the motion</p>	
3.	<p>The Lagrange's equation for a simple pendulum is, (where symbols have their usual meanings)</p> <p>(1) $\dot{\theta} + \frac{g}{\ell} \sin \theta = 0$ (2) $\ddot{\theta} + \frac{g}{\ell} \sin \theta = 0$ (3) $\ddot{\theta} + \frac{g}{\ell} \tan \theta = 0$ (4) None of these</p>	

Question No.	Questions
4.	<p>Let q_i and \dot{q}_i respectively are the generalized coordinates and velocity of a mechanical system and p_i are its generalized momenta. If H is the Hamiltonian of the system, then Hamilton's equations of motion are</p> <p>(1) $\dot{q}_i = \frac{\partial H}{\partial p_i}, \dot{p}_i = \frac{\partial H}{\partial q_i}$ (2) $\dot{q}_i = -\frac{\partial H}{\partial p_i}, \dot{p}_i = \frac{\partial H}{\partial q_i}$</p> <p>(3) $\dot{q}_i = \frac{\partial H}{\partial p_i}, \dot{p}_i = -\frac{\partial H}{\partial q_i}$ (4) $\dot{q}_i = -\frac{\partial H}{\partial p_i}, \dot{p}_i = -\frac{\partial H}{\partial q_i}$</p>
5.	<p>If $L = [L_1, L_2, L_3]$ is the vector moment of the external forces about a fixed point O, $\omega = [\omega_1, \omega_2, \omega_3]$ the angular velocity, and A, B, C are the principal moments of inertia, then Euler's dynamical equations of motion are</p> <p>(1) $L_1 = A\omega_1 - (B - C)\omega_2\omega_3, L_2 = B\omega_2 - (C - A)\omega_3\omega_1, L_3 = C\omega_3 - (A - B)\omega_1\omega_2$</p> <p>(2) $L_1 = A\dot{\omega}_1 - (B - C)\omega_2\omega_3, L_2 = B\dot{\omega}_2 - (C - A)\omega_3\omega_1, L_3 = C\dot{\omega}_3 - (A - B)\omega_1\omega_2$</p> <p>(3) $L_1 = A\dot{\omega}_1 + (B - C)\omega_2\omega_3, L_2 = B\dot{\omega}_2 + (C - A)\omega_3\omega_1, L_3 = C\dot{\omega}_3 + (A - B)\omega_1\omega_2$</p> <p>(4) None of these</p>
6.	<p>An integer is chosen at random from two hundred digits. Then the probability that the integer is divisible is divisible by 6 or 8 is</p> <p>(1) $3/4$ (2) $1/2$</p> <p>(3) $3/8$ (4) $1/4$</p>
7.	<p>In a bolts factory, machines I, II and III manufacture respectively 25%, 35% and 40% of the total bolts. Of their total output 5%, 4% and 2% are respectively defective bolts. A bolt is drawn at random from the product. If the bolt drawn is found to be defective, what is the probability that it is manufactured by machine II ?</p> <p>(1) 0.41 (2) 0.27</p> <p>(3) 0.13 (4) None of these</p>

Question No.	Questions
11.	<p>For the initial value problem (IVP) $y' = f(x, y)$, $y(0) = 0$, which of the following statement is true ?</p> <p>(1) $f(x, y) = \sqrt{xy}$ satisfies Lipschitz's condition and so IVP has unique solution.</p> <p>(2) $f(x, y) = \sqrt{xy}$ does not satisfy Lipschitz's condition and so IVP has no solution.</p> <p>(3) $f(x, y) = y$ satisfies Lipschitz's condition and so IVP has unique solution.</p> <p>(4) $f(x, y) = y$ does not satisfy Lipschitz's condition still the IVP has unique solution.</p>
12.	<p>The solution of the differential equation $\frac{dy}{dx} = 2xy$, $y(0) = 1$ by Picard's method upto third approximation is</p> <p>(1) $1 + x^2 + \frac{x^4}{2} + \frac{x^6}{6}$ (2) $1 + x^2 + \frac{3x^4}{2} + \frac{x^6}{6}$</p> <p>(3) $1 + x^2 + \frac{x^4}{4} + \frac{x^6}{6}$ (4) None of these</p>
13.	<p>Using method of variation of parameters, the solution of the differential equation $y'' - 2y' = e^x \sin x$, is</p> <p>(1) $y = c_1 + c_2 e^{2x} - \frac{1}{2} e^x \sin x$ (2) $y = c_1 e^x + c_2 e^{-x} - \frac{1}{2} e^x \sin x$</p> <p>(3) $y = c_1 + c_2 e^x + \frac{1}{2} e^x \sin x$ (4) None of these</p>

Question No.	Questions
18.	<p>The partial differential equation $(x+1)\frac{\partial^2 u}{\partial x^2} - 2(x+2)\frac{\partial^2 u}{\partial x \partial y} + (x+3)\frac{\partial^2 u}{\partial y^2} = 0$ is</p> <p>(1) Elliptic (2) Hyperbolic (3) Parabolic (4) None of these</p>
19.	<p>Using method of separation of variables, the solution of Partial differential equation</p> $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ <p>subject to the boundary conditions :</p> <p>$u(0, y) = \sin y$ for all y</p> <p>and $u(\infty, y) = 0$ for all y</p> <p>is given by</p> <p>(1) $u(x, y) = e^{-x} \sin y$ (2) $u(x, y) = e^x \sin y$ (3) $u(x, y) = e^{-2x} \sin y$ (4) None of these</p>
20.	<p>Let $u(x, t) = e^{iw x} v(t)$ with $v(0) = 1$ be a solution to $\frac{\partial u}{\partial t} = \frac{\partial^3 u}{\partial x^3}$. Then</p> <p>(1) $u(x, t) = e^{iw(x-w^2t)}$ (2) $u(x, t) = e^{iw x - w^2t}$ (3) $u(x, t) = e^{iw(x+w^2t)}$ (4) $u(x, t) = e^{iw^3(x-t)}$</p>

Question No.	Questions
21.	<p>The Laurent series expansion of $f(z) = \frac{z}{(z-1)(z-3)}$ for $0 < z-1 < 2$, is equal to</p> <p>(1) $\frac{-3}{4} \sum_{n=0}^{\infty} \frac{(z-1)^n}{2^n} - \frac{1}{2(z-1)}$ (2) $\frac{3}{4} \sum_{n=0}^{\infty} \frac{(z-1)^n}{2^n} - \frac{1}{4(z-1)}$</p> <p>(3) $\frac{1}{4} \sum_{n=0}^{\infty} \frac{(z-1)^{-n}}{2^n} + \frac{(z-1)}{2}$ (4) None of these</p>
22.	<p>The residue of the function $f(z) = \frac{1}{(z^2+1)^3}$ at $z=i$, is</p> <p>(1) $\frac{3}{16i}$ (2) $\frac{3}{2i}$</p> <p>(3) $\frac{4}{3i}$ (4) None of these</p>
23.	<p>The fixed points of the Mobius transformation $w = \frac{(2+i)z-2}{z+i}$ are</p> <p>(1) $i, -i$ (2) $0, 1$</p> <p>(3) $-1, 1$ (4) $1+i, 1-i$</p>
24.	<p>The image of circle $z-2 =2$ under the Mobius transformation $w = \frac{z}{z+1}$ is a circle in w-plane with</p> <p>(1) Centre $\left(\frac{1}{5}, 0\right)$ and radius $\frac{2}{5}$ (2) Centre $\left(\frac{2}{5}, 0\right)$ and radius $\frac{1}{5}$</p> <p>(3) Centre $\left(\frac{1}{5}, 0\right)$ and radius $\frac{1}{5}$ (4) Centre $\left(\frac{2}{5}, 0\right)$ and radius $\frac{2}{5}$</p>

Question No.	Questions	Question No.
25.	If 9 colours are used to paint 100 houses, then atleast _____ houses will be of the same colour. (1) 18 (2) 15 (3) 12 (4) 10	25.
26.	The congruence $35x \equiv 14 \pmod{21}$ has (1) 5 solutions (2) 6 solutions (3) 7 solutions (4) No solution	26.
27.	The primitive roots of 3^2 are (1) 3, 7 (2) 2, 5 (3) 5, 7 (4) None of these	27.
28.	Let $G = \{0, 1, 2, 3, 4, 5, 6, 7\}$ be a group under the binary operation 'addition modulo 8', then the order of element 5, is (1) 1 (2) 2 (3) 4 (4) 8	28.
29.	The centre of a non-abelian group of order 343 always has _____ elements in its centre. (1) 3 (2) 7 (3) 5 (4) None of these	29.

Question No.	Questions
30.	<p>Let G be a group of order 20449. Then</p> <p>(1) G has only one Sylow-11 subgroup</p> <p>(2) G has only two Sylow-11 subgroups</p> <p>(3) G has only four Sylow-11 subgroups</p> <p>(4) None of these</p>
31.	<p>Which of the following functions is not a function of bounded variation ?</p> <p>(1) $f(x) = \begin{cases} x \sin(\pi/x), & \text{if } 0 < x \leq 1 \\ 0, & \text{if } x = 0 \end{cases}$</p> <p>(2) $f(x) = \begin{cases} x^2 \sin(1/x), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$</p> <p>(3) $f(x) = 3x^2 - 2x^3, -2 \leq x \leq 2$</p> <p>(4) None of these</p>
32.	<p>Choose the incorrect statement.</p> <p>(1) The set of all irrational numbers in $[0, 1]$ is measurable.</p> <p>(2) Every non-empty one set has positive measure.</p> <p>(3) Every subset of a set of measure zero is not of measure zero.</p> <p>(4) None of these</p>
33.	<p>The directional derivative of the function $\phi(x, y) = \frac{xy}{x^2 + y^2}$ at the point $(0, 1)$ along a line making an angle of 30° with positive direction of x-axis is</p> <p>(1) $\frac{2}{\sqrt{3}}$</p> <p>(2) $\frac{\sqrt{3}}{2}$</p> <p>(3) $\sqrt{3}$</p> <p>(4) None of these</p>

Question No.	Questions
34.	<p>The metric space (R, d), where d is a usual metric, is</p> <p>(1) compact (2) disconnected (3) connected but not compact (4) compact and connected</p>
35.	<p>In a metric space $(0, 1]$ with usual metric $d(x, y) = x - y$, the sequence $\langle \frac{1}{n} \rangle$ is a</p> <p>(1) Not a Cauchy sequence (2) Cauchy sequence but does not converge in $(0, 1]$ (3) Cauchy sequence that is convergent in $(0, 1]$ (4) None of these</p>
36.	<p>Let V be a vector space over R^3. Which one of the following is not a subspace of V?</p> <p>(1) $\{(x, y, z) : 3x + y - z = 0, x, y, z \in R\}$ (2) $\{(x, y, z) : x + y - z = 0, 2x + 3y - z = 0, x, y, z \in R\}$ (3) $\{(x, y, z) : x - 3y + 4z = 0, x, y, z \in R\}$ (4) $\{(x, y, z) : x + y \geq 0, x, y, z \in R\}$</p>
37.	<p>The value of k for which the vector $u = (1, k, 5)$ in $V_3(R)$ can be expressed as a linear combination of vectors $v = (1, -3, 2)$ and $w = (2, -1, 1)$ is</p> <p>(1) 3 (2) -8 (3) -2 (4) None of these</p>

Question No.	Questions
43.	<p>If $X_1, X_2, X_3, \dots, X_n$ is a random sample from a normal population $N(\mu, 1)$. Then $t = \frac{1}{n} \sum_{i=1}^n X_i^2$ is a unbiased estimator of</p> <p>(1) μ^2 (2) $\mu^2 - 1$ (3) $1 - \mu^2$ (4) $\mu^2 + 1$</p>
44.	<p>Let X_1, X_2, \dots, X_n be a random sample of size n from a p-variate Normal distribution with mean μ and positive definite covariance matrix Σ. Choose the correct statement</p> <p>(1) $(X_1 - \mu)' \Sigma^{-1} (X_1 - \mu)$ has chi-square distribution with 1 d.f. (2) $\bar{X} \bar{X}'$ has Wishart distribution with p d.f. (3) $\sum_{i=1}^n (X_i - \mu)(X_i - \mu)'$ has Wishart distribution with n d.f. (4) $X_1 + X_2$ and $X_1 - X_2$ are independently distributed.</p>
45.	<p>In a trivariate distribution : $\sigma_1 = 2, \sigma_2 = \sigma_3 = 3, r_{12} = 0.7, r_{23} = r_{31} = 0.5$. Then $R_{1,23}$ is</p> <p>(1) 0.1423 (2) 0.7211 (3) 0.4892 (4) None of these</p>
46.	<p>The total number of all possible Latin squares of order 3 is</p> <p>(1) 12 (2) 9 (3) 6 (4) None of these</p>

Question No.	Questions	Question No.
58.	<p>The extremal of $\int_1^2 \frac{\dot{x}^2}{t^3} dt$; $x(1) = 3, x(2) = 18$ (where $\dot{x} \equiv \frac{dx}{dt}$) using Lagrange's equation is given by which of the following ?</p> <p>(1) $x = \frac{15}{7}t^3 + \frac{6}{7}$ (2) $x = 5t^2 - 2$</p> <p>(3) $x = 5t^3 + 3$ (4) $x = t^4 + 2$</p>	
59.	<p>The solution of the linear integral equation $\phi(x) = (1+x)^2 + \int_{-1}^1 (x\xi + x^2\xi^2)\phi(\xi) d\xi$, is</p> <p>(1) $\phi(x) = 1 + 6x + \frac{25}{9}x^2$ (2) $\phi(x) = 1 - 6x + \frac{5}{9}x^2$</p> <p>(3) $\phi(x) = 1 + 3x - \frac{25}{9}x^2$ (4) None of these</p>	
60.	<p>The solution to the integral equation $\phi(x) = x + \int_0^x \sin(x-\xi)\phi(\xi) d\xi$ is given by</p> <p>(1) $x^2 + \frac{x^3}{3}$ (2) $x - \frac{x^3}{3!}$</p> <p>(3) $x^2 - \frac{x^3}{3!}$ (4) $x + \frac{x^3}{3!}$</p>	

Question No.	Questions
69.	<p>If X and Y are independent normal variates with zero expectations and variances σ_1^2 and σ_2^2, then $Z = \frac{XY}{\sqrt{X^2 + Y^2}}$ is normal with variance</p> <p>(1) $\sigma_z^2 = \frac{1}{\left(\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}\right)}$ (2) $\sigma_z^2 = \sigma_1^2 + \sigma_2^2$</p> <p>(3) $\sigma_z^2 = \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}$ (4) None of these</p>
70.	<p>In testing $H : \mu = 100$ against $A : \mu \neq 100$ at the 10% level of significance, H is rejected if</p> <p>(1) 100 is contained in the 90% confidence interval</p> <p>(2) The value of the test statistic is in the acceptance region</p> <p>(3) The p-value is less than 0.10</p> <p>(4) The p-value is greater than 0.10</p>
71.	<p>The non-isomorphic abelian groups of order 20 are</p> <p>(1) $Z_4, Z_2 \times Z_2 \times Z_5$ (2) $Z_8, Z_2 \times Z_5$</p> <p>(3) $Z_4 \times Z_5, Z_2 \times Z_2 \times Z_5$ (4) None of these</p>

Question No.	Questions
72.	<p>Let $Z[x]$ be the ring of polynomials over the ring of integers. Then</p> <ol style="list-style-type: none">(1) the ideal $\langle x \rangle$ is a prime ideal but not a maximal ideal.(2) the ideal $\langle x \rangle$ is not a prime ideal but a maximal ideal.(3) the ideal $\langle x \rangle$ is a prime ideal as well as a maximal ideal.(4) the ideal $\langle x \rangle$ is neither a prime ideal nor a maximal ideal.
73.	<p>Which of the following is not a unique factorization domain ?</p> <ol style="list-style-type: none">(1) A Euclidean ring.(2) The ring $\langle Z, +, \cdot \rangle$ of integers.(3) $Z[\sqrt{-5}]$(4) None of these
74.	<p>Which one of the following polynomial is irreducible over the field Q of rational numbers ?</p> <ol style="list-style-type: none">(1) $2x^3 + 15x^2 + 3x + 6$(2) $x^3 + 3x + 1$(3) $8x^3 - 6x - 1$(4) All of the above
75.	<p>The degree of splitting field of $x^4 - x^2 - 2$ over the field Q of rational numbers, is</p> <ol style="list-style-type: none">(1) 3(2) 4(3) 2(4) None of these

Question No.	Questions
76.	<p>Let F be a finite field and let K/F be a field extension of degree 6. Then the Galois group of K/F is isomorphic to</p> <ol style="list-style-type: none"> (1) the cyclic group of order 6 (2) the permutation group on $\{1, 2, 3\}$ (3) the permutation group on $\{1, 2, 3, 4, 5, 6\}$ (4) the permutation group on $\{1\}$
77.	<p>Which of the following spaces is not separable ?</p> <ol style="list-style-type: none"> (1) \mathbb{R} with the trivial topology (2) The Cantor set as a subspace of \mathbb{R} (3) \mathbb{R} with the discrete topology (4) None of these
78.	<p>Which one of the following topological spaces is not compact ?</p> <ol style="list-style-type: none"> (1) Indiscrete topological space (2) Infinite discrete topological space (3) A topological space with cofinite topology (4) None of these
79.	<p>Let X be a topological space and U be a proper dense open subset of X. Then which of the following statement is true ?</p> <ol style="list-style-type: none"> (1) If X is connected, then U is connected. (2) If X is compact, then U is compact. (3) If $X \setminus U$ is compact, then X is compact. (4) If X is compact, then $X \setminus U$ is compact.

Question No.	Questions
80.	<p>Let X and Y be two topological spaces and let $f : X \rightarrow Y$ be a continuous function. Then</p> <ol style="list-style-type: none">(1) $f^{-1}(K)$ is connected if $K \subset Y$ is connected(2) $f^{-1}(K)$ is compact if $K \subset Y$ is compact(3) $f(K)$ is connected if $K \subset X$ is connected(4) None of these
81.	<p>Consider the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$, then</p> <ol style="list-style-type: none">(1) A has no real eigen values.(2) A has both positive and negative real eigen values.(3) All real eigen values of A are positive.(4) All real eigen values of A are negative.
82.	<p>Which of the following is a linear transformation ?</p> <ol style="list-style-type: none">(1) $T(x, y) = (1 + x, y)$ for all $(x, y) \in \mathbb{R}^2$(2) $T(x, y) = (x, x + y)$ for all $(x, y) \in \mathbb{R}^2$(3) $T(x, y) = (x^2, y)$ for all $(x, y) \in \mathbb{R}^2$(4) None of these

Question No.	Questions
83.	<p>The matrix representing the linear transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined by $T(x_1, x_2) = (2x_2, 3x_1 - x_2)$ relative to the basis $\{(1, 3), (2, 5)\}$ is</p> <p>(1) $\begin{bmatrix} 30 & 48 \\ 18 & 29 \end{bmatrix}$ (2) $\begin{bmatrix} -30 & 48 \\ 18 & -29 \end{bmatrix}$</p> <p>(3) $\begin{bmatrix} 30 & 48 \\ -18 & -29 \end{bmatrix}$ (4) $\begin{bmatrix} -30 & -48 \\ 18 & 29 \end{bmatrix}$</p>
84.	<p>Let C^3 be a complex inner product space.</p> <p>If the vectors $u_1 = (1, 2i, i)$, $u_2 = (0, 1 + i, 1)$, $u_3 = (2, 1 - i, i) \in C^3$, then the vector orthogonal to both u_1 and u_3 is</p> <p>(1) $(-3 + i, -i, 1 - 5i)$ (2) $(-3 + i, -i, 1 + 5i)$</p> <p>(3) $(3 - i, -i, 1 + 5i)$ (4) None of these</p>
85.	<p>The quadratic form corresponding to symmetric matrix $\begin{bmatrix} 9 & 3 & -3 \\ 3 & 2 & -4 \\ -3 & -4 & 2 \end{bmatrix}$ is</p> <p>(1) $9x^2 - 2y^2 + 2z^2 - 6xy - 6xz - 8yz$</p> <p>(2) $9x^2 + 2y^2 + 2z^2 + 6xy - 6xz - 8yz$</p> <p>(3) $9x^2 + 2y^2 - 2z^2 + 6xy + 6xz + 8yz$</p> <p>(4) None of these</p>

Question No.	Questions
93.	<p>Consider the statements :</p> <p>(a) The series $\frac{1}{3} - \frac{2}{5} + \frac{3}{7} - \frac{4}{9} + \dots$ is convergent.</p> <p>(b) The series $1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \dots$ is convergent.</p> <p>Then</p> <p>(1) Both the statements (a) and (b) are true</p> <p>(2) The statement (a) is true and (b) is false</p> <p>(3) The statement (a) is false and (b) is true</p> <p>(4) Neither (a) nor (b) is true</p>
94.	<p>Every bounded sequence has at least one limit point. This represents</p> <p>(1) Archimedean Property (2) Heine-Borel theorem</p> <p>(3) Bolzano-Weierstrass theorem (4) Denseness Property</p>
95.	<p>Let $f : [0, \infty) \rightarrow \mathbb{R}$ be a function, where \mathbb{R} denotes the set of all real numbers. Then which one of the following statements is true ?</p> <p>(1) If f is continuous and $\lim_{x \rightarrow \infty} f(x)$ is finite, then f is uniformly continuous.</p> <p>(2) If f is bounded and continuous, then f is uniformly continuous.</p> <p>(3) If f is uniformly continuous, then $\lim_{x \rightarrow \infty} f(x)$ exists.</p> <p>(4) None of these</p>

Question No.	Questions
96.	<p>Which of the following is false ?</p> <p>(1) The sequence $\{f_n\}$, where $f_n(x) = \frac{1}{x+n}$, is uniformly convergent in any interval $[0, b]$, $b > 0$.</p> <p>(2) The sequence $\{f_n\}$, where $f_n(x) = \frac{n^2 x}{1+n^3 x^2}$, is uniformly convergent on the interval $[0, 1]$.</p> <p>(3) The sequence $\{f_n\}$, where $f_n(x) = \tan^{-1} nx$, $x \geq 0$, is uniformly convergent in any interval $[a, b]$, $a > 0$.</p> <p>(4) None of these</p>
97.	<p>For which of the following function, Rolle's theorem is not applicable ?</p> <p>(1) $f(x) = \cos 2x$ in $[-\pi/4, \pi/4]$</p> <p>(2) $f(x) = \frac{\sin x}{e^x}$ in $[0, \pi]$</p> <p>(3) $f(x) = x^3 - 6x^2 + 11x - 6$ in $[1, 3]$</p> <p>(4) $f(x) = x$ in $[-1, 1]$</p>
98.	<p>If $f(x) = x$, $x \in [0, 1]$ and let $P = \left\{0, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, 1\right\}$ be the partition of $[0, 1]$, then $U(f, P)$ is</p> <p>(1) $23/36$</p> <p>(2) $31/36$</p> <p>(3) $49/36$</p> <p>(4) None of these</p>

Question No.	Questions	Question No.
99.	<p>If a function f defined on $[0, 1]$ as $f(x) = \begin{cases} \sin\left(\frac{1}{x}\right), & \text{if } x \text{ is irrational} \\ 0, & \text{otherwise} \end{cases}$, then</p> <p>(1) f is not bounded (2) f is R-integrable (3) f is not R-integrable since f is not bounded (4) f is not R-integrable since lower and upper integrals of f are unequal</p>	99.
100.	<p>Consider the following improper integrals</p> $I_1 = \int_0^{\infty} \frac{\sin^2 x}{x^2} dx \text{ and } I_2 = \int_1^{\infty} \frac{x^3}{(1+x)^5} dx, \text{ then}$ <p>(1) Both are divergent (2) I_1 converges but not I_2 (3) I_2 converges but not I_1 (4) Both are convergent</p>	100.

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MATHEMATICS

Sr. No. _____

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6. There will be no negative marking. Each correct answer will be awarded one full mark. Cutting, erasing, overwriting and more than one answer in OMR Answer-Sheet will be treated as incorrect answer.
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Question No.	Questions
1.	<p>The non-isomorphic abelian groups of order 20 are</p> <p>(1) $Z_4, Z_2 \times Z_2 \times Z_5$ (2) $Z_8, Z_2 \times Z_5$</p> <p>(3) $Z_4 \times Z_5, Z_2 \times Z_2 \times Z_5$ (4) None of these</p>
2.	<p>Let $Z[x]$ be the ring of polynomials over the ring of integers. Then</p> <p>(1) the ideal $\langle x \rangle$ is a prime ideal but not a maximal ideal.</p> <p>(2) the ideal $\langle x \rangle$ is not a prime ideal but a maximal ideal.</p> <p>(3) the ideal $\langle x \rangle$ is a prime ideal as well as a maximal ideal.</p> <p>(4) the ideal $\langle x \rangle$ is neither a prime ideal nor a maximal ideal.</p>
3.	<p>Which of the following is not a unique factorization domain ?</p> <p>(1) A Euclidean ring. (2) The ring $\langle Z, +, \cdot \rangle$ of integers.</p> <p>(3) $Z[\sqrt{-5}]$ (4) None of these</p>
4.	<p>Which one of the following polynomial is irreducible over the field Q of rational numbers ?</p> <p>(1) $2x^5 + 15x^3 + 3x + 6$ (2) $x^3 + 3x + 1$</p> <p>(3) $8x^3 - 6x - 1$ (4) All of the above</p>
5.	<p>The degree of splitting field of $x^4 - x^2 - 2$ over the field Q of rational numbers, is</p> <p>(1) 3 (2) 4</p> <p>(3) 2 (4) None of these</p>

Question No.	Questions
6.	<p>Let F be a finite field and let K/F be a field extension of degree 6. Then the Galois group of K/F is isomorphic to</p> <ol style="list-style-type: none"> (1) the cyclic group of order 6 (2) the permutation group on $\{1, 2, 3\}$ (3) the permutation group on $\{1, 2, 3, 4, 5, 6\}$ (4) the permutation group on $\{1\}$
7.	<p>Which of the following spaces is not separable?</p> <ol style="list-style-type: none"> (1) \mathbb{R} with the trivial topology (2) The Cantor set as a subspace of \mathbb{R} (3) \mathbb{R} with the discrete topology (4) None of these
8.	<p>Which one of the following topological spaces is not compact?</p> <ol style="list-style-type: none"> (1) Indiscrete topological space (2) Infinite discrete topological space (3) A topological space with cofinite topology (4) None of these
9.	<p>Let X be a topological space and U be a proper dense open subset of X. Then which of the following statement is true?</p> <ol style="list-style-type: none"> (1) If X is connected, then U is connected. (2) If X is compact, then U is compact. (3) If $X \setminus U$ is compact, then X is compact. (4) If X is compact, then $X \setminus U$ is compact.

Question No.	Questions
10.	<p>Let X and Y be two topological spaces and let $f : X \rightarrow Y$ be a continuous function. Then</p> <p>(1) $f^{-1}(K)$ is connected if $K \subset Y$ is connected</p> <p>(2) $f^{-1}(K)$ is compact if $K \subset Y$ is compact</p> <p>(3) $f(K)$ is connected if $K \subset X$ is connected</p> <p>(4) None of these</p>
11.	<p>Consider the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$, then</p> <p>(1) A has no real eigen values.</p> <p>(2) A has both positive and negative real eigen values.</p> <p>(3) All real eigen values of A are positive.</p> <p>(4) All real eigen values of A are negative.</p>
12.	<p>Which of the following is a linear transformation ?</p> <p>(1) $T(x, y) = (1 + x, y)$ for all $(x, y) \in \mathbb{R}^2$</p> <p>(2) $T(x, y) = (x, x + y)$ for all $(x, y) \in \mathbb{R}^2$</p> <p>(3) $T(x, y) = (x^2, y)$ for all $(x, y) \in \mathbb{R}^2$</p> <p>(4) None of these</p>

Question No.	Questions
13.	<p>The matrix representing the linear transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined by $T(x_1, x_2) = (2x_2, 3x_1 - x_2)$ relative to the basis $\{(1, 3), (2, 5)\}$ is</p> <p>(1) $\begin{bmatrix} 30 & 48 \\ 18 & 29 \end{bmatrix}$ (2) $\begin{bmatrix} -30 & 48 \\ 18 & -29 \end{bmatrix}$</p> <p>(3) $\begin{bmatrix} 30 & 48 \\ -18 & -29 \end{bmatrix}$ (4) $\begin{bmatrix} -30 & -48 \\ 18 & 29 \end{bmatrix}$</p>
14.	<p>Let C^3 be a complex inner product space.</p> <p>If the vectors $u_1 = (1, 2i, i)$, $u_2 = (0, 1 + i, 1)$, $u_3 = (2, 1 - i, i) \in C^3$, then the vector orthogonal to both u_1 and u_3 is</p> <p>(1) $(-3 + i, -i, 1 - 5i)$ (2) $(-3 + i, -i, 1 + 5i)$</p> <p>(3) $(3 + i, -i, 1 + 5i)$ (4) None of these</p>
15.	<p>The quadratic form corresponding to symmetric matrix $\begin{bmatrix} 9 & 3 & -3 \\ 3 & 2 & -4 \\ -3 & -4 & 2 \end{bmatrix}$ is</p> <p>(1) $9x^2 - 2y^2 + 2z^2 - 6xy - 6xz - 8yz$</p> <p>(2) $9x^2 + 2y^2 + 2z^2 + 6xy - 6xz - 8yz$</p> <p>(3) $9x^2 + 2y^2 - 2z^2 + 6xy + 6xz + 8yz$</p> <p>(4) None of these</p>
16.	<p>The radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!} z^n$ is</p> <p>(1) 4 (2) 3</p> <p>(3) 2 (4) None of these</p>

Question No.	Questions
25.	<p>Let $f : [0, \infty) \rightarrow \mathbb{R}$ be a function, where \mathbb{R} denotes the set of all real numbers. Then which one of the following statements is true ?</p> <p>(1) If f is continuous and $\lim_{x \rightarrow \infty} f(x)$ is finite, then f is uniformly continuous.</p> <p>(2) If f is bounded and continuous, then f is uniformly continuous.</p> <p>(3) If f is uniformly continuous, then $\lim_{x \rightarrow \infty} f(x)$ exists.</p> <p>(4) None of these</p>
26.	<p>Which of the following is false ?</p> <p>(1) The sequence $\{f_n\}$, where $f_n(x) = \frac{1}{x+n}$, is uniformly convergent in any interval $[0, b]$, $b > 0$.</p> <p>(2) The sequence $\{f_n\}$, where $f_n(x) = \frac{n^2 x}{1+n^3 x^2}$, is uniformly convergent on the interval $[0, 1]$.</p> <p>(3) The sequence $\{f_n\}$, where $f_n(x) = \tan^{-1} nx$, $x \geq 0$, is uniformly convergent in any interval $[a, b]$, $a > 0$.</p> <p>(4) None of these</p>
27.	<p>For which of the following function, Rolle's theorem is not applicable ?</p> <p>(1) $f(x) = \cos 2x$ in $[-\pi/4, \pi/4]$ (2) $f(x) = \frac{\sin x}{e^x}$ in $[0, \pi]$</p> <p>(3) $f(x) = x^3 - 6x^2 + 11x - 6$ in $[1, 3]$ (4) $f(x) = x$ in $[-1, 1]$</p>

Question No.	Questions
28.	<p>If $f(x) = x$, $x \in [0, 1]$ and let $P = \left\{0, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, 1\right\}$ be the partition of $[0, 1]$, then $U(f, P)$ is</p> <p>(1) $23/36$ (2) $31/36$ (3) $49/36$ (4) None of these</p>
29.	<p>If a function f defined on $[0, 1]$ as $f(x) = \begin{cases} \sin\left(\frac{1}{x}\right), & \text{if } x \text{ is irrational} \\ 0, & \text{otherwise} \end{cases}$, then</p> <p>(1) f is not bounded (2) f is R-integrable (3) f is not R-integrable since f is not bounded (4) f is not R-integrable since lower and upper integrals of f are unequal</p>
30.	<p>Consider the following improper integrals</p> $I_1 = \int_0^{\infty} \frac{\sin^2 x}{x^2} dx \text{ and } I_2 = \int_1^{\infty} \frac{x^3}{(1+x)^5} dx, \text{ then}$ <p>(1) Both are divergent (2) I_1 converges but not I_2 (3) I_2 converges but not I_1 (4) Both are convergent</p>
31.	<p>Let p be the probability that a coin will fall head in a single toss in order to test $H_0 : p = \frac{1}{2}$ against $H_1 : p = \frac{3}{4}$. The coin is tossed 5 times and H_0 is rejected if more than 3 heads are obtained. Then the probability of type I error is</p> <p>(1) $1/16$ (2) $3/16$ (3) $5/16$ (4) None of these</p>

Question No.	Questions
47.	<p>The extremal of the functional</p> $I[y(x)] = \int_1^2 (y'^2 - 2xy) dx \quad \text{subject to } y(1) = 0, y(2) = -1 \text{ is}$ <p>(1) $y = \frac{1}{6}(x - x^3)$ (2) $y = x^2 - 1$</p> <p>(3) $y = \frac{1}{6}(7x - x^3)$ (4) None of these</p>
48.	<p>The extremal of $\int_1^2 \frac{\dot{x}^2}{t^3} dt$; $x(1) = 3, x(2) = 18$ (where $\dot{x} \equiv \frac{dx}{dt}$) using Lagrange's equation is given by which of the following ?</p> <p>(1) $x = \frac{15}{7}t^3 + \frac{6}{7}$ (2) $x = 5t^2 - 2$</p> <p>(3) $x = 5t^3 + 3$ (4) $x = t^4 + 2$</p>
49.	<p>The solution of the linear integral equation</p> $\phi(x) = (1+x)^2 + \int_{-1}^1 (x\xi + x^2\xi^2)\phi(\xi) d\xi, \text{ is}$ <p>(1) $\phi(x) = 1 + 6x + \frac{25}{9}x^2$ (2) $\phi(x) = 1 - 6x + \frac{5}{9}x^2$</p> <p>(3) $\phi(x) = 1 + 3x - \frac{25}{9}x^2$ (4) None of these</p>

Question No.	Questions
61.	<p>The resolvent kernel for the integral equation</p> $\phi(x) = 29 + 6x + \int_0^x [5 - 6(x - \xi)] \phi(\xi) d\xi, \text{ is}$ <p>(1) $9 e^{3(x-\xi)} - 4 e^{2(x-\xi)}$ (2) $9 e^{2(x-\xi)} - 4 e^{3(x-\xi)}$ (3) $9 e^{3(x-\xi)} - e^{-2(x-\xi)}$ (4) None of these</p>
62.	<p>Consider the two dimensional motion of a mass m attached to one end of a spring whose other end is fixed. Let k be the spring constant. The kinetic energy T and the potential energy V of the system are given by</p> $T = \frac{1}{2} m (\dot{r}^2 + (r\dot{\theta})^2) \text{ and } V = \frac{1}{2} k r^2,$ <p>where $\dot{r} = \frac{dr}{dt}$ and $\dot{\theta} = \frac{d\theta}{dt}$ with t as time.</p> <p>Then which of the following statement is correct ?</p> <p>(1) r is an ignorable coordinate (2) θ is not an ignorable coordinate (3) $r^2 \dot{\theta}$ remains constant throughout the motion (4) $r \dot{\theta}$ remains constant throughout the motion</p>
63.	<p>The Lagrange's equation for a simple pendulum is, (where symbols have their usual meanings)</p> <p>(1) $\ddot{\theta} + \frac{g}{l} \sin \theta = 0$ (2) $\ddot{\theta} + \frac{g}{l} \sin \theta = 0$ (3) $\ddot{\theta} + \frac{g}{l} \tan \theta = 0$ (4) None of these</p>

Question No.	Questions
64.	<p>Let q_i and \dot{q}_i respectively are the generalized coordinates and velocity of a mechanical system and p_i are its generalized momenta. If H is the Hamiltonian of the system, then Hamilton's equations of motion are</p> <p>(1) $\dot{q}_i = \frac{\partial H}{\partial p_i}, \dot{p}_i = \frac{\partial H}{\partial q_i}$ (2) $\dot{q}_i = -\frac{\partial H}{\partial p_i}, \dot{p}_i = \frac{\partial H}{\partial q_i}$</p> <p>(3) $\dot{q}_i = \frac{\partial H}{\partial p_i}, \dot{p}_i = -\frac{\partial H}{\partial q_i}$ (4) $\dot{q}_i = -\frac{\partial H}{\partial p_i}, \dot{p}_i = -\frac{\partial H}{\partial q_i}$</p>
65.	<p>If $L = [L_1, L_2, L_3]$ is the vector moment of the external forces about a fixed point O, $\omega = [\omega_1, \omega_2, \omega_3]$ the angular velocity, and A, B, C are the principal moments of inertia, then Euler's dynamical equations of motion are</p> <p>(1) $L_1 = A\omega_1 - (B - C)\omega_2\omega_3, L_2 = B\omega_2 - (C - A)\omega_3\omega_1, L_3 = C\omega_3 - (A - B)\omega_1\omega_2$</p> <p>(2) $L_1 = A\dot{\omega}_1 - (B - C)\omega_2\omega_3, L_2 = B\dot{\omega}_2 - (C - A)\omega_3\omega_1, L_3 = C\dot{\omega}_3 - (A - B)\omega_1\omega_2$</p> <p>(3) $L_1 = A\dot{\omega}_1 + (B - C)\omega_2\omega_3, L_2 = B\dot{\omega}_2 + (C - A)\omega_3\omega_1, L_3 = C\dot{\omega}_3 + (A - B)\omega_1\omega_2$</p> <p>(4) None of these</p>
66.	<p>An integer is chosen at random from two hundred digits. Then the probability that the integer is divisible by 6 or 8 is</p> <p>(1) $3/4$ (2) $1/2$</p> <p>(3) $3/8$ (4) $1/4$</p>

Question No.	Questions
78.	<p>Let X and Y be independent and identically distributed (i.i.d.) random variables uniformly distributed on (0, 4). Then $P(X > Y \mid X < 2Y)$ is</p> <p>(1) $2/3$ (2) $5/6$ (3) $1/4$ (4) $1/3$</p>
79.	<p>If X and Y are independent normal variates with zero expectations and variances σ_1^2 and σ_2^2, then $Z = \frac{XY}{\sqrt{X^2 + Y^2}}$ is normal with variance</p> <p>(1) $\sigma_z^2 = \frac{1}{\left(\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}\right)}$ (2) $\sigma_z^2 = \sigma_1^2 + \sigma_2^2$ (3) $\sigma_z^2 = \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}$ (4) None of these</p>
80.	<p>In testing $H: \mu = 100$ against $A: \mu \neq 100$ at the 10% level of significance, H is rejected if</p> <p>(1) 100 is contained in the 90% confidence interval (2) The value of the test statistic is in the acceptance region (3) The p-value is less than 0.10 (4) The p-value is greater than 0.10</p>

Question No.	Questions
81.	<p>Which of the following functions is not a function of bounded variation ?</p> <p>(1) $f(x) = \begin{cases} x \sin(\pi/x), & \text{if } 0 < x \leq 1 \\ 0 & , \text{if } x = 0 \end{cases}$ (2) $f(x) = \begin{cases} x^2 \sin(1/x), & \text{if } x \neq 0 \\ 0 & , \text{if } x = 0 \end{cases}$</p> <p>(3) $f(x) = 3x^2 - 2x^3, -2 \leq x \leq 2$ (4) None of these</p>
82.	<p>Choose the incorrect statement.</p> <p>(1) The set of all irrational numbers in $[0, 1]$ is measurable.</p> <p>(2) Every non-empty one set has positive measure.</p> <p>(3) Every subset of a set of measure zero is not of measure zero.</p> <p>(4) None of these</p>
83.	<p>The directional derivative of the function $\phi(x, y) = \frac{xy}{x^2 + y^2}$ at the point $(0, 1)$ along a line making an angle of 30° with positive direction of x-axis is</p> <p>(1) $\frac{2}{\sqrt{3}}$ (2) $\frac{\sqrt{3}}{2}$</p> <p>(3) $\sqrt{3}$ (4) None of these</p>
84.	<p>The metric space (\mathbb{R}, d), where d is a usual metric, is</p> <p>(1) compact (2) disconnected</p> <p>(3) connected but not compact (4) compact and connected</p>

Question No.	Questions
85.	<p>In a metric space $(0, 1]$ with usual metric $d(x, y) = x - y$, the sequence $\langle \frac{1}{n} \rangle$ is a</p> <p>(1) Not a Cauchy sequence</p> <p>(2) Cauchy sequence but does not converge in $(0, 1]$</p> <p>(3) Cauchy sequence that is convergent in $(0, 1]$</p> <p>(4) None of these</p>
86.	<p>Let V be a vector space over \mathbb{R}^3. Which one of the following is not a subspace of V?</p> <p>(1) $\{(x, y, z) : 3x + y - z = 0, x, y, z \in \mathbb{R}\}$</p> <p>(2) $\{(x, y, z) : x + y - z = 0, 2x + 3y - z = 0, x, y, z \in \mathbb{R}\}$</p> <p>(3) $\{(x, y, z) : x - 3y + 4z = 0, x, y, z \in \mathbb{R}\}$</p> <p>(4) $\{(x, y, z) : x + y \geq 0, x, y, z \in \mathbb{R}\}$</p>
87.	<p>The value of k for which the vector $u = (1, k, 5)$ in $V_3(\mathbb{R})$ can be expressed as a linear combination of vectors $v = (1, -3, 2)$ and $w = (2, -1, 1)$ is</p> <p>(1) 3</p> <p>(2) -8</p> <p>(3) -2</p> <p>(4) None of these</p>

Question No.	Questions
92.	<p>The solution of the differential equation $\frac{dy}{dx} = 2xy$, $y(0) = 1$ by Picard's method upto third approximation is</p> <p>(1) $1+x^2 + \frac{x^4}{2} + \frac{x^6}{6}$ (2) $1+x^2 + \frac{3x^4}{2} + \frac{x^6}{6}$</p> <p>(3) $1+x^2 + \frac{x^4}{4} + \frac{x^6}{6}$ (4) None of these</p>
93.	<p>Using method of variation of parameters, the solution of the differential equation $y'' - 2y' = e^x \sin x$, is</p> <p>(1) $y = c_1 + c_2 e^{2x} - \frac{1}{2} e^x \sin x$ (2) $y = c_1 e^x + c_2 e^{-x} - \frac{1}{2} e^x \sin x$</p> <p>(3) $y = c_1 + c_2 e^x + \frac{1}{2} e^x \sin x$ (4) None of these</p>
94.	<p>Let n be non-negative integer. The eigen values of the Sturm-Liouville problem $y'' + \lambda y = 0$ with boundary conditions $y(0) = y(2\pi)$, $y'(0) = y'(2\pi)$ are</p> <p>(1) n (2) $n^2 \pi^2$</p> <p>(3) $n \pi$ (4) n^2</p>

Question No.	Questions
95.	<p>Green's function of the boundary value problem</p> $\frac{d^2u}{dx^2} + u = 0, u(0) = 0, u\left(\frac{\pi}{2}\right) = 0$ <p>is given by</p> <p>(1) $G(x, \xi) = \cos \xi \sin x, 0 \leq x < \xi$</p> <p>(2) $G(x, \xi) = \cosh \xi \sinh x, 0 \leq x < \xi$</p> <p>(3) $G(x, \xi) = x(1 - \xi), 0 \leq x < \xi$</p> <p>(4) None of these</p>
96.	<p>The solution of Partial differential equation $xz p + yz q = x y$ is</p> <p>(1) $\phi(xy, yz - y^2) = 0$ (2) $\phi(x/y, z + y^2) = 0$</p> <p>(3) $\phi(x/y, xy - z^2) = 0$ (4) None of these</p>
97.	<p>The integral surface for the Cauchy problem $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 1$ which passes through the circle $z = 0, x^2 + y^2 = 1$ is</p> <p>(1) $x^2 + y^2 + 2z^2 + 2zx - 2yz - 1 = 0$</p> <p>(2) $x^2 + y^2 + 2z^2 + 2zx + 2yz - 1 = 0$</p> <p>(3) $x^2 + y^2 + 2z^2 - 2zx - 2yz - 1 = 0$</p> <p>(4) None of these</p>
98.	<p>The partial differential equation $(x+1)\frac{\partial^2 u}{\partial x^2} - 2(x+2)\frac{\partial^2 u}{\partial x \partial y} + (x+3)\frac{\partial^2 u}{\partial y^2} = 0$ is</p> <p>(1) Elliptic (2) Hyperbolic</p> <p>(3) Parabolic (4) None of these</p>

Question No.	Questions
99.	<p>Using method of separation of variables, the solution of Partial differential equation</p> $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ <p>subject to the boundary conditions :</p> <p>$u(0, y) = \sin y$ for all y</p> <p>and $u(\infty, y) = 0$ for all y</p> <p>is given by</p> <p>(1) $u(x, y) = e^{-x} \sin y$ (2) $u(x, y) = e^x \sin y$</p> <p>(3) $u(x, y) = e^{-2x} \sin y$ (4) None of these</p>
100.	<p>Let $u(x, t) = e^{iw x} v(t)$ with $v(0) = 1$ be a solution to $\frac{\partial u}{\partial t} = \frac{\partial^3 u}{\partial x^3}$. Then</p> <p>(1) $u(x, t) = e^{iw(x-w^2t)}$ (2) $u(x, t) = e^{iw x - w^2t}$</p> <p>(3) $u(x, t) = e^{iw(x+w^2t)}$ (4) $u(x, t) = e^{iw^3(x-t)}$</p>

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Sr. No. 10052

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Question No.	Questions
1.	<p>Which of the following functions is not a function of bounded variation ?</p> <p>(1) $f(x) = \begin{cases} x \sin(\pi/x), & \text{if } 0 < x \leq 1 \\ 0, & \text{if } x = 0 \end{cases}$ (2) $f(x) = \begin{cases} x^2 \sin(1/x), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$</p> <p>(3) $f(x) = 3x^2 - 2x^3, -2 \leq x \leq 2$ (4) None of these</p>
2.	<p>Choose the incorrect statement.</p> <p>(1) The set of all irrational numbers in $[0, 1]$ is measurable.</p> <p>(2) Every non-empty one set has positive measure.</p> <p>(3) Every subset of a set of measure zero is not of measure zero.</p> <p>(4) None of these</p>
3.	<p>The directional derivative of the function $\phi(x, y) = \frac{xy}{x^2 + y^2}$ at the point $(0, 1)$ along a line making an angle of 30° with positive direction of x-axis is</p> <p>(1) $\frac{2}{\sqrt{3}}$ (2) $\frac{\sqrt{3}}{2}$</p> <p>(3) $\sqrt{3}$ (4) None of these</p>
4.	<p>The metric space (\mathbb{R}, d), where d is a usual metric, is</p> <p>(1) compact (2) disconnected</p> <p>(3) connected but not compact (4) compact and connected</p>

Question No.	Questions
5.	<p>In a metric space $(0, 1]$ with usual metric $d(x, y) = x - y$, the sequence $\langle \frac{1}{n} \rangle$ is a</p> <p>(1) Not a Cauchy sequence (2) Cauchy sequence but does not converge in $(0, 1]$ (3) Cauchy sequence that is convergent in $(0, 1]$ (4) None of these</p>
6.	<p>Let V be a vector space over \mathbb{R}^3. Which one of the following is not a subspace of V?</p> <p>(1) $\{(x, y, z) : 3x + y - z = 0, x, y, z \in \mathbb{R}\}$ (2) $\{(x, y, z) : x + y - z = 0, 2x + 3y - z = 0, x, y, z \in \mathbb{R}\}$ (3) $\{(x, y, z) : x - 3y + 4z = 0, x, y, z \in \mathbb{R}\}$ (4) $\{(x, y, z) : x + y \geq 0, x, y, z \in \mathbb{R}\}$</p>
7.	<p>The value of k for which the vector $u = (1, k, 5)$ in $V_3(\mathbb{R})$ can be expressed as a linear combination of vectors $v = (1, -3, 2)$ and $w = (2, -1, 1)$ is</p> <p>(1) 3 (2) -8 (3) -2 (4) None of these</p>
8.	<p>The dimension of the subspace W of \mathbb{R}^4 generated by $\{(3, 8, -3, -5), (1, -2, 5, -3), (2, 3, 1, -4)\}$ is</p> <p>(1) 1 (2) 3 (3) 2 (4) None of these</p>

Question No.	Questions
18.	<p>The maximum value of $Z = 2x + 3y$ subject to the constraints : $x + y \leq 30$; $3 \leq y \leq 12$; $x - y \geq 0$; $0 \leq x \leq 20$, is</p> <p>(1) 72 (2) 60 (3) 49 (4) None of these</p>
19.	<p>Men arrive in a queue according to a Poisson process with rate λ_1 and women arrive in the same queue according to another Poisson process with rate λ_2. The arrivals of men and women are independent. The probability that the first arrival in the queue is a man is</p> <p>(1) $\frac{\lambda_1}{\lambda_1 + \lambda_2}$ (2) $\frac{\lambda_2}{\lambda_1 + \lambda_2}$ (3) $\frac{\lambda_1}{\lambda_2}$ (4) $\frac{\lambda_2}{\lambda_1}$</p>
20.	<p>Arrivals at a telephone booth are considered to be Poisson with an average time of 12 minutes between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 4 minutes. Then the probability that a person arriving at the booth will have to wait in the queue is</p> <p>(1) $\frac{2}{3}$ (2) $\frac{1}{3}$ (3) $\frac{1}{6}$ (4) None of these</p>

Question No.	Questions
21.	<p>The resolvent kernel for the integral equation</p> $\phi(x) = 29 + 6x + \int_0^x [5 - 6(x - \xi)] \phi(\xi) d\xi, \text{ is}$ <p>(1) $9 e^{3(x-\xi)} - 4 e^{2(x-\xi)}$ (2) $9 e^{2(x-\xi)} - 4 e^{3(x-\xi)}$ (3) $9 e^{3(x-\xi)} - e^{-2(x-\xi)}$ (4) None of these</p>
22.	<p>Consider the two dimensional motion of a mass m attached to one end of a spring whose other end is fixed. Let k be the spring constant. The kinetic energy T and the potential energy V of the system are given by</p> $T = \frac{1}{2} m (\dot{r}^2 + (r\dot{\theta})^2) \text{ and } V = \frac{1}{2} k r^2,$ <p>where $\dot{r} = \frac{dr}{dt}$ and $\dot{\theta} = \frac{d\theta}{dt}$ with t as time.</p> <p>Then which of the following statement is correct ?</p> <p>(1) r is an ignorable coordinate (2) θ is not an ignorable coordinate (3) $r^2 \dot{\theta}$ remains constant throughout the motion (4) $r \dot{\theta}$ remains constant throughout the motion</p>
23.	<p>The Lagrange's equation for a simple pendulum is, (where symbols have their usual meanings)</p> <p>(1) $\dot{\theta} + \frac{g}{\ell} \sin \theta = 0$ (2) $\ddot{\theta} + \frac{g}{\ell} \sin \theta = 0$ (3) $\ddot{\theta} + \frac{g}{\ell} \tan \theta = 0$ (4) None of these</p>

Question No.	Questions
24.	<p>Let q_i and \dot{q}_i respectively are the generalized coordinates and velocity of a mechanical system and p_i are its generalized momenta. If H is the Hamiltonian of the system, then Hamilton's equations of motion are</p> <p>(1) $\dot{q}_i = \frac{\partial H}{\partial p_i}, \dot{p}_i = \frac{\partial H}{\partial q_i}$ (2) $\dot{q}_i = -\frac{\partial H}{\partial p_i}, \dot{p}_i = \frac{\partial H}{\partial q_i}$</p> <p>(3) $\dot{q}_i = \frac{\partial H}{\partial p_i}, \dot{p}_i = -\frac{\partial H}{\partial q_i}$ (4) $\dot{q}_i = -\frac{\partial H}{\partial p_i}, \dot{p}_i = -\frac{\partial H}{\partial q_i}$</p>
25.	<p>If $L = [L_1, L_2, L_3]$ is the vector moment of the external forces about a fixed point O, $\omega = [\omega_1, \omega_2, \omega_3]$ the angular velocity, and A, B, C are the principal moments of inertia, then Euler's dynamical equations of motion are</p> <p>(1) $L_1 = A\omega_1 - (B - C)\omega_2\omega_3, L_2 = B\omega_2 - (C - A)\omega_3\omega_1, L_3 = C\omega_3 - (A - B)\omega_1\omega_2$</p> <p>(2) $L_1 = A\dot{\omega}_1 - (B - C)\omega_2\omega_3, L_2 = B\dot{\omega}_2 - (C - A)\omega_3\omega_1, L_3 = C\dot{\omega}_3 - (A - B)\omega_1\omega_2$</p> <p>(3) $L_1 = A\dot{\omega}_1 + (B - C)\omega_2\omega_3, L_2 = B\dot{\omega}_2 + (C - A)\omega_3\omega_1, L_3 = C\dot{\omega}_3 + (A - B)\omega_1\omega_2$</p> <p>(4) None of these</p>
26.	<p>An integer is chosen at random from two hundred digits. Then the probability that the integer is divisible by 6 or 8 is</p> <p>(1) $3/4$ (2) $1/2$</p> <p>(3) $3/8$ (4) $1/4$</p>
27.	<p>In a bolts factory, machines I, II and III manufacture respectively 25%, 35% and 40% of the total bolts. Of their total output 5%, 4% and 2% are respectively defective bolts. A bolt is drawn at random from the product. If the bolt drawn is found to be defective, what is the probability that it is manufactured by machine II?</p> <p>(1) 0.41 (2) 0.27</p> <p>(3) 0.13 (4) None of these</p>

Question No.	Questions
28.	<p>Let X be a continuous random variable with probability density function (p.d.f.) defined as $f(x) = 6x(1-x)$, $0 \leq x \leq 1$.</p> <p>Then the value of number b such that $P(X < b) = P(X > b)$ is</p> <p>(1) $1/4$ (2) $3/4$ (3) $1/2$ (4) None of these</p>
29.	<p>If X and Y are two random variables having joint density function given by</p> $f(x, y) = \begin{cases} 6x^2y & ; 0 < x < 1, 0 < y < 1 \\ 0 & ; \text{otherwise} \end{cases}$ <p>Then $P(X + Y < 1)$ is</p> <p>(1) $1/4$ (2) $1/10$ (3) $3/8$ (4) None of these</p>
30.	<p>For $k = 1, 2, \dots, 10$, let the probability density function of the random variable X_k is given by</p> $f_{X_k}(x) = \begin{cases} e^{-x/k} & , x > 0 \\ k & , \text{otherwise} \end{cases}$ <p>Then the value of $E\left(\sum_{k=1}^{10} k X_k\right)$ is equal to</p> <p>(1) 385 (2) 256 (3) 144 (4) 110</p>

Question No.	Questions
31.	<p>For the initial value problem (IVP) $y' = f(x, y)$, $y(0) = 0$, which of the following statement is true ?</p> <p>(1) $f(x, y) = \sqrt{xy}$ satisfies Lipschitz's condition and so IVP has unique solution.</p> <p>(2) $f(x, y) = \sqrt{xy}$ does not satisfy Lipschitz's condition and so IVP has no solution.</p> <p>(3) $f(x, y) = y$ satisfies Lipschitz's condition and so IVP has unique solution.</p> <p>(4) $f(x, y) = y$ does not satisfy Lipschitz's condition still the IVP has unique solution.</p>
32.	<p>The solution of the differential equation $\frac{dy}{dx} = 2xy$, $y(0) = 1$ by Picard's method upto third approximation is</p> <p>(1) $1 + x^2 + \frac{x^4}{2} + \frac{x^6}{6}$ (2) $1 + x^2 + \frac{3x^4}{2} + \frac{x^6}{6}$</p> <p>(3) $1 + x^2 + \frac{x^4}{4} + \frac{x^6}{6}$ (4) None of these</p>
33.	<p>Using method of variation of parameters, the solution of the differential equation $y'' - 2y' = e^x \sin x$, is</p> <p>(1) $y = c_1 + c_2 e^{2x} - \frac{1}{2} e^x \sin x$ (2) $y = c_1 e^x + c_2 e^{-x} - \frac{1}{2} e^x \sin x$</p> <p>(3) $y = c_1 + c_2 e^x + \frac{1}{2} e^x \sin x$ (4) None of these</p>

Question No.	Questions
38.	<p>The partial differential equation $(x+1)\frac{\partial^2 u}{\partial x^2} - 2(x+2)\frac{\partial^2 u}{\partial x \partial y} + (x+3)\frac{\partial^2 u}{\partial y^2} = 0$ is</p> <p>(1) Elliptic (2) Hyperbolic (3) Parabolic (4) None of these</p>
39.	<p>Using method of separation of variables, the solution of Partial differential equation</p> $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ <p>subject to the boundary conditions :</p> <p>$u(0, y) = \sin y$ for all y and $u(\infty, y) = 0$ for all y</p> <p>is given by</p> <p>(1) $u(x, y) = e^{-x} \sin y$ (2) $u(x, y) = e^x \sin y$ (3) $u(x, y) = e^{-2x} \sin y$ (4) None of these</p>
40.	<p>Let $u(x, t) = e^{iwx} v(t)$ with $v(0) = 1$ be a solution to $\frac{\partial u}{\partial t} = \frac{\partial^3 u}{\partial x^3}$. Then</p> <p>(1) $u(x, t) = e^{i w(x - w^2 t)}$ (2) $u(x, t) = e^{i w x - w^2 t}$ (3) $u(x, t) = e^{i w(x + w^2 t)}$ (4) $u(x, t) = e^{i w^3(x - t)}$</p>

Question No.	Questions
41.	<p>The Laurent series expansion of $f(z) = \frac{z}{(z-1)(z-3)}$ for $0 < z-1 < 2$, is equal to</p> <p>(1) $\frac{-3}{4} \sum_{n=0}^{\infty} \frac{(z-1)^n}{2^n} - \frac{1}{2(z-1)}$ (2) $\frac{3}{4} \sum_{n=0}^{\infty} \frac{(z-1)^n}{2^n} - \frac{1}{4(z-1)}$</p> <p>(3) $\frac{1}{4} \sum_{n=0}^{\infty} \frac{(z-1)^{-n}}{2^n} + \frac{(z-1)}{2}$ (4) None of these</p>
42.	<p>The residue of the function $f(z) = \frac{1}{(z^2+1)^3}$ at $z = i$, is</p> <p>(1) $\frac{3}{16i}$ (2) $\frac{3}{2i}$</p> <p>(3) $\frac{4}{3i}$ (4) None of these</p>
43.	<p>The fixed points of the Mobius transformation $w = \frac{(2+i)z-2}{z+i}$ are</p> <p>(1) $i, -i$ (2) $0, 1$</p> <p>(3) $-1, 1$ (4) $1+i, 1-i$</p>

Question No.	Questions
44.	<p>The image of circle $z - 2 = 2$ under the Mobius transformation $w = \frac{z}{z+1}$ is a circle in w-plane with</p> <p>(1) Centre $\left(\frac{1}{5}, 0\right)$ and radius $\frac{2}{5}$ (2) Centre $\left(\frac{2}{5}, 0\right)$ and radius $\frac{1}{5}$</p> <p>(3) Centre $\left(\frac{1}{5}, 0\right)$ and radius $\frac{1}{5}$ (4) Centre $\left(\frac{2}{5}, 0\right)$ and radius $\frac{2}{5}$</p>
45.	<p>If 9 colours are used to paint 100 houses, then atleast _____ houses will be of the same colour.</p> <p>(1) 18 (2) 15</p> <p>(3) 12 (4) 10</p>
46.	<p>The congruence $35x \equiv 14 \pmod{21}$ has</p> <p>(1) 5 solutions (2) 6 solutions</p> <p>(3) 7 solutions (4) No solution</p>
47.	<p>The primitive roots of 3^2 are</p> <p>(1) 3, 7 (2) 2, 5</p> <p>(3) 5, 7 (4) None of these</p>

Question No.	Questions
52.	<p>Which of the following is a linear transformation ?</p> <p>(1) $T(x, y) = (1 + x, y)$ for all $(x, y) \in \mathbb{R}^2$</p> <p>(2) $T(x, y) = (x, x + y)$ for all $(x, y) \in \mathbb{R}^2$</p> <p>(3) $T(x, y) = (x^2, y)$ for all $(x, y) \in \mathbb{R}^2$</p> <p>(4) None of these</p>
53.	<p>The matrix representing the linear transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined by $T(x_1, x_2) = (2x_2, 3x_1 - x_2)$ relative to the basis $\{(1, 3), (2, 5)\}$ is</p> <p>(1) $\begin{bmatrix} 30 & 48 \\ 18 & 29 \end{bmatrix}$ (2) $\begin{bmatrix} -30 & 48 \\ 18 & -29 \end{bmatrix}$</p> <p>(3) $\begin{bmatrix} 30 & 48 \\ -18 & -29 \end{bmatrix}$ (4) $\begin{bmatrix} -30 & -48 \\ 18 & 29 \end{bmatrix}$</p>
54.	<p>Let C^3 be a complex inner product space.</p> <p>If the vectors $u_1 = (1, 2i, i)$, $u_2 = (0, 1 + i, 1)$, $u_3 = (2, 1 - i, i) \in C^3$, then the vector orthogonal to both u_1 and u_3 is</p> <p>(1) $(-3 + i, -i, 1 - 5i)$ (2) $(-3 + i, -i, 1 + 5i)$</p> <p>(3) $(3 + i, -i, 1 + 5i)$ (4) None of these</p>
55.	<p>The quadratic form corresponding to symmetric matrix $\begin{bmatrix} 9 & 3 & -3 \\ 3 & 2 & -4 \\ -3 & -4 & 2 \end{bmatrix}$ is</p> <p>(1) $9x^2 - 2y^2 + 2z^2 - 6xy - 6xz - 8yz$</p> <p>(2) $9x^2 + 2y^2 + 2z^2 + 6xy - 6xz - 8yz$</p> <p>(3) $9x^2 + 2y^2 - 2z^2 + 6xy + 6xz + 8yz$</p> <p>(4) None of these</p>

Question No.	Questions
64.	<p>Which one of the following polynomial is irreducible over the field \mathbb{Q} of rational numbers?</p> <p>(1) $2x^5 + 15x^3 + 3x + 6$ (2) $x^3 + 3x + 1$ (3) $8x^3 - 6x - 1$ (4) All of the above</p>
65.	<p>The degree of splitting field of $x^4 - x^2 - 2$ over the field \mathbb{Q} of rational numbers, is</p> <p>(1) 3 (2) 4 (3) 2 (4) None of these</p>
66.	<p>Let F be a finite field and let K/F be a field extension of degree 6. Then the Galois group of K/F is isomorphic to</p> <p>(1) the cyclic group of order 6 (2) the permutation group on $\{1, 2, 3\}$ (3) the permutation group on $\{1, 2, 3, 4, 5, 6\}$ (4) the permutation group on $\{1\}$</p>
67.	<p>Which of the following spaces is not separable?</p> <p>(1) \mathbb{R} with the trivial topology (2) The Cantor set as a subspace of \mathbb{R} (3) \mathbb{R} with the discrete topology (4) None of these</p>

Question No.	Questions																				
72.	<p>Using Gauss Elimination method, the solution of following equations</p> $4x + 3y + 2z = 8, \quad x + y + 2z = 7, \quad 3x + 2y + 4z = 13$ <p>is given by</p> <p>(1) $x = -1, y = 2, z = 3$ (2) $x = 1, y = 2, z = 3$</p> <p>(3) $x = -1, y = -2, z = 3$ (4) $x = 1, y = -2, z = 3$</p>																				
73.	<p>Given that</p> <table border="1" data-bbox="359 806 1141 929"> <tr> <td>x</td> <td>3</td> <td>7</td> <td>9</td> <td>10</td> </tr> <tr> <td>f(x)</td> <td>168</td> <td>120</td> <td>72</td> <td>63</td> </tr> </table> <p>The value of third divided difference of the function f(x) is</p> <p>(1) 5 (2) 1</p> <p>(3) -1 (4) -2</p>	x	3	7	9	10	f(x)	168	120	72	63										
x	3	7	9	10																	
f(x)	168	120	72	63																	
74.	<p>Consider the data given below :</p> <table border="1" data-bbox="351 1310 1492 1444"> <tr> <td>x.</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> <td>7</td> <td>8</td> </tr> <tr> <td>f(x)</td> <td>0</td> <td>4.13</td> <td>7.20</td> <td>9.25</td> <td>10.25</td> <td>10.00</td> <td>9.05</td> <td>7.12</td> <td>6.2</td> </tr> </table> <p>Using Simpson's one third rule, the value of $\int_0^8 f(x) dx$ is</p> <p>(1) 34.5 (2) 47.3</p> <p>(3) 52.8 (4) 60.4</p>	x.	0	1	2	3	4	5	6	7	8	f(x)	0	4.13	7.20	9.25	10.25	10.00	9.05	7.12	6.2
x.	0	1	2	3	4	5	6	7	8												
f(x)	0	4.13	7.20	9.25	10.25	10.00	9.05	7.12	6.2												

Question No.	Questions
78.	<p>The extremal of $\int_1^2 \frac{\dot{x}^2}{t^3} dt$; $x(1) = 3, x(2) = 18$ (where $\dot{x} \equiv \frac{dx}{dt}$) using Lagrange's equation is given by which of the following ?</p> <p>(1) $x = \frac{15}{7}t^3 + \frac{6}{7}$ (2) $x = 5t^2 - 2$ (3) $x = 5t^3 + 3$ (4) $x = t^4 + 2$</p>
79.	<p>The solution of the linear integral equation</p> $\phi(x) = (1+x)^2 + \int_{-1}^1 (x\xi + x^2\xi^2) \phi(\xi) d\xi, \text{ is}$ <p>(1) $\phi(x) = 1 + 6x + \frac{25}{9}x^2$ (2) $\phi(x) = 1 - 6x + \frac{5}{9}x^2$ (3) $\phi(x) = 1 + 3x - \frac{25}{9}x^2$ (4) None of these</p>
80.	<p>The solution to the integral equation $\phi(x) = x + \int_0^x \sin(x-\xi) \phi(\xi) d\xi$ is given by</p> <p>(1) $x^2 + \frac{x^3}{3}$ (2) $x - \frac{x^3}{3!}$ (3) $x^2 - \frac{x^3}{3!}$ (4) $x + \frac{x^3}{3!}$</p>

Question No.	Questions
84.	<p>Every bounded sequence has at least one limit point. This represents</p> <p>(1) Archimedean Property (2) Heine-Borel theorem</p> <p>(3) Bolzano-Weierstress theorem (4) Denseness Property</p>
85.	<p>Let $f : [0, \infty) \rightarrow \mathbb{R}$ be a function, where \mathbb{R} denotes the set of all real numbers. Then which one of the following statements is true ?</p> <p>(1) If f is continuous and $\lim_{x \rightarrow \infty} f(x)$ is finite, then f is uniformly continuous.</p> <p>(2) If f is bounded and continuous, then f is uniformly continuous.</p> <p>(3) If f is uniformly continuous, then $\lim_{x \rightarrow \infty} f(x)$ exists.</p> <p>(4) None of these</p>
86.	<p>Which of the following is false ?</p> <p>(1) The sequence $\{f_n\}$, where $f_n(x) = \frac{1}{x+n}$, is uniformly convergent in any interval $[0, b]$, $b > 0$.</p> <p>(2) The sequence $\{f_n\}$, where $f_n(x) = \frac{n^2 x}{1+n^3 x^2}$, is uniformly convergent on the interval $[0, 1]$.</p> <p>(3) The sequence $\{f_n\}$, where $f_n(x) = \tan^{-1} nx$, $x \geq 0$, is uniformly convergent in any interval $[a, b]$, $a > 0$.</p> <p>(4) None of these</p>

Question No.	Questions
87.	<p>For which of the following function, Rolle's theorem is not applicable?</p> <p>(1) $f(x) = \cos 2x$ in $[-\pi/4, \pi/4]$ (2) $f(x) = \frac{\sin x}{e^x}$ in $[0, \pi]$</p> <p>(3) $f(x) = x^3 - 6x^2 + 11x - 6$ in $[1, 3]$ (4) $f(x) = x$ in $[-1, 1]$</p>
88.	<p>If $f(x) = x$, $x \in [0, 1]$ and let $P = \left\{0, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, 1\right\}$ be the partition of $[0, 1]$, then $U(f, P)$ is</p> <p>(1) $23/36$ (2) $31/36$</p> <p>(3) $49/36$ (4) None of these</p>
89.	<p>If a function f defined on $[0, 1]$ as $f(x) = \begin{cases} \sin\left(\frac{1}{x}\right), & \text{if } x \text{ is irrational} \\ 0, & \text{otherwise} \end{cases}$, then</p> <p>(1) f is not bounded</p> <p>(2) f is R-integrable</p> <p>(3) f is not R-integrable since f is not bounded</p> <p>(4) f is not R-integrable since lower and upper integrals of f are unequal</p>

Question No.	Questions
90.	<p>Consider the following improper integrals</p> $I_1 = \int_0^{\infty} \frac{\sin^2 x}{x^2} dx \text{ and } I_2 = \int_1^{\infty} \frac{x^3}{(1+x)^5} dx, \text{ then}$ <p>(1) Both are divergent (2) I_1 converges but not I_2 (3) I_2 converges but not I_1 (4) Both are convergent</p>
91.	<p>If 2% of the items manufactured by a factory are defective, then the probability that there are 3 defective items in a sample of 100 items is</p> <p>(1) 0.48 (2) 0.33 (3) 0.27 (4) 0.18</p>
92.	<p>For a certain normal distribution, the first moment about 10 is 40 and the fourth moment about 50 is 48. Then the mean and standard deviation of the distribution respectively are</p> <p>(1) 20, 3 (2) 30, 4 (3) 50, 2 (4) None of these</p>
93.	<p>The characteristic function of χ^2-distribution with n degrees of freedom is</p> <p>(1) $(1 - 2it)^{n/2}$ (2) $(1 + 2it)^{n/2}$ (3) $(1 - 2it)^{-n/2}$ (4) None of these</p>
94.	<p>If Tchebycheff's inequality for a random variable X with mean 12 is $P\{6 < X < 18\} \geq \frac{3}{4}$, then the standard deviation of X is</p> <p>(1) 2 (2) 3 (3) 8 (4) None of these</p>

Question No.	Questions	
99.	<p>If X and Y are independent normal variates with zero expectations and variances σ_1^2 and σ_2^2, then $Z = \frac{XY}{\sqrt{X^2 + Y^2}}$ is normal with variance</p> <p>(1) $\sigma_z^2 = \frac{1}{\left(\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}\right)}$ (2) $\sigma_z^2 = \sigma_1^2 + \sigma_2^2$</p> <p>(3) $\sigma_z^2 = \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}$ (4) None of these</p>	
100.	<p>In testing $H_0 : \mu = 100$ against $A : \mu \neq 100$ at the 10% level of significance, H_0 is rejected if</p> <p>(1) 100 is contained in the 90% confidence interval</p> <p>(2) The value of the test statistic is in the acceptance region</p> <p>(3) The p-value is less than 0.10</p> <p>(4) The p-value is greater than 0.10</p>	

ANSWER KEYS OF MATHEMATICS FOR SESSION 2022-23				
Q. NO.	A	B	C	D
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2	4	3	1	3
3	3	2	3	2
4	3	3	4	3
5	1	2	2	2
6	2	4	1	4
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48	2	1	4	4
49	4	1	1	2
50	3	2	4	1

S. Kalus
13/12/2022

Michael Poomoy

ANSWER KEYS OF MATHEMATICS FOR SESSION 2022-23				
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91	2	2	3	4
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94	3	3	4	2
95	2	1	1	2
96	1	2	3	3
97	3	4	3	2
98	1	1	2	4
99	1	4	1	1
100	2	4	1	3

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13/12/2022

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